Math 4620/5620

Examples of Modeling Real-life Situations as Linear Programs

1. Portfolio selection.

National Insurance Associates carries an investment portfolio of a variety of stocks, bonds, and other investment alternatives. Currently $200,000 of funds have become available and must be considered for new investment opportunities. The four stock options National is considering and the relevant financial data are as follows:

	Investment Alternative			
	A	B	C	D
Price per share	$100	$50	$80	$40
Projected annual rate of return	0.12	0.08	0.06	0.10
Risk measure per dollar invested (*Higher values indicate greater risk*)	0.10	0.07	0.05	0.08

The risk measure provided by the firm's top financial advisor, indicates the relative uncertainty associated with the stock regarding the realization of its projected annual rate of return.

National's top management has stipulated the following investment guidelines:

1. The projected annual rate of return for the portfolio must be at least 9%.
2. No one stock can count for more than 50% of the total dollar investment.

(a) For this overall situation, develop a linear programming model to yield an investment portfolio which minimizes total risk.

(b) Revise the model in (a) to ignore risk and maximize projected return on investment. Why might the company prefer the model developed in (a)?

2.Work Scheduling.

A post office requires different numbers of full-time employees on different days of the week. The number of full-time employees required on each day is given in the table below.

	Number of full-time employees required
Day 1=Monday	17
Day 2=Tuesday	13
Day 3=Wednesday	15
Day 4=Thursday	19
Day 5=Friday	14
Day 6=Saturday	16
Day 7=Sunday	11

(a) Union rules state that each full-time employee must work five consecutive days and then receive two days off. For example, an employee who works Thursday to Monday must be off on Tuesday and Wednesday. The post office wants to meet its daily requirements using only full-time employees. Formulate an LP that the post office can use to minimize the number of full-time employees that must be hired.
(b) Suppose that the post office can force employees to work one day of overtime each week. For example, an employee whose regular shift is Monday to Friday can also be required to work on Saturday. Each employee is paid $50 a day for each of the first five days worked during a week and $62 for the overtime day (if any). Formulate an LP whose solution will enable the post office to minimize the cost of meeting its weekly work requirements.

3.Blending problem.

Sunco Oil manufactures three types of gasoline (gas 1, gas 2, and gas 3). Each type of gasoline is produced by blending together three types of crude oil (crude 1, crude 2, and crude 3). The sales price per barrel of gasoline and the purchase price per barrel of crude oil are given in the following table. Sunco can purchase up to 5000 barrels of each type of crude oil daily.

	Sales price per barrel		Purchase price per barrel
Gas 1	$70	Crude 1	$45
Gas 2	$60	Crude 2	$35
Gas 3	$50	Crude 3	$25

The three types of gasoline differ in their octane rating and sulfur content. The crude oil blended to form gas 1 must have an average octane rating of at least 10 and contain at most 1% sulfur. The crude oil blended to form gas 2 must have an average octane rating of at least 8 and contain at most 2% sulfur. The crude oil blended to form gas 3 must have an average octane rating of at least 6 and contain at most 1% sulfur. The octane rating and the sulfur content of the three types of oil are given in the following table. It costs $4 to transform one barrel of oil into one barrel of gasoline, and Sunco's refinery can produce up to 14,000 barrels of gasoline daily.

	Octane rating	Sulfur content
Crude 1	12	0.5%
Crude 2	6	2.0%
Crude 3	8	3.0%

Sunco's customers require the following amounts of each gasoline: gas 1 – 3000 barrels per day; gas 2 – 2000 barrels per day; gas 3 – 1000 barrels per day. The company considers it an obligation to meet these demands.
Formulate an LP that will enable Sunco to maximize daily profits (profits = revenues - costs).