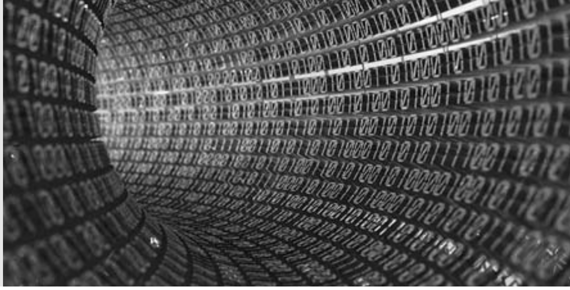


Answers and Hints to Selected Problems

CHAPTER 1

Section 1.1

1. $\mathbf{A}$ is 4×5, $\mathbf{B}$ is 3×3, $\mathbf{C}$ is 3×4,
$\mathbf{D}$ is 4×4, $\mathbf{E}$ is 2×3, $\mathbf{F}$ is 5×1,
$\mathbf{G}$ is 4×2, $\mathbf{H}$ is 2×2, $\mathbf{J}$ is 1×3.

2. $a_{13} = -2,\ a_{21} = 2,$
$b_{13} = 3,\ b_{21} = 0,$
$c_{13} = 3,\ c_{21} = 5,$
$d_{13} = t^2,\ d_{21} = t - 2,$
$e_{13} = \frac{1}{4},\ e_{21} = \frac{2}{3},$
$f_{13} =$ does not exist, $f_{21} = 5,$
$g_{13} =$ does not exist, $g_{21} = 2\pi,$
$h_{13} =$ does not exist, $h_{21} = 0,$
$j_{13} = -30,\ j_{21}$ does not exist.

3. $a_{23} = -6,\ a_{32} = 3,\ b_{31} = 4,$
$b_{32} = 3,\ c_{11} = 1,\ d = 22\,t^4,\ e_{13} = \frac{1}{4},$
$g_{22} = 18,\ g_{23}$ and h_{32} do not exist.

4. $\mathbf{A} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$

5. $\mathbf{A} = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 1 & \frac{2}{3} \\ 3 & \frac{3}{2} & 1 \end{bmatrix}.$

6. $\mathbf{B} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & -2 \\ -1 & -2 & -3 \end{bmatrix}.$

7. $\mathbf{C} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}.$

8. $\mathbf{D} = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 3 & 0 & -1 & -2 \\ 4 & 5 & 0 & -1 \end{bmatrix}.$

9. (a) $\begin{bmatrix} 9 & 15 \end{bmatrix}$, (b) $\begin{bmatrix} 12 & 0 \end{bmatrix}$, (c) $\begin{bmatrix} 13 & 30 \end{bmatrix}$, (d) $\begin{bmatrix} 21 & 15 \end{bmatrix}$.

10. (a) $\begin{bmatrix} 7 & 4 & 1776 \end{bmatrix}$, (b) $\begin{bmatrix} 12 & 7 & 1941 \end{bmatrix}$, (c) $\begin{bmatrix} 4 & 23 & 1809 \end{bmatrix}$, (d) $\begin{bmatrix} 10 & 31 & 1688 \end{bmatrix}$.

11. $\begin{bmatrix} 950 & 1253 & 98 \end{bmatrix}$. **12.** $\begin{bmatrix} 3 & 5 & 3 & 4 \\ 0 & 2 & 9 & 5 \\ 4 & 2 & 0 & 0 \end{bmatrix}$. **13.** $\begin{bmatrix} 72 & 12 & 16 \\ 45 & 32 & 16 \\ 81 & 10 & 35 \end{bmatrix}$.

14. $\begin{bmatrix} 100 & 150 & 50 & 500 \\ 27 & 45 & 116 & 2 \\ 29 & 41 & 116 & 3 \end{bmatrix}$.

15. (a) $\begin{bmatrix} 1000 & 2000 & 3000 \\ 0.07 & 0.075 & 0.0725 \end{bmatrix}$. (b) $\begin{bmatrix} 1070.00 & 2150.00 & 3217.50 \\ 0.075 & 0.08 & 0.0775 \end{bmatrix}$.

16. $\begin{bmatrix} 0.95 & 0.05 \\ 0.01 & 0.99 \end{bmatrix}$. **17.** $\begin{bmatrix} 0.6 & 0.4 \\ 0.7 & 0.3 \end{bmatrix}$. **18.** $\begin{bmatrix} 0.10 & 0.50 & 0.40 \\ 0.20 & 0.60 & 0.20 \\ 0.25 & 0.65 & 0.10 \end{bmatrix}$.

19. $\begin{bmatrix} 0.80 & 0.15 & 0.05 \\ 0.10 & 0.88 & 0.02 \\ 0.25 & 0.30 & 0.45 \end{bmatrix}$.

Section 1.2

1. $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$. **2.** $\begin{bmatrix} -5 & -10 \\ -15 & -20 \end{bmatrix}$. **3.** $\begin{bmatrix} 9 & 3 \\ -3 & 6 \\ 9 & -6 \\ 6 & 18 \end{bmatrix}$. **4.** $\begin{bmatrix} -20 & 20 \\ 0 & -20 \\ 50 & -30 \\ 50 & 10 \end{bmatrix}$.

5. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \\ 0 & 0 \\ -2 & -2 \end{bmatrix}$. **6.** $\begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$. **7.** $\begin{bmatrix} 0 & 2 \\ 6 & 1 \end{bmatrix}$. **8.** $\begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 8 & -5 \\ 7 & 7 \end{bmatrix}$.

9. $\begin{bmatrix} 3 & 2 \\ -2 & 2 \\ 3 & -2 \\ 4 & 8 \end{bmatrix}$. **10.** Does not exist. **11.** $\begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix}$. **12.** $\begin{bmatrix} -2 & -2 \\ 0 & -7 \end{bmatrix}$.

13. $\begin{bmatrix} 5 & -1 \\ -1 & 4 \\ -2 & 1 \\ -3 & 5 \end{bmatrix}$. **14.** $\begin{bmatrix} 3 & 0 \\ 0 & 2 \\ 3 & -2 \\ 0 & 4 \end{bmatrix}$. **15.** $\begin{bmatrix} 17 & 22 \\ 27 & 32 \end{bmatrix}$. **16.** $\begin{bmatrix} 5 & 6 \\ 3 & 18 \end{bmatrix}$.

17. $\begin{bmatrix} -0.1 & 0.2 \\ 0.9 & -0.2 \end{bmatrix}$.

18. $\begin{bmatrix} 4 & -3 \\ -1 & 4 \\ -10 & 6 \\ -8 & 0 \end{bmatrix}$.

19. $\mathbf{X} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$.

20. $\mathbf{Y} = \begin{bmatrix} -11 & -12 \\ -11 & -19 \end{bmatrix}$.

21. $\mathbf{X} = \begin{bmatrix} 11 & 1 \\ -3 & 8 \\ 4 & -3 \\ 1 & 17 \end{bmatrix}$.

22. $\mathbf{Y} = \begin{bmatrix} -1.0 & 0.5 \\ 0.5 & -1.0 \\ 2.5 & -1.5 \\ 1.5 & -0.5 \end{bmatrix}$.

23. $\mathbf{R} = \begin{bmatrix} -2.8 & -1.6 \\ 3.6 & -9.2 \end{bmatrix}$.

24. $\mathbf{S} = \begin{bmatrix} -1.5 & 1.0 \\ -1.0 & -1.0 \\ -1.5 & 1.0 \\ 2.0 & 0 \end{bmatrix}$.

25. $\begin{bmatrix} 5 & 8 \\ 13 & 9 \end{bmatrix}$.

27. $\begin{bmatrix} -\theta^3 + 6\theta^2 + \theta & 6\theta - 6 \\ 21 & -\theta^4 - 2\theta^2 - \theta + 6/\theta \end{bmatrix}$.

32. (a) $\begin{bmatrix} 200 & 150 \end{bmatrix}$, (b) $\begin{bmatrix} 600 & 450 \end{bmatrix}$, (c) $\begin{bmatrix} 550 & 550 \end{bmatrix}$.

33. (b) $\begin{bmatrix} 11 & 2 & 6 & 3 \end{bmatrix}$, (c) $\begin{bmatrix} 9 & 4 & 10 & 8 \end{bmatrix}$.

34. (b) $\begin{bmatrix} 10{,}500 & 6{,}000 & 4{,}500 \end{bmatrix}$, (c) $\begin{bmatrix} 35{,}500 & 14{,}500 & 3{,}300 \end{bmatrix}$.

Section 1.3

1. (a) 2×2, (b) 4×4, (c) 2×1, (d) Not defined, (e) 4×2, (f) 2×4, (g) 4×2, (h) Not defined, (i) Not defined, (j) 1×4, (k) 4×4, (l) 4×2.

2. $\begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$.

3. $\begin{bmatrix} 23 & 34 \\ 31 & 46 \end{bmatrix}$.

4. $\begin{bmatrix} 5 & -4 & 3 \\ 9 & -8 & 7 \end{bmatrix}$.

5. $\mathbf{A} = \begin{bmatrix} 13 & -12 & 11 \\ 17 & -16 & 15 \end{bmatrix}$.

6. Not defined.

7. $\begin{bmatrix} -5 & -6 \end{bmatrix}$.

8. $\begin{bmatrix} -9 & -10 \end{bmatrix}$.

9. $\begin{bmatrix} -7 & 4 & -1 \end{bmatrix}$.

10. Not defined.

11. $\begin{bmatrix} 1 & -3 \\ 7 & -3 \end{bmatrix}$.

12. $\begin{bmatrix} 2 & -2 & 2 \\ 7 & -4 & 1 \\ -8 & 4 & 0 \end{bmatrix}$.

13. $\begin{bmatrix} 1 & 3 \end{bmatrix}$.

14. Not defined.

15. Not defined.

16. Not defined.

17. $\begin{bmatrix} -1 & -2 & -1 \\ 1 & 0 & -3 \\ 1 & 3 & 5 \end{bmatrix}$.

18. $\begin{bmatrix} 2 & -2 & 1 \\ -2 & 0 & 0 \\ 1 & -2 & 2 \end{bmatrix}$.

19. $\begin{bmatrix} -1 & 1 & 5 \end{bmatrix}$.

22. $\begin{bmatrix} x+2y \\ 3x+4y \end{bmatrix}$.

23. $\begin{bmatrix} x-z \\ 3x+y+z \\ x+3y \end{bmatrix}$.

24. $\begin{bmatrix} a_{11}x+a_{12}y \\ a_{21}x+a_{22}y \end{bmatrix}$.

25. $\begin{bmatrix} 2b_{11}-b_{12}+3b_{13} \\ 2b_{21}-b_{22}+3b_{23} \end{bmatrix}$.

26. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

27. $\begin{bmatrix} 0 & 40 \\ -16 & 8 \end{bmatrix}$.

28. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

29. $\begin{bmatrix} 7 & 5 \\ 11 & 10 \end{bmatrix}$.

32. $\begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 10 \\ 11 \end{bmatrix}$.

33. $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$.

34. $\begin{bmatrix} 5 & 3 & 2 & 4 \\ 1 & 1 & 0 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}=\begin{bmatrix} 5 \\ 0 \\ -3 \\ 4 \end{bmatrix}$.

35. (a) $\mathbf{PN}=[38{,}000]$, which represents the total revenue for that flight.

(b) $\mathbf{NP}=\begin{bmatrix} 26{,}000 & 45{,}5000 & 65{,}000 \\ 4{,}000 & 7{,}000 & 10{,}000 \\ 2{,}000 & 3{,}500 & 5{,}000 \end{bmatrix}$,

which has no physical significance.

36. (a) $\mathbf{HP}=[9{,}625 \quad 9{,}762.50 \quad 9{,}887.50 \quad 10{,}100 \quad 9{,}887.50]$, which represents the portfolio value each day.

(b) **PH** does not exist.

37. $\mathbf{TW}=[14.00 \ \ 65.625 \ \ 66.50]^{\mathsf{T}}$, which denotes the cost of producing each product.

38. $\mathbf{OTW}=[33{,}862.50]$, which denotes the cost of producing all items on order.

39. $\mathbf{FC}=\begin{bmatrix} 613 & 625 \\ 887 & 960 \\ 1870 & 1915 \end{bmatrix}$,

which represents the number of each sex in each state of sickness.

Section 1.4

1. $\begin{bmatrix} 7 & 4 & -1 \\ 6 & 1 & 0 \\ 2 & 2 & -6 \end{bmatrix}$.

2. $\begin{bmatrix} t^3+3t & 2t^2+3 & 3 \\ 2t^3+t^2 & 4t^2+t & t \\ t^4+t^2+t & 2t^3+t+1 & t+1 \\ t^5 & 2t^4 & 0 \end{bmatrix}$.

3. (a) $\mathbf{BA}^{\mathsf{T}}$, (b) $2\mathbf{A}^{\mathsf{T}}+\mathbf{B}$, (c) $(\mathbf{B}^{\mathsf{T}}+\mathbf{C})\mathbf{A}=\mathbf{B}^{\mathsf{T}}\mathbf{A}+\mathbf{CA}$, (d) $\mathbf{AB}+\mathbf{C}^{\mathsf{T}}$,

(e) $\mathbf{A}^{\mathsf{T}}\mathbf{A}^{\mathsf{T}}+\mathbf{A}^{\mathsf{T}}\mathbf{A}-\mathbf{AA}^{\mathsf{T}}-\mathbf{AA}$.

4. $\mathbf{X}^{\mathsf{T}}\mathbf{X}=[29]$, and $\mathbf{XX}^{\mathsf{T}}=\begin{bmatrix} 4 & 6 & 8 \\ 6 & 9 & 12 \\ 8 & 12 & 16 \end{bmatrix}$.

5. $\mathbf{X}^{\mathsf{T}}\mathbf{X} = \begin{bmatrix} 1 & -2 & 3 & -4 \\ -2 & 4 & -6 & 8 \\ 3 & -6 & 9 & -12 \\ -4 & 8 & -12 & 16 \end{bmatrix}$, and $\mathbf{X}\mathbf{X}^{\mathsf{T}} = [30]$.

6. $\left[2x^2 + 6xy + 4y^2\right]$. **7.** **A**, **B**, **D**, **F**, **M**, **N**, **R**, and **T**.

8. **E**, **F**, **H**, **K**, **L**, **M**, **N**, **R**, and **T**. **9.** Yes.

10. No, see **H** and **L** in Problem 7. **11.** Yes, see **L** in Problem 7.

12. $\begin{bmatrix} -5 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. **14.** No.

19. $\mathbf{D}^2$ is a diagonal matrix with diagonal elements 4, 9, and 25; $\mathbf{D}^3$ is a diagonal matrix with diagonal elements 8, 27, and −125.

20. A diagonal matrix with diagonal elements 1, 8, 27.

23. A diagonal matrix with diagonal elements 4, 0, 10. **25.** 4.

28. $\mathbf{A} = \mathbf{B} + \mathbf{C}$. **29.** $\begin{bmatrix} 1 & \frac{7}{2} & -\frac{1}{2} \\ \frac{7}{2} & 1 & 5 \\ -\frac{1}{2} & 5 & -8 \end{bmatrix} + \begin{bmatrix} 0 & \frac{3}{2} & -\frac{1}{2} \\ -\frac{3}{2} & 0 & -2 \\ \frac{1}{2} & 2 & 0 \end{bmatrix}$.

30. $\begin{bmatrix} 6 & \frac{3}{2} & 1 \\ \frac{3}{2} & 0 & -4 \\ 1 & -4 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{2} & 2 \\ \frac{1}{2} & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}$.

34. (a) $\mathbf{P}^2 = \begin{bmatrix} 0.37 & 0.63 \\ 0.28 & 0.72 \end{bmatrix}$ and $\mathbf{P}^3 = \begin{bmatrix} 0.289 & 0.711 \\ 0.316 & 0.684 \end{bmatrix}$,

(b) 0.37, (c) 0.63, (d) 0.711, (e) 0.684.

35. $1 \to 1 \to 1 \to 1, 1 \to 1 \to 2 \to 1, 1 \to 2 \to 1 \to 1, 1 \to 2 \to 2 \to 1$.

36. (a) 0.097, (b) 0.0194. **37.** (a) 0.64, (b) 0.636.

38. (a) 0.1, (b) 0.21. **39.** (a) 0.6675, (b) 0.577075, (c) 0.267.

40. $\mathbf{M} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$.

41. (a) $\mathbf{M} = \begin{bmatrix} 0 & 2 & 0 & 1 & 0 \\ 2 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$,

(b) 3 paths consisting of 2 arcs connecting node 1 to node 5.

42. (a) $\mathbf{M} = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$,

(b) $\mathbf{M}^3$ has a path from node 1 to node 7; it is the first integral power of $\mathbf{M}$ having m_{17} positive. The minimum number of *intermediate* cities is two.

Section 1.5

1. (a), (b), and (d) are submatrices.

3. $\left[\begin{array}{ccc|c} 4 & 5 & -1 & 9 \\ \hline 15 & 10 & 4 & 22 \\ 1 & 1 & 5 & 9 \end{array}\right]$.

4. Partition $\mathbf{A}$ and $\mathbf{B}$ into four 2×2 submatrices each. Then,

$$\mathbf{AB} = \left[\begin{array}{cc|cc} 11 & 9 & 0 & 0 \\ 4 & 6 & 0 & 0 \\ \hline 0 & 0 & 2 & 1 \\ 0 & 0 & 4 & -1 \end{array}\right].$$

5. $\left[\begin{array}{cc|cc} 18 & 6 & 0 & 0 \\ 12 & 6 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 4 \end{array}\right]$.

6. $\left[\begin{array}{cc|cc} 7 & 8 & 0 & 0 \\ -4 & -1 & 0 & 0 \\ \hline 0 & 0 & 5 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$.

7. $\mathbf{A}^2 = \left[\begin{array}{cc|cccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$. $\mathbf{A}^3 = \left[\begin{array}{cc|cccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$.

8. $\mathbf{A}^n = \mathbf{A}$ when n is odd.

Section 1.6

1. $p = 1.$ **2.** $\begin{bmatrix} -4/3 \\ -1 \\ -8/3 \\ 1/3 \end{bmatrix}.$ **3.** $\begin{bmatrix} 1 & -0.4 & 1 \end{bmatrix}.$

4. (a) Not defined, (b) $\begin{bmatrix} 6 & -3 & 12 & 3 \\ 2 & -1 & 4 & 1 \\ 12 & -6 & 24 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, (c) [29], (d) [29].

5. (a) $\begin{bmatrix} 4 & -1 & 1 \end{bmatrix}$, (b) [−1], (c) $\begin{bmatrix} 2 & 0 & -2 \\ -1 & 0 & 1 \\ 3 & 0 & -3 \end{bmatrix}$, (d) $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$.

6. (c), (d), (f), (g), (h), and (i).

7. (a) $\sqrt{2}$, (b) 5, (c) $\sqrt{3}$, (d) $\frac{1}{2}\sqrt{3}$, (e) $\sqrt{15}$, (f) $\sqrt{39}$.

8. (a) $\sqrt{2}$, (b) $\sqrt{5}$, (c) $\sqrt{3}$, (d) 2, (e) $\sqrt{30}$, (f) $\sqrt{2}$.

9. (a) $\sqrt{15}$, (b) $\sqrt{39}$. **12.** $x\begin{bmatrix} 2 \\ 4 \end{bmatrix} + y\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 10 \\ 11 \end{bmatrix}.$

13. $x\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} + y\begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix} + z\begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix} + w\begin{bmatrix} 6 \\ 8 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$ **16.** $\begin{bmatrix} 0.5 & 0.3 & 0.2 \end{bmatrix}.$

17. (a) There is a 0.6 probability that an individual chosen at random initially will live in the city; thus, 60% of the population initially lives in the city, while 40% lives in the suburbs.

(b) $\mathbf{d}^{(1)} = [0.574 \;\; 0.426]$. (c) $\mathbf{d}^{(2)} = [0.54956 \;\; 0.45044]$.

18. (a) 40% of customers now use brand X, 50% use brand Y, and 10% use other brands.

(b) $\mathbf{d}_1 = [0.395 \;\; 0.530 \;\; 0.075]$. (c) $\mathbf{d}_2 = [0.38775 \;\; 0.54815 \;\; 0.06410]$.

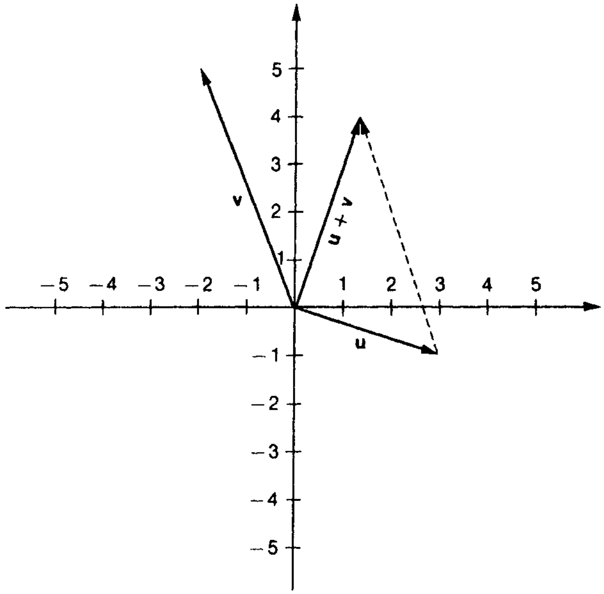
19. (a) $\mathbf{d}^{(0)} = [0 \;\; 1]$. (b) $\mathbf{d}^{(1)} = [0.7 \;\; 0.3]$.

20. (a) $\mathbf{d}^{(0)} = [1 \;\; 0 \;\; 0]$.

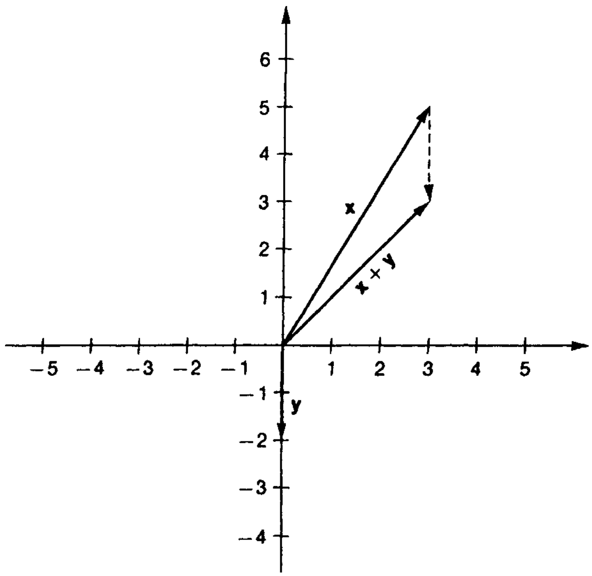
(b) $\mathbf{d}^{(2)} = [0.21 \;\; 0.61 \;\; 0.18]$. A probability of 0.18 that the harvest will be good in two years.

Section 1.7

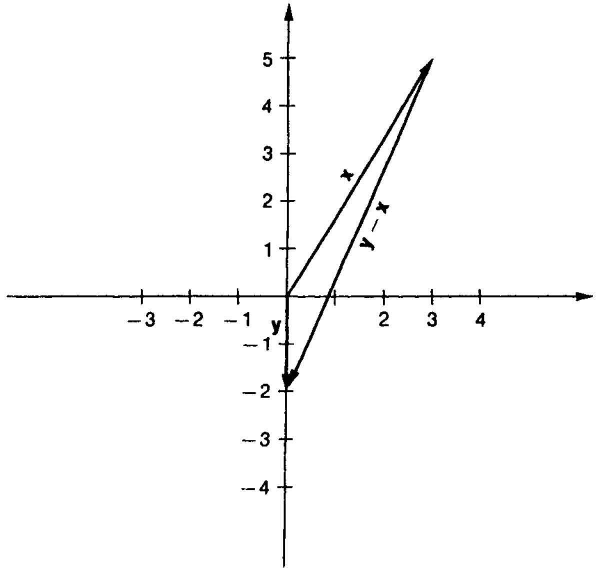
1.



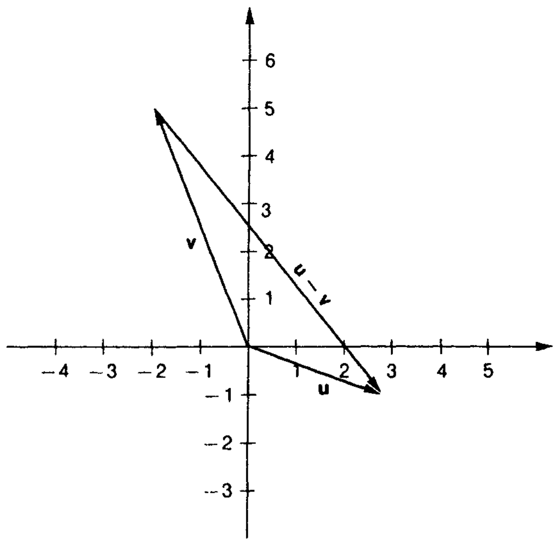
4.



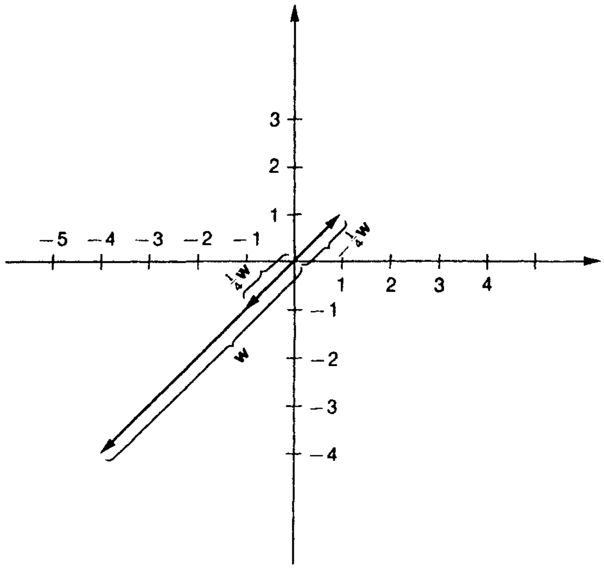
6.



7.



16.



17. 341.57°. **18.** 111.80°. **19.** 225°. **20.** 59.04°. **21.** 270°.

CHAPTER 2

Section 2.1

1. (a) No. (b) Yes. **2.** (a) Yes. (b) No. (c) Yes.

3. No value of k will work. **4.** $k = 1$. **5.** $k = 1/12$.

6. k is arbitrary; any value will work. **7.** No value of k will work.

8. $\begin{bmatrix} 3 & 5 \\ 2 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 11 \\ -3 \end{bmatrix}$. **9.** $\begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & -2 \\ 1 & 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$.

10. $\begin{bmatrix} 1 & 2 & 3 \\ 1 & -3 & 2 \\ 3 & -4 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 6 \end{bmatrix}$. **11.** $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 2 \\ -3 & -6 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

12. $50r + 60s = 70{,}000,$
$30r + 40s = 45{,}000.$

13. $5d + 0.25b = 200,$
$10d + b = 500.$

14. $8{,}000A + 3{,}000B + 1{,}000C = 70{,}000,$
$5{,}000A + 12{,}000B + 10{,}000C = 181{,}000,$
$1{,}000A + 3{,}000B + 2{,}000C = 41{,}000.$

15. $$\begin{aligned} 5A + 4B + 8C + 12D &= 80, \\ 20A + 30B + 15C + 5D &= 200, \\ 3A + 3B + 10C + 7D &= 50. \end{aligned}$$

16. $$\begin{aligned} b + 0.05c + 0.05s &= 20{,}000, \\ c &= 8{,}000, \\ 0.03c + s &= 12{,}000. \end{aligned}$$

17. (a) $C = 800{,}000 + 30B$, (b) Add the additional equation $S = C$.

18. $$\begin{aligned} -0.60p_1 + 0.30p_2 + 0.50p_3 &= 0, \\ 0.40p_1 - 0.75p_2 + 0.35p_3 &= 0, \\ 0.20p_1 + 0.45p_2 - 0.85p_3 &= 0. \end{aligned}$$

19. $$\begin{aligned} -\tfrac{1}{2}p_1 + \tfrac{1}{3}p_2 + \tfrac{1}{6}p_3 &= 0, \\ \tfrac{1}{4}p_1 - \tfrac{2}{3}p_2 + \tfrac{1}{3}p_3 &= 0, \\ \tfrac{1}{4}p_1 + \tfrac{1}{3}p_2 - \tfrac{1}{2}p_3 &= 0. \end{aligned}$$

20. $$\begin{aligned} -0.85p_1 + 0.10p_2 + \phantom{\tfrac{1}{3}p_3 +{}} 0.15p_4 &= 0, \\ 0.20p_1 - 0.60p_2 + \tfrac{1}{3}p_3 + 0.40p_4 &= 0, \\ 0.30p_1 + 0.15p_2 - \tfrac{2}{3}p_3 + 0.45p_4 &= 0, \\ 0.35p_1 + 0.35p_2 + \tfrac{1}{3}p_3 - p_4 &= 0. \end{aligned}$$

22. $\mathbf{A} = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{3} & 0 \end{bmatrix}$ and $\mathbf{d} = \begin{bmatrix} 20{,}000 \\ 30{,}000 \end{bmatrix}$.

23. $\mathbf{A} = \begin{bmatrix} 0 & 0.02 & 0.50 \\ 0.20 & 0 & 0.30 \\ 0.10 & 0.35 & 0.10 \end{bmatrix}$ and $\mathbf{d} = \begin{bmatrix} 50{,}000 \\ 80{,}000 \\ 30{,}000 \end{bmatrix}$.

24. $\mathbf{A} = \begin{bmatrix} 0.20 & 0.15 & 0.40 & 0.25 \\ 0 & 0.20 & 0 & 0 \\ 0.10 & 0.05 & 0 & 0.10 \\ 0.30 & 0.30 & 0.10 & 0.05 \end{bmatrix}$ and $\mathbf{d} = \begin{bmatrix} 0 \\ 5{,}000{,}000 \\ 0 \\ 0 \end{bmatrix}$.

Section 2.2

1. $x = 1, y = 1, z = 2.$

2. $x = -6z, y = 7z, z$ is arbitrary.

3. $x = y = 1.$

4. $r = t + 13/7, s = 2t + 15/7, t$ is arbitrary.

5. $l = \dfrac{1}{5}(-n + 1), m = \dfrac{1}{5}(3n - 5p - 3), n$ and p are arbitrary.

6. $x = 0, y = 0, z = 0.$

7. $x = 2, y = 1, z = -1.$

8. $x = 1, y = 1, z = 0, w = 1.$

Section 2.3

1. $\mathbf{A}^{\mathbf{b}} = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 1 & 1 \end{bmatrix}$.

2. $\mathbf{A}^{\mathbf{b}} = \begin{bmatrix} 1 & 2 & -1 & -1 \\ 2 & -3 & 2 & 4 \end{bmatrix}$.

3. $\mathbf{A}^{\mathbf{b}} = \begin{bmatrix} 1 & 2 & 5 \\ -3 & 1 & 13 \\ 4 & 3 & 0 \end{bmatrix}.$

4. $\mathbf{A}^{\mathbf{b}} = \begin{bmatrix} 2 & 4 & 0 & 2 \\ 3 & 2 & 1 & 8 \\ 5 & -3 & 7 & 15 \end{bmatrix}.$

5. $\mathbf{A}^{\mathbf{b}} = \begin{bmatrix} 2 & 3 & -4 & 12 \\ 3 & -2 & 0 & -1 \\ 8 & -1 & -4 & 10 \end{bmatrix}.$

6. $\begin{aligned} x + 2y &= 5, \\ y &= 8. \end{aligned}$

7. $\begin{aligned} x - 2y + 3z &= 10, \\ y - 5z &= -3, \\ z &= 4. \end{aligned}$

8. $\begin{aligned} r - 3s + 12t &= 40, \\ s - 6t &= -200, \\ t &= 25. \end{aligned}$

9. $\begin{aligned} x + 3y \quad &= -8, \\ y + 4z &= 2, \\ 0 &= 0. \end{aligned}$

10. $\begin{aligned} a - 7b + 2c &= 0, \\ b - c &= 0, \\ 0 &= 0. \end{aligned}$

11. $\begin{aligned} u - v \quad &= -1, \\ v - 2w &= 2, \\ w &= -3, \\ 0 &= 1. \end{aligned}$

12. $x = -11, y = 8.$

13. $x = 32, y = 17, z = 4.$

14. $r = -410, s = -50, t = 25.$

15. $x = -14 + 12z, y = 2 - 4z, z$ is arbitrary.

16. $a = 5c, b = c, c$ is arbitrary.

17. No solution.

18. $\begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & 23 \end{bmatrix}.$

19. $\begin{bmatrix} 1 & 6 & 5 \\ 0 & 1 & 18 \end{bmatrix}.$

20. $\begin{bmatrix} 1 & 3.5 & -2.5 \\ 0 & 1 & -6 \end{bmatrix}.$

21. $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 7 \\ 0 & 0 & 1 & 41/29 \end{bmatrix}.$

22. $\begin{bmatrix} 1 & 3 & 2 & 1 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & 1 & -32/23 \end{bmatrix}.$

23. $\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & 1 & -5 & 1 \\ 0 & 0 & 1 & -9/35 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$

24. $\begin{bmatrix} 1 & 3/2 & 2 & 3 & 0 & 5 \\ 0 & 1 & -50 & -32 & -6 & -130 \\ 0 & 0 & 1 & 53/76 & 5/76 & 190/76 \end{bmatrix}.$

25. $x = 1, y = -2.$

26. $x = 5/7 - (1/7)z, y = -6/7 + (4/7)z, z$ is arbitrary.

27. $a = -3, b = 4.$

28. $r = 13/3, s = t = -5/3.$

29. $r = \frac{1}{13}(21 + 8t), s = \frac{1}{13}(38 + 12t)$, t is arbitrary.

30. $x = 1, y = 1, z = 2.$ **31.** $x = -6z, y = 7z, z$ is arbitrary.

32. $x = y = 1.$ **33.** $r = t + 13/7, s = 2t + 15/7, t$ is arbitrary.

34. $l = \frac{1}{5}(-n + 1), m = \frac{1}{5}(3n - 5p - 3), n$ and p are arbitrary.

35. $r = 500, s = 750.$ **36.** $d = 30, b = 200.$ **37.** $A = 5, B = 8, C = 6.$

38. $A = 19.759 - 4.145D$, $B = -7.108 + 2.735D$,
$C = 1.205 - 0.277D$, D is arbitrary.

39. $b = \$19{,}012.$

40. 80,000 barrels. **41.** $p_1 = (48/33)p_3$, $p_2 = (41/33)p_3$, p_3 is arbitrary.

42. $p_1 = (8/9)p_3$, $p_2 = (5/6)p_3$, p_3 is arbitrary.

43. $p_1 = 0.3435p_4$, $p_2 = 1.4195p_4$, $p_3 = 1.1489p_4$, p_4 is arbitrary.

44. $x_1 = \$66{,}000$; $x_2 = \$52{,}000$.

45. To construct an elementary matrix that will interchange the ith and jth rows, simply interchange those rows in the identity matrix of appropriate order.

46. To construct an elementary matrix that will multiply the ith row of a matrix by the scalar r, simply replace the unity element in the i–i position of an identity matrix of appropriate order by r.

47. To construct an elementary matrix that will add r times the ith row to the jth row, simply do the identical process to an identity matrix of appropriate order.

48. $\mathbf{x}^{(0)} = \begin{bmatrix} 40{,}000 \\ 60{,}000 \end{bmatrix}, \quad \mathbf{x}^{(1)} = \begin{bmatrix} 55{,}000 \\ 43{,}333 \end{bmatrix}, \quad \mathbf{x}^{(2)} = \begin{bmatrix} 58{,}333 \\ 48{,}333 \end{bmatrix}.$

49. $\mathbf{x}^{(0)} = \begin{bmatrix} 100{,}000 \\ 160{,}000 \\ 60{,}000 \end{bmatrix}, \quad \mathbf{x}^{(1)} = \begin{bmatrix} 83{,}200 \\ 118{,}000 \\ 102{,}000 \end{bmatrix}, \quad \mathbf{x}^{(2)} = \begin{bmatrix} 103{,}360 \\ 127{,}240 \\ 89{,}820 \end{bmatrix}.$

The solution is $x_1 = \$99{,}702$; $x_2 = \$128{,}223$; and $x_3 = \$94{,}276$, rounded to the nearest dollar.

50. $\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 10{,}000{,}000 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{x}^{(1)} = \begin{bmatrix} 1{,}500{,}000 \\ 7{,}000{,}000 \\ 500{,}000 \\ 3{,}000{,}000 \end{bmatrix}, \quad \mathbf{x}^{(2)} = \begin{bmatrix} 2{,}300{,}00 \\ 6{,}400{,}000 \\ 800{,}000 \\ 2{,}750{,}000 \end{bmatrix}.$

The solution is: energy = \$2,484,488; tourism = \$6,250,000; transportation = \$845,677; and construction = \$2,847,278, all rounded to the nearest dollar.

Section 2.4

1. (a) 4, (b) 4, (c) 8.
2. (a) 5, (b) 5, (c) 5.
3. (a) 3, (b) 3, (c) 8.
4. (a) 4, (b) −3, (c) 8.
5. (a) 9, (b) 9, (c) 11.
6. (a) 4, (b) 1, (c) 10.
7. $a = -3, b = 4$.
8. $r = 13/3, s = t = -5/3$.

9. Depending on the roundoff procedure used, the last equation may not be $0 = 0$, but rather numbers very close to zero. Then only one answer is obtained.

Section 2.5

1. Independent.
2. Independent.
3. Dependent.
4. Dependent.
5. Independent.
6. Dependent.
7. Independent.
8. Dependent.
9. Dependent.
10. Dependent.
11. Independent.
12. Dependent.
13. Independent.
14. Independent.
15. Dependent.
16. Independent.
17. Dependent.
18. Dependent.
19. Dependent.

20. $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = (-2)\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + (1)\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + (3)\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$

21. (a) $[2 \quad 3] = 2[1 \quad 0] + 3[0 \quad 1]$, (b) $[2 \quad 3] = \frac{5}{2}[1 \quad 1] + \left(-\frac{1}{2}\right)[1 \quad -1]$, (c) No.

22. (a) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \left(\frac{1}{2}\right)\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \left(\frac{1}{2}\right)\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \left(\frac{1}{2}\right)\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, (b) No,

(c) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = (0)\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + (1)\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + (0)\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}.$

23. $\begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} = (1)\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + (1)\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + (0)\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}.$

24. $[a \quad b] = (a)[1 \quad 0] + (b)[0 \quad 1]$.

25. $[a \quad b] = \left(\dfrac{a+b}{2}\right)[1 \quad 1] + \left(\dfrac{a-b}{2}\right)[1 \quad -1]$.

26. $[1 \quad 0]$ cannot be written as a linear combination of these vectors.

27. $[a \quad -2a] = (a/2)[2 \quad -4] + (0)[-3 \quad 6]$.

28. $[a \quad b] = \left(\dfrac{a+2b}{7}\right)[1 \quad 3] + \left(\dfrac{3a-b}{7}\right)[2 \quad -1] + (0)[1 \quad 1]$.

29. $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \left(\dfrac{a-b+c}{2}\right)\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \left(\dfrac{a+b-c}{2}\right)\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \left(\dfrac{-a+b+c}{2}\right)\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

30. No, impossible to write any vector with a nonzero second component as a linear combination of these vectors.

31. $\begin{bmatrix} a \\ 0 \\ a \end{bmatrix} = (a)\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + (0)\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + (0)\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$.

32. 1 and 2 are bases.

33. 7 and 11 are bases.

39. $(-k)x + (1)k\mathbf{x} = \mathbf{0}$.

42. $\mathbf{0} = \mathbf{A0} = \mathbf{A}(c_1\mathbf{x}_1 + c_2\mathbf{x}_2 + \cdots + c_k\mathbf{x}_k) = c_1\mathbf{Ax}_1 + c_2\mathbf{Ax}_2 + \cdots + c_k\mathbf{Ax}_k$
$= c_1\mathbf{y}_1 + c_2\mathbf{y}_2 + \cdots + c_k\mathbf{y}_k)$.

Section 2.6

1. 2. **2.** 2. **3.** 1. **4.** 2. **5.** 3.

6. Independent. **7.** Independent. **8.** Dependent.

9. Dependent. **10.** Independent. **11.** Dependent.

12. Independent. **13.** Dependent. **14.** Dependent.

15. Dependent. **16.** Independent. **17.** Dependent.

18. Independent. **19.** Dependent. **20.** Independent.

21. Dependent. **22.** Dependent.

23. (a) Yes, (b) Yes, (c) No. **24.** (a) Yes, (b) No, (c) Yes.

25. Yes. **26.** Yes. **27.** No. **28.** First two.

29. First two. **30.** First and third. **31.** 0.

Section 2.7

1. Consistent with no arbitrary unknowns; $x = 2/3$, $y = 1/3$.

2. Inconsistent.

3. Consistent with one arbitrary unknown; $x = (1/2)(3 - 2z)$, $y = -1/2$.

4. Consistent with two arbitrary unknowns; $x = (1/7)(11 - 5z - 2w)$, $y = (1/7)(1 - 3z + 3w)$.

5. Consistent with no arbitrary unknowns; $x = y = 1$, $z = -1$.

6. Consistent with no arbitrary unknowns; $x = y = 0$.

7. Consistent with no arbitrary unknowns; $x = y = z = 0$.

8. Consistent with no arbitrary unknowns; $x = y = z = 0$.

9. Consistent with two arbitrary unknowns; $x = z - 7w$, $y = 2z - 2w$.

CHAPTER 3

Section 3.1

1. (c).

2. None.

3. $\begin{bmatrix} \frac{3}{14} & \frac{-2}{14} \\ \frac{-5}{14} & \frac{8}{14} \end{bmatrix}$.

4. $\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$.

5. $\mathbf{D}$ has no inverse.

7. $\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}$.

8. $\begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -2 & 1 \end{bmatrix}$.

9. $\begin{bmatrix} 4 & -6 \\ -6 & 12 \end{bmatrix}$.

10. $\begin{bmatrix} \frac{-1}{5} & \frac{1}{10} \\ \frac{3}{20} & \frac{-1}{20} \end{bmatrix}$.

11. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

12. $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$.

13. $\begin{bmatrix} 1 & 0 \\ 0 & -5 \end{bmatrix}$.

14. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

15. $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$.

16. $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$.

17. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$.

18. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 0 & 1 \end{bmatrix}$.

19. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

20. $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$.

21. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$.

22. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$.

23. $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$.

24. $\begin{bmatrix} 1 & 0 \\ 0 & 7 \end{bmatrix}$. **25.** $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. **26.** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -0.2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

27. $\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}$. **28.** $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$. **29.** $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$. **30.** $\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$.

31. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$. **32.** $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. **33.** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$.

34. $\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. **35.** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$. **36.** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{4} \end{bmatrix}$.

37. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. **38.** $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. **39.** $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

40. $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$. **41.** $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. **42.** $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

43. $\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$. **44.** No inverse. **45.** $\begin{bmatrix} \frac{1}{3} & 0 \\ 0 & -\frac{1}{3} \end{bmatrix}$. **46.** $\begin{bmatrix} 2 & 0 \\ 0 & -\frac{3}{2} \end{bmatrix}$.

47. $\begin{bmatrix} \frac{1}{10} & 0 & 0 \\ 0 & \frac{1}{5} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$. **48.** $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$. **49.** $\begin{bmatrix} -\frac{1}{4} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & \frac{5}{3} \end{bmatrix}$.

50. $\begin{bmatrix} 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{bmatrix}$. **51.** $\begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. **52.** $\begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{5} & 0 & 0 \\ 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

53. $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. **54.** $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{7} \end{bmatrix}$. **55.** $\begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{5} & 0 & 0 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Section 3.2

1. $\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}$. **2.** $\frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. **3.** Does not exist.

4. $\frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$. **5.** $\begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix}$. **6.** $\begin{bmatrix} 4 & -6 \\ -6 & 12 \end{bmatrix}$.

7. $\frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$. **8.** $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. **9.** $\begin{bmatrix} -1 & -1 & 1 \\ 6 & 5 & -4 \\ -3 & -2 & 2 \end{bmatrix}$.

10. Does not exist. **11.** $\frac{1}{2}\begin{bmatrix} 1 & 0 & 0 \\ -5 & 2 & 0 \\ 1 & -2 & 2 \end{bmatrix}$. **12.** $\frac{1}{6}\begin{bmatrix} 3 & -1 & -8 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$.

13. $\begin{bmatrix} 9 & -5 & -2 \\ 5 & -3 & -1 \\ -36 & 21 & 8 \end{bmatrix}$. **14.** $\frac{1}{17}\begin{bmatrix} 1 & 7 & -2 \\ 7 & -2 & 3 \\ -2 & 3 & 4 \end{bmatrix}$.

15. $\frac{1}{17}\begin{bmatrix} 14 & 5 & -6 \\ -5 & -3 & 7 \\ 13 & 1 & -8 \end{bmatrix}$. **16.** Does not exist.

17. $\frac{1}{33}\begin{bmatrix} 5 & 3 & 1 \\ -6 & 3 & 12 \\ -8 & 15 & 5 \end{bmatrix}$. **18.** $\frac{1}{4}\begin{bmatrix} 0 & -4 & 4 \\ 1 & 5 & -4 \\ 3 & 7 & -8 \end{bmatrix}$.

19. $\frac{1}{4}\begin{bmatrix} 4 & -4 & -4 & -4 \\ 0 & 4 & 2 & 5 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & -2 \end{bmatrix}$. **20.** $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ -8 & 3 & \frac{1}{2} & 0 \\ -25 & 10 & 2 & -1 \end{bmatrix}$.

21. Inverse of a nonsingular lower triangular matrix is lower triangular.

22. Inverse of a nonsingular upper triangular matrix is upper triangular.

23. 35 62 5 10 47 75 2 3 38 57 15 25 18 36.

24. 14 116 10 20 −39 131 −3 5 −57 95 −5 45 36 72.

25. 3 5 48 81 14 28 47 75 2 3 28 42 27 41 5 10.

26. HI THERE. **27.** THIS IS FUN.

28. 24 13 27 19 28 9 0 1 1 24 10 24 18 0 18.

Section 3.3

1. $x = 1, y = -2$. **2.** $a = -3, b = 4$. **3.** $x = 2, y = -1$.
4. $l = 1, p = 3$. **5.** Not possible; A is singular.
6. $x = -8, y = 5, z = 3$. **7.** $x = y = z = 1$.
8. $l = 1, m = -2, n = 0$. **9.** $r = 4.333, s = t = -1.667$.
10. $r = 3.767, s = -1.133, t = -1.033$. **11.** Not possible; A is singular.
12. $x = y = 1, z = 2$. **13.** $r = 500, s = 750$. **14.** $d = 30, b = 200$.
15. $A = 5, B = 8, C = 6$. **16.** $B = \$19{,}012$.
17. 80,000 barrels. **18.** $x_1 = 66{,}000$; $x_2 = 52{,}000$.
19. $x_1 = 99{,}702$; $x_2 = 128{,}223$; $x_3 = 94{,}276$.

Section 3.4

11. $\mathbf{A}^{-2} = \begin{bmatrix} 11 & -4 \\ -8 & 3 \end{bmatrix}, \quad \mathbf{B}^{-2} = \begin{bmatrix} 9 & -20 \\ -4 & 9 \end{bmatrix}.$

12. $\mathbf{A}^{-3} = \begin{bmatrix} 41 & -15 \\ -30 & 11 \end{bmatrix}, \quad \mathbf{B}^{-3} = \begin{bmatrix} -38 & 85 \\ 17 & -38 \end{bmatrix}.$

13. $\mathbf{A}^{-2} = \frac{1}{4}\begin{bmatrix} 22 & -10 \\ -15 & 7 \end{bmatrix}, \quad \mathbf{B}^{-4} = \frac{1}{512}\begin{bmatrix} 47 & 15 \\ -45 & -13 \end{bmatrix}.$

14. $\mathbf{A}^{-2} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{B}^{-2} = \begin{bmatrix} 1 & -4 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$

15. $\mathbf{A}^{-3} = \begin{bmatrix} 1 & -3 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{B}^{-3} = \begin{bmatrix} 1 & -6 & -9 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}.$

16. $\frac{1}{125} = \begin{bmatrix} -11 & -2 \\ 2 & -11 \end{bmatrix}.$

17. First show that $\left(\mathbf{BA}^{-1}\right)^{\mathsf{T}} = \mathbf{A}^{-1}\mathbf{B}^{\mathsf{T}}$ and that $\left(\mathbf{A}^{-1}\mathbf{B}^{\mathsf{T}}\right)^{-1} = \left(\mathbf{B}^{\mathsf{T}}\right)^{-1}\mathbf{A}$.

Section 3.5

1. $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 10 \\ -9 \end{bmatrix}.$

2. $\begin{bmatrix} 1 & 0 \\ 0.5 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 1.5 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 8 \\ -5 \end{bmatrix}.$

3. $\begin{bmatrix} 1 & 0 \\ 0.625 & 1 \end{bmatrix}\begin{bmatrix} 8 & 3 \\ 0 & 0.125 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} -400 \\ 1275 \end{bmatrix}.$

4. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}.$

5. $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & -2 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 5 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}.$

6. $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 3 \\ 0 & -1 & -6 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} -10 \\ 0 \\ 10 \end{bmatrix}.$

7. $\begin{bmatrix} 1 & 0 & 0 \\ \frac{4}{3} & 1 & 0 \\ 1 & -\frac{21}{8} & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 & 1 \\ 0 & -\frac{8}{3} & -\frac{1}{3} \\ 0 & 0 & \frac{1}{8} \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 10 \\ -10 \\ 40 \end{bmatrix}.$

8. $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -0.75 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & -1 \\ 0 & -4 & 3 \\ 0 & 0 & 4.25 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 79 \\ 1 \\ 1 \end{bmatrix}.$

9. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 19 \\ -3 \\ 5 \end{bmatrix}.$

10. $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & \frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 2 \\ -1 \\ \frac{1}{2} \end{bmatrix}.$

11. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 3 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 1 \\ -5 \\ 2 \\ 1 \end{bmatrix}.$

12. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{2}{7} & \frac{5}{7} & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & -1 & 3 \\ 0 & \frac{7}{2} & \frac{5}{2} & -\frac{1}{2} \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & \frac{3}{7} \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 266.67 \\ -166.67 \\ 166.67 \\ 266.67 \end{bmatrix}.$

13. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 10 \\ 10 \\ 10 \\ -10 \end{bmatrix}$.

14. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -2 & 1.5 & 1 & 0 \\ 0.5 & 0 & 0.25 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 2 & 0 \\ 0 & 2 & -2 & 6 \\ 0 & 0 & 8 & -8 \\ 0 & 0 & 0 & 3 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} -2.5 \\ -1.5 \\ 1.5 \\ 2.0 \end{bmatrix}$.

15. (a) $x = 5$, $y = -2$; (b) $x = -5/7$, $y = 1/7$.

16. (a) $x = 1$, $y = 0$, $z = 2$; (b) $x = 140$, $y = -50$, $z = -20$.

17. (a) $\begin{bmatrix} 8 \\ -3 \\ -1 \end{bmatrix}$, (b) $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$, (c) $\begin{bmatrix} 35 \\ 5 \\ 15 \end{bmatrix}$, (d) $\begin{bmatrix} -0.5 \\ 1.5 \\ 1.5 \end{bmatrix}$.

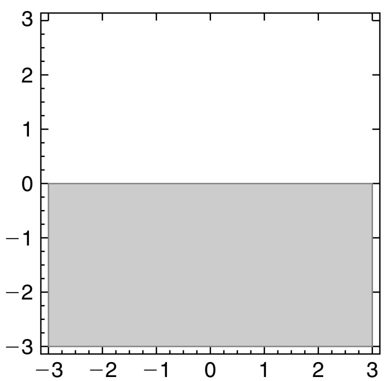
18. (a) $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$, (b) $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, (c) $\begin{bmatrix} 80 \\ 50 \\ -10 \\ 20 \end{bmatrix}$, (d) $\begin{bmatrix} -\frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$.

21. (d) **A** is singular.

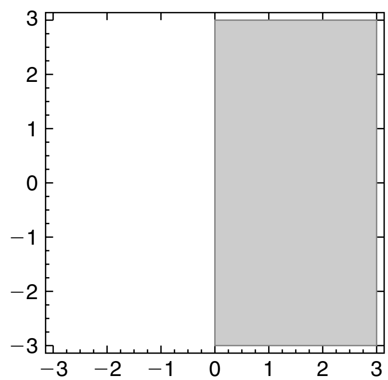
CHAPTER 4

Section 4.1

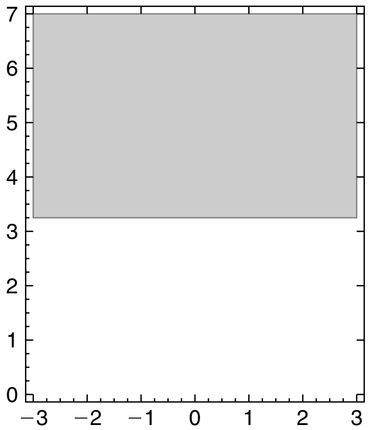
1.



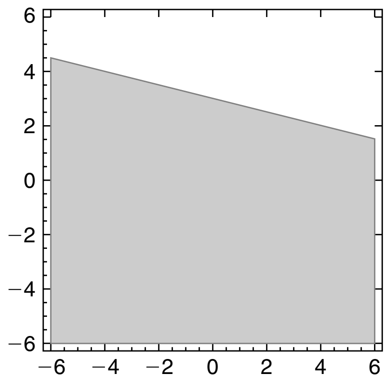
2.



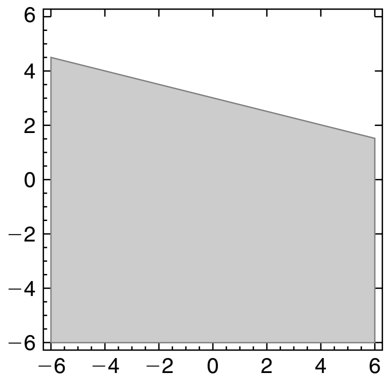
3.


7
6
5
4
3
2
1
0
−3 −2 −1 0 1 2 3

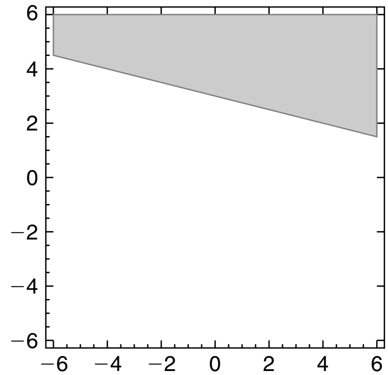
4.


6
4
2
0
−2
−4
−6
−6 −4 −2 0 2 4 6

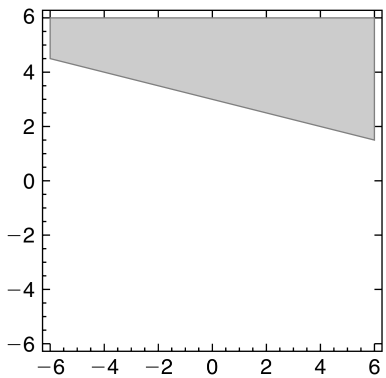
5.


6
4
2
0
−2
−4
−6
−6 −4 −2 0 2 4 6

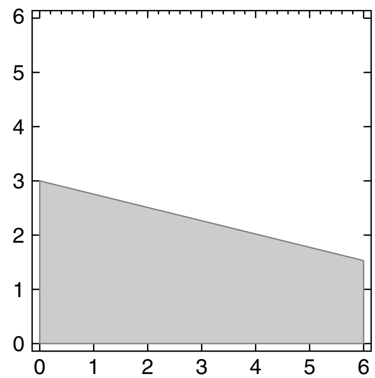
6.


6
4
2
0
−2
−4
−6
−6 −4 −2 0 2 4 6

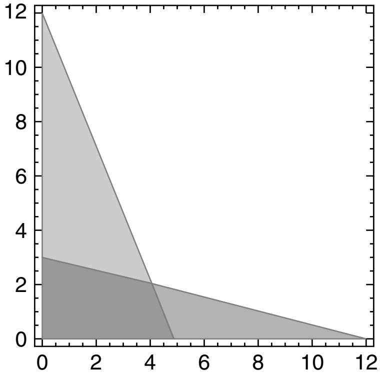
7.


6
4
2
0
−2
−4
−6
−6 −4 −2 0 2 4 6

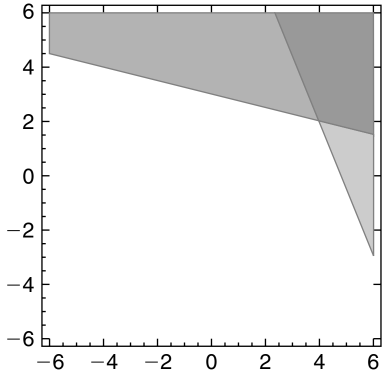
8.


6
5
4
3
2
1
0
0 1 2 3 4 5 6

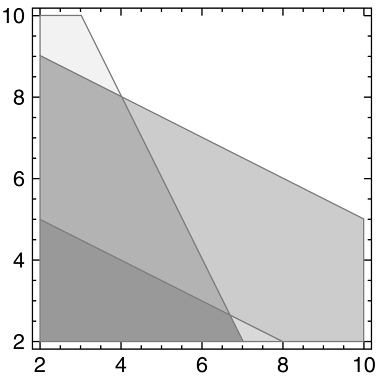
9.



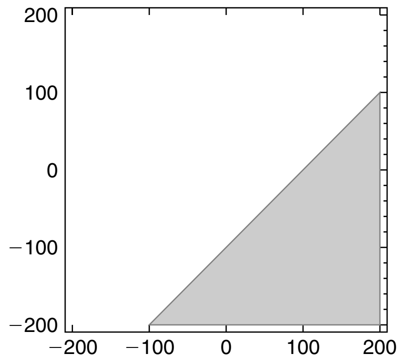
10.



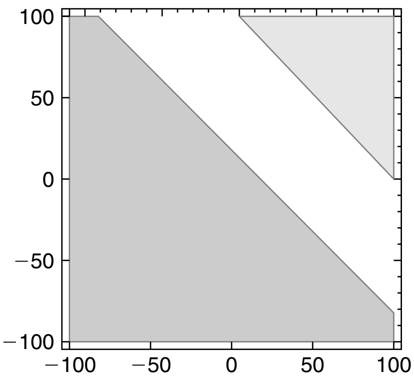
11.



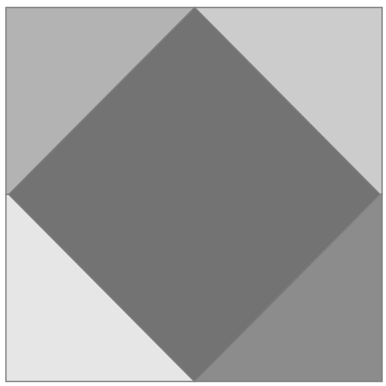
12.



13.



14.



Section 4.2

Note: Assume all variables are non-negative for (1) through (8).

1. Let $x =$ the number of trucks of wheat; $y =$ the number of trucks of corn. $2x + 3y \leq 23, 3x + y \leq 17$.

   The objective function is $5000x + 6000y$.

2. The objective function is $8000x + 5000y$.
3. Let $x =$ the number of units of X; $y =$ the number of units of Y. $2x + 3y \geq 180$, $3x + 2y \geq 240$. The objective function is $500x + 750y$.
4. The objective function is $750x + 500y$.
5. Add the third constraint $10x + 10y \geq 210$.
6. Let $x =$ the number of ounces of Zinc and $y =$ the number of ounces of Calcium. $2x + y \geq 10,\ x + 4y \geq 15$. The objective function is $.04x + .05y$.
7. Add the third constraint $3x + 2y \geq 12$.
8. The objective function is $.07x + .08y$.
9. The *Richard Nardone Emporium* needs at least 1800 cases of regular scotch and at least 750 cases of premium scotch. Each foreign shipment from distributor "x" can deliver two cases of the former and three cases of the latter, while distributor "y" can produce nine cases of the former and one case of the latter for each foreign shipment. Minimize the cost if each "x" shipment costs \$400 and each "y" shipment costs \$1100. Note that the units for $K(x, y)$ is in \$100's.

   (g) Three components are required to produce a special force (in pounds): mechanical, chemical, and electrical. The following constraints are imposed:

   - Every x force requires one mechanical unit, two chemical units and one electrical unit;
   - Every y force needs one mechanical unit, one chemical unit and three electrical units;
   - Every z force requires two mechanical units, one chemical unit and one electrical unit.

   The respective limits on these components is 12, 14, and 15 units, respectively. The *Cafone Force Machine* uses $2x$ plus $3y$ plus $4z$ pounds of force; maximize the sum of these forces.

Section 4.3

1. \$50,000. **2.** \$57,000.
3. \$45,000. Note that the minimum occurs at every point on the line segment connecting (72,12) and (90,0).

4. \$60,000. Note that the minimum occurs at every point on the line segment connecting (72,12) and (0,120).

5. $X = 72$, $Y = 12$ is one solution, $X = 90$, $Y = 0$ is another solution.

6. About 29 cents.

9. 400.

12. 3280.

14. 60,468.8.

15. 3018.8.

Section 4.4

1. \$50,000.

2. \$57,000.

3. 30.

4. 20.

5. 72.

7.
$$\begin{array}{ccccc|c} x_1 & x_2 & s_1 & s_2 & z & \\ 2 & 5 & 1 & 0 & 0 & 10 \\ 3 & 4 & 0 & 1 & 0 & 12 \\ \hline -100 & -55 & 0 & 0.5 & 1 & 0 \end{array}$$

CHAPTER 5

Section 5.1

1. -2.

2. 38.

3. 38.

4. -2.

5. 82.

6. -82.

7. 9.

8. -20.

9. 21.

10. 2.

11. 20.

12. 0.

13. 0.

14. 0.

15. -93.

16. $4t - 6$.

17. $2t^2 + 6$.

18. $5t^2$.

19. 0 and 2.

20. -1 and 4.

21. 2 and 3.

22. $\pm\sqrt{6}$.

23. $\lambda^2 - 9\lambda - 2$.

24. $\lambda^2 - 9\lambda + 38$.

25. $\lambda^2 - 13\lambda - 2$.

26. $\lambda^2 - 8\lambda + 9$.

27. $|\mathbf{A}||\mathbf{B}| = |\mathbf{AB}|$.

28. They differ by a sign.

29. The new determinants are the chosen constant times the old determinants, respectively.

30. No change.

31. Zero.

32. Identical.

33. Zero.

Section 5.2

1. −6. **2.** 22. **3.** 0. **4.** −9. **5.** −33.
6. 15. **7.** −5. **8.** −10. **9.** 0. **10.** 0.
11. 0. **12.** 119. **13.** −8. **14.** 22. **15.** −7.
16. −40. **17.** 52. **18.** 25. **19.** 0. **20.** 0.
21. −11. **22.** 0. **23.** Product of diagonal elements.
24. Always zero. **25.** $-\lambda^3 + 7\lambda + 22$.
26. $-\lambda^3 + 4\lambda^2 - 17\lambda$. **27.** $-\lambda^3 + 6\lambda - 9$.
28. $-\lambda^3 + 10\lambda^2 - 22\lambda - 33$.

Section 5.3

2. For an upper triangular matrix, expand by the first column at each step.

3. Use the third column to simplify both the first and second columns.

6. Factor the numbers −1, 2, 2, and 3 from the third row, second row, first column, and second column, respectively.

7. Factor a five from the third row. Then use this new third row to simplify the second row and the new second row to simplify the first row.

8. Interchange the second and third rows, and then transpose.

9. Multiply the first row by 2, the second row by −1, and the second column by 2.

10. Apply the third elementary row operation with the third row to make the first two rows identical.

11. Multiply the first column by 1/2, the second column by 1/3, to obtain identical columns.

13. $1 = \det(\mathbf{I}) = \det(\mathbf{A}\mathbf{A}^{-1}) = \det(\mathbf{A})\det(\mathbf{A}^{-1})$.

Section 5.4

1. −1. **2.** 0. **3.** −311. **4.** −10. **5.** 0.
6. −5. **7.** 0. **8.** 0. **9.** 119. **10.** −9.
11. −33. **12.** 15. **13.** 2187. **14.** 52. **15.** 25.
16. 0. **17.** 0. **18.** 152. **19.** 0. **20.** 0.

Section 5.5

1. Does not exist. **2.** $\begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}$. **3.** $\begin{bmatrix} 4 & -6 \\ -6 & 12 \end{bmatrix}$.

4. $\frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}.$ **5.** $\begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix}.$ **6.** Does not exist.

7. $\frac{1}{2}\begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}.$ **8.** $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$ **9.** $\begin{bmatrix} -1 & -1 & 1 \\ 6 & 5 & -4 \\ -3 & -2 & 2 \end{bmatrix}.$

10. Does not exist. **11.** $\frac{1}{2}\begin{bmatrix} 1 & 0 & 0 \\ -5 & 2 & 0 \\ 1 & -2 & 2 \end{bmatrix}.$ **12.** $\frac{1}{17}\begin{bmatrix} 14 & 5 & -6 \\ -5 & -3 & 7 \\ 13 & 1 & -8 \end{bmatrix}.$

13. Does not exist. **14.** $\frac{1}{33}\begin{bmatrix} 5 & 3 & 1 \\ -6 & 3 & 12 \\ -8 & 15 & 5 \end{bmatrix}.$ **15.** $\frac{1}{4}\begin{bmatrix} 0 & -4 & 4 \\ 1 & 5 & -4 \\ 3 & 7 & -8 \end{bmatrix}.$

16. $\frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$ **17.** $\det(\mathbf{AB}) = \det(\mathbf{A})\det(\mathbf{B})$.

19. Equals the number of rows in the matrix.

Section 5.6

1. $x = 1, y = -2$. **2.** $x = 3, y = -3$. **3.** $a = 10/11, b = -20/11$.

4. $s = 50, t = 30$. **5.** Determinant of coefficient matrix is zero.

6. System is not square. **7.** $x = 10, y = z = 5$.

8. $x = 1, y = -4, z = 5$. **9.** $x = y = 1, z = 2$. **10.** $a = b = c = 1$.

11. Determinant of coefficient matrix is zero. **12.** $r = 3, s = -2, t = 3$.

13. $x = 1, y = 2, z = 5, w = -3$.

CHAPTER 6

Section 6.1

1. (a), (d), (e), (f), and (h). **2.** (a) 3, (d) 5, (e) 3, (f) 3, (h) 5.

3. (c), (e), (f), and (g). **4.** (c) 0, (e) 0, (f) −4, (g) −4.

5. (b), (c), (d), (e), and (g). **6.** (b) 2, (c) 1, (d) 1, (e) 3, (g) 3.

7. (a), (b), and (d). **8.** (a) −2, (b) −1, (d) 2.

Section 6.2

1. 2, 3. **2.** 1, 4. **3.** 0, 8. **4.** −3, 12.

5. 3, 3. **6.** 3, −3. **7.** $\pm\sqrt{34}$. **8.** $\pm 4i$.

9. $\pm i$. **10.** 1, 1. **11.** 0, 0. **12.** 0, 0.

13. $\pm\sqrt{2}$. **14.** 10, −11. **15.** −10, 11. **16.** $t, -2t$.

17. $2t, 2t$. **18.** $2\theta, 3\theta$. **19.** 2, 4, −2. **20.** 1, 2, 3.

21. 1, 1, 3. **22.** 0, 2, 2. **23.** 2, 3, 9. **24.** 1, −2, 5.

25. 2, 3, 6. **26.** 0, 0, 14. **27.** 0, 10, 14. **28.** 2, 2, 5.

29. 0, 0, 6. **30.** 3, 3, 9. **31.** $3, \pm 2i$. **32.** $0, \pm i$.

33. 3, 3, 3. **34.** $2, 4, 1, \pm i\sqrt{5}$. **35.** 1, 1, 2, 2.

Section 6.3

1. $\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. **2.** $\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. **3.** $\begin{bmatrix} 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

4. $\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. **5.** $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix}$. **6.** $\begin{bmatrix} -5 \\ 3-\sqrt{34} \end{bmatrix}, \begin{bmatrix} -5 \\ 3+\sqrt{34} \end{bmatrix}$.

7. $\begin{bmatrix} -5 \\ 3-4i \end{bmatrix}, \begin{bmatrix} -5 \\ 3+4i \end{bmatrix}$. **8.** $\begin{bmatrix} -5 \\ 2-i \end{bmatrix}, \begin{bmatrix} -5 \\ 2+i \end{bmatrix}$. **9.** $\begin{bmatrix} -2-\sqrt{2} \\ 1 \end{bmatrix}, \begin{bmatrix} -2+\sqrt{2} \\ 1 \end{bmatrix}$.

10. $\begin{bmatrix} 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \end{bmatrix}$. **11.** $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. **12.** $\begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix}$.

13. $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$. **14.** $\begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$. **15.** $\begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

16. $\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix}$. **17.** $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. **18.** $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$.

19. $\begin{bmatrix} 9 \\ 1 \\ 13 \end{bmatrix}, \begin{bmatrix} 5 \\ -1+2i \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ -1-2i \\ 0 \end{bmatrix}$. **20.** $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -i \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ i \\ 1 \end{bmatrix}$.

21. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$. **22.** $\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \end{bmatrix}$.

23. $\begin{bmatrix} 10 \\ -6 \\ 11 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \\ -1 \end{bmatrix}.$ **24.** $\begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}.$

25. $\begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}.$ **26.** $\begin{bmatrix} 3/\sqrt{13} \\ -2/\sqrt{13} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}.$

27. $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}, \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}.$ **28.** $\begin{bmatrix} 1/\sqrt{18} \\ -4/\sqrt{18} \\ 1/\sqrt{18} \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}.$

29. $\begin{bmatrix} 0 \\ -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}, \begin{bmatrix} 3/\sqrt{13} \\ -2/\sqrt{13} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4/5 \\ 3/5 \end{bmatrix}.$ **30.** $[1 \ \ -1], [-1 \ \ 2].$

31. $[-2 \ \ 1], [1 \ \ 1].$ **32.** $[-2 \ \ 1], [2 \ \ 3].$

33. $[-3 \ \ 2], [1 \ \ 1].$ **34.** $[1 \ \ -2 \ \ 1], [1 \ \ 0 \ \ 1], [-1 \ \ 0 \ \ 1].$

35. $[1 \ \ 0 \ \ 1], [2 \ \ 1 \ \ 2], [-1 \ \ 0 \ \ 1].$ **36.** $[-2 \ \ -3 \ \ 4], [1 \ \ 0 \ \ 0], [2 \ \ 3 \ \ 3].$

37. $[1 \ \ -1 \ \ 0], [1 \ \ 1 \ \ 1], [1 \ \ 1 \ \ -2].$

38. $\mathbf{Ax} = \lambda\mathbf{x}$, so $(\mathbf{Ax})^{\mathsf{T}} = (\lambda\mathbf{x})^{\mathsf{T}}$, and $\mathbf{x}^{\mathsf{T}}\mathbf{A} = \lambda\mathbf{x}^{\mathsf{T}}$. **39.** $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}.$

40. $\begin{bmatrix} \frac{2}{5} & \frac{3}{5} \end{bmatrix}.$ **41.** $\begin{bmatrix} \frac{1}{8} & \frac{2}{8} & \frac{5}{8} \end{bmatrix}.$ **42.** (a) $\begin{bmatrix} \frac{1}{6} & \frac{5}{6} \end{bmatrix}.$ (b) $\frac{1}{6}$.

43. $[7/11 \ \ 4/11]$; probability of having a Republican is $7/11 = 0.636$.

44. $[23/120 \ \ 71/120 \ \ 26/120]$; probability of a good harvest is $26/120 = 0.217$.

45. $[40/111 \ \ 65/111 \ \ 6/111]$; probability of a person using brand **Y** is $65/111 = 0.586$

Section 6.4

1. 9. **2.** 9.2426. **3.** $5 + 8 + \lambda = -4, \lambda = -17$.

4. $(5)(8)\lambda = -4, \lambda = -0.1$. **5.** Their product is –24.

6. (a) $-6, 8$; (b) $-15, 20$; (c) $-6, 1$; (d) $1, 8$.

7. (a) $4, 4, 16$; (b) $-8, 8, 64$; (c) $6, -6, -12$; (d) $1, 5, 7$.

8. (a) $2\mathbf{A}$, (b) $5\mathbf{A}$, (c) $\mathbf{A}^2$, (d) $\mathbf{A} + 3\mathbf{I}$.

9. (a) $2\mathbf{A}$, (b) $\mathbf{A}^2$, (c) $\mathbf{A}^3$, (d) $\mathbf{A} - 2\mathbf{I}$.

Section 6.5

1. $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$. **2.** $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$. **3.** $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. **4.** $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

5. $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. **6.** $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$. **7.** $\begin{bmatrix} 5 \\ -4 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

8. $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. **9.** $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$. **10.** $\begin{bmatrix} 1 \\ 3 \\ 9 \end{bmatrix}$.

11. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. **12.** $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$. **13.** $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

14. $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$. **15.** $\begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

16. $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$.

Section 6.6

1.

Iteration	Eigenvector components		Eigenvalue
0	1.0000	1.0000	
1	0.6000	1.0000	5.0000
2	0.5238	1.0000	4.2000
3	0.5059	1.0000	4.0476
4	0.5015	1.0000	4.0118
5	0.5004	1.0000	4.0029

2.

Iteration	Eigenvector components		Eigenvalue
0	1.0000	1.0000	
1	0.5000	1.0000	10.0000
2	0.5000	1.0000	8.0000
3	0.5000	1.0000	8.0000

3.

Iteration	Eigenvector components		Eigenvalue
0	1.0000	1.0000	
1	0.6000	1.0000	15.0000
2	0.6842	1.0000	11.4000
3	0.6623	1.0000	12.1579
4	0.6678	1.0000	11.9610
5	0.6664	1.0000	12.0098

4.

Iteration	Eigenvector components		Eigenvalue
0	1.0000	1.0000	
1	0.5000	1.0000	2.0000
2	0.2500	1.0000	4.0000
3	0.2000	1.0000	5.0000
4	0.1923	1.0000	5.2000
5	0.1912	1.0000	5.2308

5.

Iteration	Eigenvector components		Eigenvalue
0	1.0000	1.0000	
1	1.0000	0.6000	10.0000
2	1.0000	0.5217	9.2000
3	1.0000	0.5048	9.0435
4	1.0000	0.5011	9.0096
5	1.0000	0.5002	9.0021

6.

Iteration	Eigenvector components		Eigenvalue
0	1.0000	1.0000	
1	1.0000	0.4545	11.0000
2	1.0000	0.4175	9.3636
3	1.0000	0.4145	9.2524
4	1.0000	0.4142	9.2434
5	1.0000	0.4142	9.2427

7.

Iteration	Eigenvector components			Eigenvalue
0	1.0000	1.0000	1.0000	
1	0.2500	1.0000	0.8333	12.0000
2	0.0763	1.0000	0.7797	9.8333
3	0.0247	1.0000	0.7605	9.2712
4	0.0081	1.0000	0.7537	9.0914
5	0.0027	1.0000	0.7513	9.0310

8.

Iteration	Eigenvector components			Eigenvalue
0	1.0000	1.0000	1.0000	
1	0.6923	0.6923	1.0000	13.0000
2	0.5586	0.7241	1.0000	11.1538
3	0.4723	0.6912	1.0000	11.3448
4	0.4206	0.6850	1.0000	11.1471
5	0.3883	0.6774	1.0000	11.1101

9.

Iteration	Eigenvector components			Eigenvalue
0	1.0000	1.0000	1.0000	
1	0.4000	0.7000	1.0000	20.0000
2	0.3415	0.6707	1.0000	16.4000
3	0.3343	0.6672	1.0000	16.0488
4	0.3335	0.6667	1.0000	16.0061
5	0.3333	0.6667	1.0000	16.0008

10.

Iteration	Eigenvector components			Eigenvalue
0	1.0000	1.0000	1.0000	
1	0.4000	1.0000	0.3000	−20.0000
2	1.0000	0.7447	0.0284	−14.1000
3	0.5244	1.0000	−0.3683	−19.9504
4	1.0000	0.7168	−0.5303	−18.5293
5	0.6814	1.0000	−0.7423	−20.3976

11. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, which are eigenvectors corresponding to $\lambda = 1$ and $\lambda = 2$, not $\lambda = 3$. Thus, the power method converges to $\lambda = 2$.

12. There is no single dominant eigenvalue. Here, $|\lambda_1| = |\lambda_2| = \sqrt{34}$.

13. Shift by $\lambda = 4$. Power method on $\mathbf{A} = \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix}$ converges after three iterations to $\mu = -3$. $\lambda + \mu = 1$.

14. Shift by $\lambda = 16$. Power method on $\mathbf{A} = \begin{bmatrix} -13 & 2 & 3 \\ 2 & -10 & 6 \\ 3 & 6 & -5 \end{bmatrix}$ converges after three iterations to $\mu = -14$. $\lambda + \mu = 2$.

15.

Iteration	Eigenvector components		Eigenvalue
0	1.0000	1.0000	
1	−0.3333	1.0000	0.6000
2	1.0000	−0.7778	0.6000
3	−0.9535	1.0000	0.9556
4	1.0000	0.9904	0.9721
5	−0.9981	1.0000	0.9981

16.

Iteration	Eigenvector components		Eigenvalue
0	1.0000	−0.5000	
1	−0.8571	1.0000	0.2917
2	1.0000	−0.9615	0.3095
3	−0.9903	1.0000	0.3301
4	1.0000	0.9976	0.3317
5	−0.9994	1.0000	0.3331

17.

Iteration	Eigenvector components		Eigenvalue
0	1.0000	1.0000	
1	0.2000	1.0000	0.2778
2	−0.1892	1.0000	0.4111
3	−0.2997	1.0000	0.4760
4	−0.3258	1.0000	0.4944
5	−0.3316	1.0000	0.4987

18.

Iteration	Eigenvector components		Eigenvalue
0	1.0000	1.0000	
1	−0.2000	1.0000	0.7143
2	−0.3953	1.0000	1.2286
3	−0.4127	1.0000	1.3123
4	−0.4141	1.0000	1.3197
5	−0.4142	1.0000	1.3203

19.

Iteration	Eigenvector components			Eigenvalue
0	1.0000	1.0000	1.0000	
1	1.0000	0.4000	−0.2000	0.3125
2	1.0000	0.2703	−0.4595	0.4625
3	1.0000	0.2526	−0.4949	0.4949
4	1.0000	0.2503	−0.4994	0.4994
5	1.0000	0.2500	−0.4999	0.4999

20.

Iteration	Eigenvector components			Eigenvalue
0	1.0000	1.0000	1.0000	
1	0.3846	1.0000	0.9487	−0.1043
2	0.5004	0.7042	1.0000	−0.0969
3	0.3296	0.7720	1.0000	−0.0916
4	0.3857	0.6633	1.0000	−0.0940
5	0.3244	0.7002	1.0000	−0.0907

21.

Iteration	Eigenvector components			Eigenvalue
0	1.0000	1.0000	1.0000	
1	−0.6667	1.0000	−0.6667	−1.5000
2	−0.3636	1.0000	−0.3636	1.8333
3	−0.2963	1.0000	−0.2963	1.2273
4	−0.2712	1.0000	−0.2712	1.0926
5	−0.2602	1.0000	−0.2602	1.0424

22. Cannot construct an **LU** decomposition. Shift as explained in Problem 13.

23. Cannot solve $\mathbf{Lx}_1 = \mathbf{y}$ uniquely for $\mathbf{x}_1$ because one eigenvalue is zero. Shift as explained in Problem 13.

24. Yes, on occasion.

25. Inverse power method applied to $\mathbf{A} = \begin{bmatrix} -7 & 2 & 3 \\ 2 & -4 & 6 \\ 3 & 6 & 1 \end{bmatrix}$ converges to $\mu = 1/6$. $\lambda + 1/\mu = 10 + 6 = 16$.

26. Inverse power method applied to $\mathbf{A} = \begin{bmatrix} 27 & -17 & 7 \\ -17 & 21 & 1 \\ 7 & 1 & 11 \end{bmatrix}$ converges to $\mu = 1/3$. $\lambda + 1/\mu = -25 + 3 = -22$.

CHAPTER 7

Section 7.1

1. (a) $\begin{bmatrix} 0 & -4 & 8 \\ 0 & 4 & -8 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 8 & -16 \\ 0 & -8 & 16 \\ 0 & 0 & 0 \end{bmatrix}$; (b) $\begin{bmatrix} 57 & 78 \\ 117 & 174 \end{bmatrix}, \begin{bmatrix} 234 & 348 \\ 522 & 756 \end{bmatrix}$.

2. $p_k(\mathbf{A}) = \begin{bmatrix} p_k(\lambda_1) & 0 & 0 \\ 0 & p_k(\lambda_2) & 0 \\ 0 & 0 & p_k(\lambda_3) \end{bmatrix}$.

4. In general, $\mathbf{AB} \neq \mathbf{BA}$.

5. Yes. **6.** $\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}$. **7.** $\begin{bmatrix} 2 \\ 0 \\ 3/2 \end{bmatrix}$.

8. 2–2 element tends to ∞, so limit diverges. **9.** a, b, d, and f.

10. f. **11.** All except c. **13.** $\begin{bmatrix} e & 0 \\ 0 & e^2 \end{bmatrix}$. **14.** $\begin{bmatrix} e^{-1} & 0 \\ 0 & e^{28} \end{bmatrix}$.

15. $\begin{bmatrix} e^2 & 0 & 0 \\ 0 & e^{-2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$. **16.** $\sin(\mathbf{A}) = \begin{bmatrix} \sin(\lambda_1) & 0 & \cdots & 0 \\ 0 & \sin(\lambda_2) & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \sin(\lambda_n) \end{bmatrix}$.

17. $\begin{bmatrix} \sin(1) & 0 \\ 0 & \sin(2) \end{bmatrix}$. **18.** $\begin{bmatrix} \sin(-1) & 0 \\ 0 & \sin(28) \end{bmatrix}$.

19. $\cos \mathbf{A} = \sum_{k=0}^{\infty} \frac{(-1)^k \mathbf{A}^{2k}}{(2k)!}$, $\cos \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} \cos(1) & 0 \\ 0 & \cos(2) \end{bmatrix}$.

20. $\begin{bmatrix} \cos(2) & 0 & 0 \\ 0 & \cos(-2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Section 7.2

1. $\mathbf{A}^{-1} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$. **2.** Since $\alpha_0 = 0$, the inverse does not exist.

3. Since $\alpha_0 = 0$, the inverse does not exist.

4. $\mathbf{A}^{-1} \begin{bmatrix} -1/3 & -1/3 & 2/3 \\ -1/3 & 1/6 & 1/6 \\ 1/2 & 1/4 & -1/4 \end{bmatrix}$. **5.** $\mathbf{A}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Section 7.3

1. $\begin{aligned} 1 &= \alpha_1 + \alpha_0, \\ -1 &= -\alpha_1 + \alpha_0; \end{aligned}$ $\begin{bmatrix} -2 & 3 \\ -1 & 2 \end{bmatrix}$. **2.** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

3. $\begin{aligned} 0 &= \alpha_0, \\ -1 &= -\alpha_1 + \alpha_0; \end{aligned}$ $\begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$.

4. $\begin{aligned} 0 &= \alpha_0 \\ 1 &= -\alpha_1 + \alpha_0; \end{aligned}$ $\begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix}$. **5.** $\begin{bmatrix} -3 & 6 \\ -1 & 2 \end{bmatrix}$. **6.** $\begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix}$.

7. $$\begin{aligned} 3^{78} &= 3\alpha_1 + \alpha_0, \\ 4^{78} = 4^{78} &= 4\alpha_1 + \alpha_0; \end{aligned} \qquad \begin{bmatrix} -4^{78} + 2\left(3^{78}\right) & -4^{78} + 3^{78} \\ 2\left(4^{78}\right) - 2\left(3^{78}\right) & 2\left(4^{78}\right) - 3^{78} \end{bmatrix}.$$

8. $$\begin{bmatrix} -4^{41} + 2\left(3^{41}\right) & -4^{41} + 3^{41} \\ 2\left(4^{41}\right) - 2\left(3^{41}\right) & 2\left(4^{41}\right) - 3^{41} \end{bmatrix}.$$

9. $$\begin{aligned} 1 &= \alpha_2 + \alpha_1 + \alpha_0, \\ 1 &= \alpha_2 - \alpha_1 + \alpha_0, \\ 2^{222} &= 4\alpha_2 + 2\alpha_1 + \alpha_0; \end{aligned} \qquad \begin{bmatrix} 1 & 0 & \left(-4 + 4\left(2^{222}\right)\right)/3 \\ 0 & 1 & \left(-2 + 2\left(2^{222}\right)\right)/3 \\ 0 & 0 & 2^{222} \end{bmatrix}.$$

10. $$\begin{aligned} 3^{17} &= 9\alpha_2 + 3\alpha_1 + \alpha_0, \\ 5^{17} &= 25\alpha_2 + 5\alpha_1 + \alpha_0, \\ 10^{17} &= 100\alpha_2 + 10\alpha_1 + \alpha_0. \end{aligned}$$

11. $$\begin{aligned} 2^{25} &= 8\alpha_3 + 4\alpha_2 + 2\alpha_1 + \alpha_0, \\ (-2)^{25} &= -8\alpha_3 + 4\alpha_2 - 2\alpha_1 + \alpha_0, \\ 3^{25} &= 27\alpha_3 + 9\alpha_2 + 3\alpha_1 + \alpha_0, \\ 4^{25} &= 64\alpha_3 + 16\alpha_2 + 4\alpha_1 + \alpha_0. \end{aligned}$$

12. $$\begin{aligned} 1 &= \alpha_3 + \alpha_2 + \alpha_1 + \alpha_0, \\ (-2)^{25} &= -8\alpha_3 + 4\alpha_2 - 2\alpha_1 + \alpha_0, \\ 3^{25} &= 27\alpha_3 + 9\alpha_2 + 3\alpha_1 + \alpha_0, \\ \left(-4^{25}\right) &= -64\alpha_3 + 16\alpha_2 - 4\alpha_1 + \alpha_0. \end{aligned}$$

13. $$\begin{aligned} 1 &= \alpha_4 + \alpha_3 + \alpha_2 + \alpha_1 + \alpha_0, \\ 1 &= \alpha_4 - \alpha_3 + \alpha_2 - \alpha_1 + \alpha_0, \\ 256 &= 16\alpha_4 + 8\alpha_3 + 4\alpha_2 + 2\alpha_1 + \alpha_0, \\ 256 &= 16\alpha_4 - 8\alpha_3 + 4\alpha_2 - 2\alpha_1 + \alpha_0, \\ 6{,}561 &= 81\alpha_4 + 27\alpha_3 + 9\alpha_2 + 3\alpha_1 + \alpha_0. \end{aligned}$$

14. $$\begin{aligned} 5{,}837 &= 9\alpha_2 + 3\alpha_1 + \alpha_0, \\ 381{,}255 &= 25\alpha_2 + 5\alpha_1 + \alpha_0, \\ 10^8 - 3\,(10)^5 + 5 &= 100\alpha_2 + 10\alpha_1 + \alpha_0. \end{aligned}$$

15. $$\begin{aligned} 165 &= 8\alpha_3 + 4\alpha_2 + 2\alpha_1 + \alpha_0, \\ 357 &= -8\alpha_3 + 4\alpha_2 - 2\alpha_1 + \alpha_0, \\ 5{,}837 &= 27\alpha_3 + 9\alpha_2 + 3\alpha_1 + \alpha_0, \\ 62{,}469 &= 64\alpha_3 + 16\alpha_2 + 4\alpha_1 + \alpha_0. \end{aligned}$$

16. $$\begin{aligned} 3 &= \alpha_3 + \alpha_2 + \alpha_1 + \alpha_0, \\ 357 &= -8\alpha_3 + 4\alpha_2 - 2\alpha_1 + \alpha_0, \\ 5{,}837 &= 27\alpha_3 + 9\alpha_2 + 3\alpha_1 + \alpha_0, \\ 68{,}613 &= -64\alpha_3 + 16\alpha_2 - 4\alpha_1 + \alpha_0. \end{aligned}$$

17. $$\begin{aligned} 15 &= \alpha_3 + \alpha_2 + \alpha_1 + \alpha_0, \\ 960 &= -8\alpha_3 + 4\alpha_2 - 2\alpha_1 + \alpha_0, \\ 59{,}235 &= 27\alpha_3 + 9\alpha_2 + 3\alpha_1 + \alpha_0, \\ 1{,}048{,}160 &= -64\alpha_3 + 16\alpha_2 - 4\alpha_1 + \alpha_0. \end{aligned}$$

18.
$$\begin{aligned} 15 &= \alpha_4 + \alpha_3 + \alpha_2 + \alpha_1 + \alpha_0, \\ -13 &= \alpha_4 - \alpha_3 + \alpha_2 - \alpha_1 + \alpha_0, \\ 1{,}088 &= 16\alpha_4 + 8\alpha_3 + 4\alpha_2 + 2\alpha_1 + \alpha_0, \\ 960 &= 16\alpha_4 - 8\alpha_3 + 4\alpha_2 - 2\alpha_1 + \alpha_0, \\ 59{,}235 &= 81\alpha_4 + 27\alpha_3 + 9\alpha_2 + 3\alpha_1 + \alpha_0. \end{aligned}$$

19. $\begin{bmatrix} 9 & -9 \\ 3 & -3 \end{bmatrix}$. **20.** $\begin{bmatrix} 6 & -9 \\ 3 & -6 \end{bmatrix}$. **21.** $\begin{bmatrix} -50{,}801 & -56{,}632 \\ 113{,}264 & 119{,}095 \end{bmatrix}$.

22. $\begin{bmatrix} 3{,}007 & -5{,}120 \\ 1{,}024 & -3{,}067 \end{bmatrix}$. **23.** $\begin{bmatrix} 938 & 160 \\ -32 & 1130 \end{bmatrix}$. **24.** $\begin{bmatrix} 2 & -4 & -3 \\ 0 & 0 & 0 \\ 1 & -5 & -2 \end{bmatrix}$.

25.
$$\begin{aligned} 2{,}569 &= 4\alpha_2 + 2\alpha_1 + \alpha_0, \\ 5{,}633 &= 4\alpha_2 - 2\alpha_1 + \alpha_0, \\ 5 &= \alpha_2 + \alpha_1 + \alpha_0. \end{aligned} \qquad \begin{bmatrix} -339 & -766 & 1110 \\ -4440 & 4101 & 344 \\ -1376 & -3064 & 4445 \end{bmatrix}.$$

26.
$$\begin{aligned} 0.814453 &= 0.25\alpha_2 + 0.5\alpha_1 + \alpha_0, \\ 0.810547 &= 0.25\alpha_2 - 0.5\alpha_1 + \alpha_0, \\ 0.988285 &= 0.0625\alpha_2 + 0.25\alpha_1 + \alpha_0. \end{aligned} \qquad \begin{bmatrix} 1.045578 & 0.003906 & -0.932312 \\ 0.058270 & 0.812500 & -0.229172 \\ 0.014323 & 0.000977 & 0.755207 \end{bmatrix}.$$

Section 7.4

1.
$$\begin{aligned} 128 &= 2\alpha_1 + \alpha_0, \\ 448 &= \alpha_1. \end{aligned}$$

2.
$$\begin{aligned} 128 &= 4\alpha_2 + 2\alpha_1 + \alpha_0, \\ 448 &= 4\alpha_2 + \alpha_1, \\ 1{,}344 &= 2\alpha_2. \end{aligned}$$

3.
$$\begin{aligned} 128 &= 4\alpha_2 + 2\alpha_1 + \alpha_0, \\ 448 &= 4\alpha_2 + \alpha_1, \\ 1 &= \alpha_2 + \alpha_1 + \alpha_0. \end{aligned}$$

4.
$$\begin{aligned} 59{,}049 &= 3\alpha_1 + \alpha_0, \\ 196{,}830 &= \alpha_1. \end{aligned}$$

5.
$$\begin{aligned} 59{,}049 &= 9\alpha_2 + 3\alpha_1 + \alpha_0, \\ 196{,}830 &= 6\alpha_2 + \alpha_1, \\ 590{,}490 &= 2\alpha_2. \end{aligned}$$

6.
$$\begin{aligned} 59{,}049 &= 27\alpha_3 + 9\alpha_2 + 3\alpha_1 + \alpha_0, \\ 196{,}830 &= 27\alpha_3 + 6\alpha_2 + \alpha_1, \\ 590{,}490 &= 18\alpha_3 + 2\alpha_2, \\ 1{,}574{,}640 &= 6\alpha_3. \end{aligned}$$

7.
$$\begin{aligned} 512 &= 8\alpha_3 + 4\alpha_2 + 2\alpha_1 + \alpha_0, \\ 2{,}304 &= 12\alpha_3 + 4\alpha_2 + \alpha_1, \\ 9{,}216 &= 12\alpha_3 + 2\alpha_2, \\ 32{,}256 &= 6\alpha_3. \end{aligned}$$

8.
$$\begin{aligned} 512 &= 8\alpha_3 + 4\alpha_2 + 2\alpha_1 + \alpha_0, \\ 2{,}304 &= 12\alpha_3 + 4\alpha_2 + \alpha_1, \\ 9{,}216 &= 12\alpha_3 + 2\alpha_2, \\ 1 &= \alpha_3 + \alpha_2 + \alpha_1 + \alpha_0. \end{aligned}$$

9.
$$\begin{aligned} 512 &= 8\alpha_3 + 4\alpha_2 + 2\alpha_1 + \alpha_0, \\ 2{,}304 &= 12\alpha_3 + 4\alpha_2 + \alpha_1, \\ 1 &= \alpha_3 + \alpha_2 + \alpha_1 + \alpha_0. \\ 9 &= 3\alpha_3 + 2\alpha_2 + \alpha_1. \end{aligned}$$

10.
$$\begin{aligned}
(5)^{10} - 3(5)^5 &= \alpha_5(5)^5 + \alpha_4(5)^4 + \alpha_3(5)^3 + \alpha_2(5)^2 + \alpha_1(5) + \alpha_0,\\
10(5)^9 - 15(5)^4 &= 5\alpha_5(5)^4 + 4\alpha_4(5)^3 + 3\alpha_3(5)^2 + 2\alpha_2(5) + \alpha_1,\\
90(5)^8 - 60(5)^3 &= 20\alpha_5(5)^3 + 12\alpha_4(5)^2 + 6\alpha_3(5) + 2\alpha_2,\\
720(5)^7 - 180(5)^2 &= 60\alpha_5(5)^2 + 24\alpha_4(5) + 6\alpha_3,\\
(2)^{10} - 3(2)^5 &= \alpha_5(2)^5 + \alpha_4(2)^4 + \alpha_3(2)^3 + \alpha_2(2)^2 + \alpha_1(2) + \alpha_0,\\
10(2)^9 - 15(2)^4 &= 5\alpha_5(2)^4 + 4\alpha_4(2)^3 + 3\alpha_3(2)^2 + 2\alpha_2(2) + \alpha_1.
\end{aligned}$$

11. $\begin{bmatrix} 729 & 0 \\ 0 & 729 \end{bmatrix}$. **12.** $\begin{bmatrix} 4 & 1 & -3 \\ 0 & -1 & 0 \\ 5 & 1 & -4 \end{bmatrix}$. **13.** $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

Section 7.5

1. $e = \alpha_1 + \alpha_0$, $e^2 = 2\alpha_1 + \alpha_0$.

2. $e^2 = 2\alpha_1 + \alpha_0$, $e^2 = \alpha_1$.

3. $e^2 = 4\alpha_2 + 2\alpha_1 + \alpha_0$, $e^2 = 4\alpha_2 + \alpha_1$, $e^2 = 2\alpha_2$.

4.
$$\begin{aligned}
e^1 &= \alpha_2 + \alpha_1 + \alpha_0,\\
e^{-2} &= 4\alpha_2 - 2\alpha_1 + \alpha_0,\\
e^3 &= 9\alpha_2 + 3\alpha_1 + \alpha_0.
\end{aligned}$$

5.
$$\begin{aligned}
e^{-2} &= 4\alpha_2 - 2\alpha_1 + \alpha_0,\\
e^{-2} &= -4\alpha_2 + \alpha_1,\\
e^1 &= \alpha_2 + \alpha_1 + \alpha_0.
\end{aligned}$$

6.
$$\begin{aligned}
\sin(1) &= \alpha_2 + \alpha_1 + \alpha_0,\\
\sin(2) &= 4\alpha_2 + 2\alpha_1 + \alpha_0,\\
\sin(3) &= 9\alpha_2 + 3\alpha_1 + \alpha_0.
\end{aligned}$$

7.
$$\begin{aligned}
\sin(-2) &= 4\alpha_2 - 2\alpha_1 + \alpha_0,\\
\cos(-2) &= -4\alpha_2 + \alpha_1,\\
\sin(1) &= \alpha_2 + \alpha_1 + \alpha_0.
\end{aligned}$$

8.
$$\begin{aligned}
e^2 &= 8\alpha_3 + 4\alpha_2 + 2\alpha_1 + \alpha_0,\\
e^2 &= 12\alpha_3 + 4\alpha_2 + \alpha_1,\\
e^2 &= 12\alpha_3 + 2\alpha_2,\\
e^2 &= 6\alpha_3.
\end{aligned}$$

9.
$$\begin{aligned}
e^2 &= 8\alpha_3 + 4\alpha_2 + 2\alpha_1 + \alpha_0,\\
e^2 &= 12\alpha_3 + 4\alpha_2 + \alpha_1,\\
e^{-2} &= -8\alpha_3 + 4\alpha_2 - 2\alpha_1 + \alpha_0,\\
e^{-2} &= 12\alpha_3 - 4\alpha_2 + \alpha_1.
\end{aligned}$$

10.
$$\begin{aligned}
\sin(2) &= 8\alpha_3 + 4\alpha_2 + 2\alpha_1 + \alpha_0,\\
\cos(2) &= 12\alpha_3 + 4\alpha_2 + \alpha_1,\\
\sin(-2) &= -8\alpha_3 + 4\alpha_2 - 2\alpha_1 + \alpha_0,\\
\cos(-2) &= 12\alpha_3 - 4\alpha_2 + \alpha_1.
\end{aligned}$$

11.
$$\begin{aligned}
e^3 &= 27\alpha_3 + 9\alpha_2 + 3\alpha_1 + \alpha_0,\\
e^3 &= 27\alpha_3 + 6\alpha_2 + \alpha_1,\\
e^3 &= 18\alpha_3 + 2\alpha_2,\\
e^{-1} &= -\alpha_3 + \alpha_2 - \alpha_1 + \alpha_0.
\end{aligned}$$

12.
$$\begin{aligned}
\cos(3) &= 27\alpha_3 + 9\alpha_2 + 3\alpha_1 + \alpha_0,\\
-\sin(3) &= 27\alpha_3 + 6\alpha_2 + \alpha_1,\\
-\cos(3) &= 18\alpha_3 + 2\alpha_2,\\
\cos(-1) &= -\alpha_3 + \alpha_2 - \alpha_1 + \alpha_0.
\end{aligned}$$

13. $\frac{1}{7}\begin{bmatrix} 3e^5+4e^{-2} & 3e^5-3e^{-2} \\ 4e^5-4e^{-2} & 4e^5+3e^{-2} \end{bmatrix}$. **14.** $e^3\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$.

15. $e^2\begin{bmatrix} 0 & 1 & 3 \\ -1 & 2 & 5 \\ 0 & 0 & 1 \end{bmatrix}$. **16.** $\frac{1}{16}\begin{bmatrix} 12e^2+4e^{-2} & 4e^2-4e^{-2} & 38e^2+2e^{-2} \\ 12e^2-12e^{-2} & 4e^2+12e^{-2} & 46e^2-6e^{-2} \\ 0 & 0 & 16e^2 \end{bmatrix}$.

17. $\frac{1}{5}\begin{bmatrix} -1 & 6 \\ 4 & 1 \end{bmatrix}$.

18. (a) $\begin{bmatrix} \log(3/2) & \log(3/2)-\log(1/2) \\ 0 & \log(1/2) \end{bmatrix}$.

(b) and (c) are not defined since they possess eigenvalues having absolute value greater than 1.

(d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Section 7.6

1. $1/7\begin{bmatrix} 3e^{8t}+4^{et} & 4e^{8t}-4e^{t} \\ 3e^{8t}-3e^{t} & 4e^{8t}+3e^{t} \end{bmatrix}$.

2. $\begin{bmatrix} (2/\sqrt{3})\sinh\sqrt{3}t+\cosh\sqrt{3}t & (1/\sqrt{3})\sinh\sqrt{3}t \\ (-1/\sqrt{3})\sinh\sqrt{3}t & (-2/\sqrt{3})\sinh\sqrt{3}t+\cosh\sqrt{3}t \end{bmatrix}$.

Note:

$\sinh\sqrt{3}t = \dfrac{e^{\sqrt{3}t}-e^{-\sqrt{3}t}}{2}$ and $\cosh\sqrt{3}t = \dfrac{e^{\sqrt{3}t}+e^{-\sqrt{3}t}}{2}$.

3. $e^{3t}\begin{bmatrix} 1+t & t \\ -t & 1-t \end{bmatrix}$. **4.** $\begin{bmatrix} 1.4e^{-2t}-0.4e^{-7t} & 0.2e^{-2t}-0.2e^{-7t} \\ -2.8e^{-2t}+2.8e^{-7t} & -0.4e^{-2t}+1.4e^{-7t} \end{bmatrix}$.

5. $\begin{bmatrix} 0.8e^{-2t}+0.2e^{-7t} & 0.4e^{-2t}-0.4e^{-7t} \\ 0.4e^{-2t}-0.4e^{-7t} & 0.2e^{-2t}+0.8e^{-7t} \end{bmatrix}$.

6. $\begin{bmatrix} 0.5e^{-4t}+0.5e^{-16t} & 0.5e^{-4t}-0.5e^{-16t} \\ 0.5e^{-4t}-0.5e^{-16t} & 0.5e^{-4t}+0.5e^{-16t} \end{bmatrix}$.

7. $\begin{bmatrix} 1 & t & t^2/2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix}$.

8. $$\frac{1}{12}\begin{bmatrix} 12e^{t} & 0 & 0 \\ -9e^{t}+14e^{3t}-5e^{-3t} & 8e^{3t}+4e^{-3t} & 4e^{3t}-4e^{-3t} \\ -24e^{t}+14e^{3t}+10e^{-3t} & 8e^{3t}-8e^{-3t} & 4e^{3t}+8e^{-3t} \end{bmatrix}.$$

Section 7.7

1. $$\begin{bmatrix} (1/2)\sin 2t+\cos 2t & (-1/2)\sin 2t \\ (5/2)\sin 2t & (-1/2)\sin 2t+\cos 2t \end{bmatrix}.$$

2. $$\begin{bmatrix} \sqrt{2}\sin\sqrt{2}t+\cos\sqrt{2}t & -\sqrt{2}\sin\sqrt{2}t \\ (3/\sqrt{2})\sin\sqrt{2}t & -\sqrt{2}\sin\sqrt{2}t+\cos\sqrt{2}t \end{bmatrix}.$$

3. $$\begin{bmatrix} \cos(8t) & \frac{1}{8}\sin(8t) \\ -8\sin(8t) & \cos(8t) \end{bmatrix}.$$

4. $$\frac{1}{4}\begin{bmatrix} 2\sin(8t)+4\cos(8t) & -4\sin(8t) \\ 5\sin(8t) & -2\sin(8t)+4\cos(8t) \end{bmatrix}.$$

5. $$\begin{bmatrix} 2\sin(t)+\cos(t) & 5\sin(t) \\ -\sin(t) & -2\sin(t)+\cos(t) \end{bmatrix}.$$

6. $$\frac{1}{3}e^{-4t}\begin{bmatrix} 4\sin(3t)+3\cos(3t) & \sin(3t) \\ -25\sin(3t) & -4\sin(3t)+3\cos(3t) \end{bmatrix}.$$

7. $$e^{4t}\begin{bmatrix} -\sin t+\cos t & \sin t \\ -2\sin t & \sin t+\cos t \end{bmatrix}.$$

8. $$\begin{bmatrix} 1 & -2+2\cos(t)+\sin(t) & -5+5\cos(t) \\ 0 & \cos(t)-2\sin(t) & -5\sin(t) \\ 0 & \sin(t) & \cos(t)+2\sin(t) \end{bmatrix}.$$

Section 7.8

3. $\mathbf{A}$ does not have an inverse.

8. $$e^{\mathbf{A}}=\begin{bmatrix} e & e-1 \\ 0 & 1 \end{bmatrix},\quad e^{\mathbf{B}}=\begin{bmatrix} 1 & e-1 \\ 0 & 1 \end{bmatrix},\quad e^{\mathbf{A}}e^{\mathbf{B}}=\begin{bmatrix} e & 2e^2-2e \\ 0 & e \end{bmatrix},$$

$$e^{\mathbf{B}}e^{\mathbf{A}}=\begin{bmatrix} e & 2e-2 \\ 0 & e \end{bmatrix},\quad e^{\mathbf{A}+\mathbf{B}}=\begin{bmatrix} e & 2e \\ 0 & e \end{bmatrix}.$$

9. $\mathbf{A}=\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, $\mathbf{B}=\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$. Also see Problem 10.

11. First show that for any integer n, $(\mathbf{P}^{-1}\mathbf{B}\mathbf{P})^n=\mathbf{P}^{-1}\mathbf{B}^n\mathbf{P}$, and then use Eq. (6) directly.

Section 7.9

1. (a) $\begin{bmatrix} -\sin t & 2t \\ 2 & e^{(t-1)} \end{bmatrix}$. (b) $\begin{bmatrix} 6t^2e^{t^3} & 2t-1 & 0 \\ 2t+3 & 2\cos 2t & 1 \\ -18t\cos^2(3t^2)\sin(3t^2) & 0 & 1/t \end{bmatrix}$.

4. $\begin{bmatrix} \sin t + c_1 & \frac{1}{3}t^3 - t + c_2 \\ t^2 + c_3 & e^{(t-1)} + c_4 \end{bmatrix}$.

CHAPTER 8

Section 8.1

1. $\mathbf{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$, $\mathbf{A}(t) = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 6 \\ 7 \end{bmatrix}$, $t_0 = 0$.

2. $\mathbf{x}(t) = \begin{bmatrix} y(t) \\ z(t) \end{bmatrix}$, $\mathbf{A}(t) = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $t_0 = 0$.

3. $\mathbf{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$, $\mathbf{A}(t) = \begin{bmatrix} -3 & 3 \\ 4 & -4 \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $t_0 = 0$.

4. $\mathbf{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$, $\mathbf{A}(t) = \begin{bmatrix} 3 & 0 \\ 2 & 0 \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} t \\ t+1 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $t_0 = 0$.

5. $\mathbf{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$, $\mathbf{A}(t) = \begin{bmatrix} 3t^2 & 7 \\ 1 & t \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} 2 \\ 2t \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$, $t_0 = 1$.

6. $\mathbf{x}(t) = \begin{bmatrix} u(t) \\ v(t) \\ w(t) \end{bmatrix}$, $\mathbf{A}(t) = \begin{bmatrix} e^t & t & 1 \\ t^2 & -3 & t+1 \\ 0 & 1 & e^{t^2} \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, $t_0 = 4$.

7. $\mathbf{x}(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$, $\mathbf{A}(t) = \begin{bmatrix} 0 & 6 & 1 \\ 1 & 0 & -3 \\ 0 & -2 & 0 \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 10 \\ 10 \\ 20 \end{bmatrix}$, $t_0 = 0$.

8. $\mathbf{x}(t) = \begin{bmatrix} r(t) \\ s(t) \\ u(t) \end{bmatrix}$, $\mathbf{A}(t) = \begin{bmatrix} t^2 & -3 & -\sin t \\ 1 & -1 & 0 \\ 2 & e^t & t^2-1 \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} \sin t \\ t^2-1 \\ \cos t \end{bmatrix}$,

$\mathbf{c} = \begin{bmatrix} 4 \\ -2 \\ 5 \end{bmatrix}$, $t_0 = 1$.

9. Only (c). **10.** Only (c). **11.** Only (b).

Section 8.2

1. $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, $\mathbf{A}(t) = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$, $t_0 = 0$.

2. $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, $\mathbf{A}(t) = \begin{bmatrix} 0 & 1 \\ t & -e^t \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $t_0 = 1$.

3. $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, $\mathbf{A}(t) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} 0 \\ t^2 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$, $t_0 = 0$.

4. $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, $\mathbf{A}(t) = \begin{bmatrix} 0 & 1 \\ 3 & 2e^t \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} 0 \\ 2e^{-t} \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $t_0 = 0$.

5. $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, $\mathbf{A}(t) = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} 0 \\ e^{-t} \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, $t_0 = 1$.

6. $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$, $\mathbf{A}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/4 & 0 & -t/4 \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 2 \\ 1 \\ -205 \end{bmatrix}$,

$t_0 = -1$.

7. $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}$, $\mathbf{A}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & e^{-t} & -te^{-t} & 0 \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ e^{-t} \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 1 \\ 2 \\ \pi \\ e^3 \end{bmatrix}$,

$t_0 = 0$.

8. $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \end{bmatrix}$, $\mathbf{A}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -4 & 0 \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ t^2 - t \end{bmatrix}$,

$\mathbf{c} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}$, $t_0 = \pi$.

Section 8.3

1. $$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ y_1(t) \end{bmatrix}, \ \mathbf{A}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 2 & 4 \\ 5 & 0 & -6 \end{bmatrix}, \ \mathbf{f}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{c} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}, \ t_0 = 0.$$

2. $$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ y_1(t) \\ y_2(t) \end{bmatrix}, \ \mathbf{A}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 \end{bmatrix}, \ \mathbf{f}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{c} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 4 \end{bmatrix}, \ t_0 = 0.$$

3. $$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ y_1(t) \\ y_2(t) \end{bmatrix}, \ \mathbf{A}(t) = \begin{bmatrix} -4 & 0 & t^2 \\ 0 & 0 & 1 \\ t^2 & t & 0 \end{bmatrix}, \ \mathbf{f}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{c} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \ t_0 = 2.$$

4. $$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ y_1(t) \\ y_2(t) \end{bmatrix}, \ \mathbf{A}(t) = \begin{bmatrix} -4 & 0 & 2 \\ 0 & 0 & 1 \\ 3 & t & 0 \end{bmatrix}, \ \mathbf{f}(t) = \begin{bmatrix} t \\ 0 \\ -1 \end{bmatrix}, \ \mathbf{c} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \ t_0 = 3.$$

5. $$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ y_1(t) \\ y_2(t) \\ y_3(t) \\ y_4(t) \end{bmatrix}, \ \mathbf{A}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ t & 0 & -t & 0 & 1 & 0 \end{bmatrix}, \ \mathbf{f}(t) = \begin{bmatrix} 0 \\ -t \\ 0 \\ 0 \\ 0 \\ -e^t \end{bmatrix},$$

$$\mathbf{c} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 3 \\ 9 \\ 4 \end{bmatrix}, \ t_0 = -1.$$

6. $$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ y_1(t) \\ y_2(t) \end{bmatrix}, \ \mathbf{A}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 2 \end{bmatrix}, \ \mathbf{f}(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$\mathbf{c} = \begin{bmatrix} 21 \\ 4 \\ -5 \\ 5 \\ 7 \end{bmatrix}, \ t_0 = 0.$$

7. $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ y_1(t) \\ y_2(t) \\ z_1(t) \end{bmatrix}$, $\mathbf{A}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} -2 \\ 0 \\ -2 \\ 0 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 1 \\ 2 \\ 17 \\ 0 \end{bmatrix}$, $t_0 = \pi$.

8. $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ y_1(t) \\ y_2(t) \\ z_1(t) \\ z_2(t) \end{bmatrix}$, $\mathbf{A}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}$, $\mathbf{f}(t) = \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$,

$\mathbf{c} = \begin{bmatrix} 4 \\ -4 \\ 5 \\ -5 \\ 9 \\ -9 \end{bmatrix}$, $t_0 = 20$.

Section 8.4

3. (a) $e^{-3t} \begin{bmatrix} 1 & -t & t^2/2 \\ 0 & 1 & -t \\ 0 & 0 & 1 \end{bmatrix}$, (b) $e^{3(t-2)} \begin{bmatrix} 1 & (t-2) & (t-2)^2/2 \\ 0 & 1 & (t-2) \\ 0 & 0 & 1 \end{bmatrix}$,

(c) $e^{3(t-s)} \begin{bmatrix} 1 & (t-2) & (t-s)^2/2 \\ 0 & 1 & (t-s) \\ 0 & 0 & 1 \end{bmatrix}$,

(d) $e^{-3(t-2)} \begin{bmatrix} 1 & -(t-2) & (t-2)^2/2 \\ 0 & 1 & -(t-s) \\ 0 & 0 & 1 \end{bmatrix}$.

5. (a) $\frac{1}{6} \begin{bmatrix} 2e^{-5t} + 4e^{t} & 2e^{-5t} - 2e^{t} \\ 4e^{-5t} - 4e^{t} & 4e^{-5t} + 2e^{t} \end{bmatrix}$, (b) $\frac{1}{6} \begin{bmatrix} 2e^{-5s} + 4e^{s} & 2e^{-5s} - 2e^{s} \\ 4e^{-5s} - 4e^{s} & 4e^{-5s} + 2e^{s} \end{bmatrix}$,

(c) $\frac{1}{6} \begin{bmatrix} 2e^{5(t-3)} + 4e^{-(t-3)} & 2e^{5(t-3)} - 2e^{-(t-3)} \\ 4e^{5(t-3)} - 4e^{-(t-3)} & 4e^{5(t-3)} + 2e^{-(t-3)} \end{bmatrix}$.

6. (a) $\frac{1}{3} \begin{bmatrix} \sin 3t + 3\cos 3t & -5\sin 3t \\ 2\sin 3t & -\sin 3t + 3\cos 3t \end{bmatrix}$,

(b) $\frac{1}{3} \begin{bmatrix} \sin 3s + 3\cos 3s & -5\sin 3s \\ 2\sin 3s & -\sin 3s + 3\cos 3s \end{bmatrix}$,

(c) $\frac{1}{3}\begin{bmatrix} \sin 3(t-s) + 3\cos 3(t-s) & -5\sin 3(t-s) \\ 2\sin 3(t-s) & -\sin 3(t-s) + 3\cos 3(t-s)t \end{bmatrix}$.

7. $x(t) = 5e^{(t-2)} - 3e^{-(t-2)}$, $y(t) = 5e^{(t-2)} - e^{-(t-2)}$.

8. $x(t) = 2e^{(t-1)} - 1$, $y(t) = 2e^{(t-1)} - 1$.

9. $x(t) = k_3e^t + 3k_4e^{-t}$, $y(t) = k_3e^t + k_4e^{-t}$.

10. $x(t) = k_3e^t + 3k_4e^{-t} - 1$, $y(t) = k_3e^t + k_4e^{-t} - 1$.

11. $x(t) = \cos 2t - (1/6)\sin 2t + (1/3)\sin t$.

12. $x(t) = t^4/24 + (5/4)t^2 - (2/3)t + 3/8$.

13. $x(t) = (4/9)\,e^{2t} + \left(5/9\right) e^{-1t} - (1/3)\,te^{-1t}$

14. $x(t) = -8\cos t - 6\sin t + 8 + 6t$,
$y(t) = 4\cos t - 2\sin t - 3$.

Section 8.5

4. First show that

$$\begin{aligned}
&\mathbf{\Phi}^{\mathsf{T}}(t_1, t_0)\left[\int_{t_0}^{t_1} \mathbf{\Phi}(t_1, s)\mathbf{\Phi}^{\mathsf{T}}(t_1, s)ds\right]^{-1} \mathbf{\Phi}(t_1, t_0) \\
&= \left[\mathbf{\Phi}(t_0, t_1)\int_{t_0}^{t_1} \mathbf{\Phi}(t_1, s)\mathbf{\Phi}'(t_1, s)ds\mathbf{\Phi}^{\mathsf{T}}(t_0, t_1)\right]^{-1} \\
&= \left[\int_{t_0}^{t_1} \mathbf{\Phi}(t_0, t_1)\mathbf{\Phi}(t_1, s)[\mathbf{\Phi}(t_0, t_1)\mathbf{\Phi}(t_1, s)]^{\mathsf{T}}ds\right]^{-1}.
\end{aligned}$$

CHAPTER 9

Section 9.1

1. (a) The English alphabet: $a, b, c, \ldots x, y, z$. 26. 5/26.

(b) The 366 days designated by a 2008 Calendar, ranging from 1 January through 31 December. 366. 1/366.

(c) A list of all 43 United States Presidents. 43. 1/43.

(d) Same as (c). 43. 2/43 (Grover Cleveland was both the 22nd and 24th President).

(e) Regular deck of 52 cards. 52. 1/52.

(f) Pinochle deck of 48 cards. 48. 2/48.

(g) See Figure 9.1 of Chapter 9. 36. 1/36.

(h) Same as (g).
(i) Same as (g). 5/36.
(j) Same as (g). 2/36.
(k) Same as (g). 18/36.
(l) Same as (g). 7/36.
(m) Same as (g). 5/36.
(n) Same as (g). 12/36.
(o) Same as (n).
(p) Same as (g). 0.

2. The sample space would consist of all 216 possibilities, ranging from rolling a "3" to tossing an "18".

3. 90. **4.** 1950.

Section 9.2

1. (a) 8/52. (b) 16/52. (c) 28/52.
(d) 2/52. (e) 28/52. (f) 26/52.
(g) 39/52. (h) 48/52. (i) 36/52.

2. (a) 18/36. (b) 15/36. (c) 10/36.
(d) 30/36. (e) 26/36. (f) 1.

3. (a) 108/216. (b) 1/216. (c) 1/216.
(d) 3/216. (e) 3/216. (f) 0.
(g) 213/216. (h) 210/216. (i) 206/216.

4. 0.75. **5.** 0.4.

6. $P(A \cup B \cup C \cup D) = P(A) + P(B) + P(C) + P(D) - P(A \cap B) - P(A \cap C) - P(A \cap D) - P(B \cap C) - P(B \cap D) - P(C \cap D) + P(A \cap B \cap C) + P(A \cap B \cap D) + P(A \cap C \cap D) + P(B \cap C \cap D) - P(A \cap B \cap C \cap D)$.

Section 9.3

1. (a) 15. (b) 7. (c) 56. (d) 190.
(e) 190. (f) 1. (g) 1. (h) 100.
(i) 1000. (j) 1.

2. 2,042,975. **3.** 5005.

4. (a) Approximately .372. (b) Approximately .104.
(c) Approximately .135. (d) Approximately .969.
(e) Approximately .767.

5. (a) $\binom{500}{123}(.65)^{123}(.35)^{377}$.

(b) $\binom{500}{485}(.65)^{485}(.35)^{15}$.

(c) $$\binom{500}{497}(.65)^{497}(.35)^{3}+\binom{500}{498}(.65)^{498}(.35)^{2}+\binom{500}{499}(.65)^{499}(.35)^{1}+\binom{500}{500}(.65)^{500}(.35)^{0}.$$

(d) $1-\binom{500}{498}(.65)^{498}(.35)^{2}-\binom{500}{499}(.65)^{499}(.35)^{1}-\binom{500}{500}(.65)^{500}(.35)^{0}$.

(e) $$\binom{500}{100}(.65)^{100}(.35)^{400}+\binom{500}{200}(.65)^{200}(.35)^{300}+\binom{500}{300}(.65)^{300}(.35)^{200}+\binom{500}{400}(.65)^{400}(.35)^{100}+\binom{500}{500}(.65)^{500}(.35)^{0}.$$

6. Approximately .267.

7. Approximately .267.

Section 9.4

1. (a) There is a negative element in the second row.

(b) The first row does not add to 1.

(c) The third row does not add to 1.

(d) It is not a square matrix.

2. (a) If it is sunny today, there is a probability of .5 that it will be sunny tomorrow and a .5 probability that it will rain tomorrow. If it rains today, there is a .7 probability that it will be sunny tomorrow and a .3 chance that it will rain tomorrow.

(b) If a parking meter works today, there is a probability of .95 that it will work tomorrow with a .05 probability that it will not work tomorrow. If the parking meter is inoperative today, there is a probability of .02 that it will be fixed tomorrow and a .98 probability that it will not be fixed tomorrow.

(c) Any scenario has a "50–50" chance at any stage.

(d) What is "good" stays "good"; what is "bad" stays "bad".

(e) What is "good" today is "bad" tomorrow; what is "bad" today is "good" tomorrow.

(f) See Example 2 in Section 9.4 and use Tinker, Evers, and Chance for Moe, Curly, and Larry and instead of visiting or staying home use “borrowing a car” or “not borrowing a car”.

3. Clearly if we raise either matrix to any power, we obtain the original matrix.

4. The even powers produce $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and the odd powers give back the original matrix. And situation repeats itself after an even number of time periods.

5. $p_{11}^{(2)} = 0.4,\ p_{21}^{(2)} = 0.15,\ p_{12}^{(3)} = 0.7,\ p_{22}^{(3)} = 0.825.$

6. (a) $\begin{bmatrix} .7 & .3 \\ .4 & .6 \end{bmatrix}$

(b) Approximately 0.5725.

(c) Approximately 0.5717.

CHAPTER 10

Section 10.1

1. 11, 5. **2.** 8, 4. **3.** −50, 74. **4.** 63, 205.

5. 64, 68. **6.** 6, 5. **7.** 26, 24. **8.** −30, 38.

9. 5/6, 7/18. **10.** $5/\sqrt{6}$, 1. **11.** 7/24, 1/3. **12.** 0, 1400.

13. 2, 3. **14.** 1, 1. **15.** −19, 147. **16.** −1/5, 1/5.

17. undefined, 6. **18.** $\begin{bmatrix} 3/5 \\ 4/5 \end{bmatrix}$. **19.** $\begin{bmatrix} 4/\sqrt{41} \\ -5/\sqrt{41} \end{bmatrix}$. **20.** $\begin{bmatrix} 7/\sqrt{58} & 3/\sqrt{58} \end{bmatrix}$.

21. $\begin{bmatrix} -4/\sqrt{34} \\ 3/\sqrt{34} \\ -3/\sqrt{34} \end{bmatrix}$. **22.** $\begin{bmatrix} 3/\sqrt{17} \\ -2/\sqrt{17} \\ -2/\sqrt{17} \end{bmatrix}$. **23.** $\begin{bmatrix} 2/\sqrt{21} & 4/\sqrt{21} & 1/\sqrt{21} \end{bmatrix}$.

24. $\begin{bmatrix} 4/\sqrt{197} \\ -6/\sqrt{197} \\ -9/\sqrt{197} \\ 8/\sqrt{197} \end{bmatrix}$. **25.** $\begin{bmatrix} 1/\sqrt{55} & 2/\sqrt{55} & -3\sqrt{55} & 4/\sqrt{55} & -5/\sqrt{55} \end{bmatrix}$.

26 $\begin{bmatrix} -3/\sqrt{259} & 8/\sqrt{259} & 11/\sqrt{259} & -4/\sqrt{259} & 7/\sqrt{259} \end{bmatrix}$.

27. No vector **x** exists. **28.** Yes, see Problem 12.

33. $\|\mathbf{x}+\mathbf{y}\|^2 = \langle \mathbf{x}+\mathbf{y}, \mathbf{x}+\mathbf{y} \rangle = \langle \mathbf{x}, \mathbf{x} \rangle + 2\langle \mathbf{x}, \mathbf{y} \rangle + \langle \mathbf{y}, \mathbf{y} \rangle$
$= \|\mathbf{x}\|^2 + 2\langle \mathbf{x}, \mathbf{y} \rangle + \|\mathbf{y}\|^2.$

34. Show that $\|\mathbf{x}-\mathbf{y}\|^2 = \|\mathbf{x}\|^2 - 2\langle \mathbf{x}, \mathbf{y} \rangle + \|\mathbf{y}\|^2$, and then use Problem 33.

37. Note that $\langle \mathbf{x}, \mathbf{y} \rangle \le |\langle \mathbf{x}, \mathbf{y} \rangle|$. **38.** $\langle \mathbf{x}, \mathbf{y} \rangle = \det(\mathbf{x}^{\mathsf{T}}\mathbf{y})$.

40. 145 **41.** 27. **42.** 32.

Section 10.2

1. $\mathbf{x}$ and $\mathbf{y}$, $\mathbf{x}$ and $\mathbf{u}$, $\mathbf{y}$ and $\mathbf{v}$, $\mathbf{u}$ and $\mathbf{v}$.

2. $\mathbf{x}$ and $\mathbf{z}$, $\mathbf{x}$ and $\mathbf{u}$, $\mathbf{y}$ and $\mathbf{u}$, $\mathbf{z}$ and $\mathbf{u}$, $\mathbf{y}$ and $\mathbf{v}$. **3.** $-20/3$.

4. -4. **5.** 0.5. **6.** $x = -3y$.

7. $x = 1,\ y = -2$. **8.** $x = y = -z$. **9.** $x = y = -z;\ z = \pm 1/\sqrt{3}$.

10. $\begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}, \begin{bmatrix} 2/\sqrt{5} \\ -1/\sqrt{5} \end{bmatrix}$. **11.** $\begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}, \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$.

12. $\begin{bmatrix} 3/\sqrt{13} \\ -2/\sqrt{13} \end{bmatrix}, \begin{bmatrix} 2/\sqrt{13} \\ 3/\sqrt{13} \end{bmatrix}$. **13.** $\begin{bmatrix} 1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}, \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$.

14. $\begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \\ 0 \end{bmatrix}, \begin{bmatrix} -2/\sqrt{45} \\ 4/\sqrt{45} \\ 5/\sqrt{45} \end{bmatrix}, \begin{bmatrix} 1/3 \\ -2/3 \\ 2/3 \end{bmatrix}$. **15.** $\begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}, \begin{bmatrix} -1/\sqrt{6} \\ 1/\sqrt{6} \\ 2/\sqrt{6} \end{bmatrix}$.

16. $\begin{bmatrix} 0 \\ 3/5 \\ 4/5 \end{bmatrix}, \begin{bmatrix} 3/5 \\ 16/25 \\ -12/25 \end{bmatrix}, \begin{bmatrix} 4/5 \\ -12/25 \\ 9/25 \end{bmatrix}$.

17. $\begin{bmatrix} 0 \\ 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}, \begin{bmatrix} 3/\sqrt{15} \\ -2/\sqrt{15} \\ 1/\sqrt{15} \\ 1/\sqrt{15} \end{bmatrix}, \begin{bmatrix} 3/\sqrt{35} \\ 3/\sqrt{35} \\ -4/\sqrt{35} \\ 1/\sqrt{35} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{7} \\ 1/\sqrt{7} \\ 1/\sqrt{7} \\ -2/\sqrt{7} \end{bmatrix}$.

18. $\begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1/\sqrt{6} \\ 1/\sqrt{6} \\ -2/\sqrt{6} \\ 0 \end{bmatrix}, \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ -1/\sqrt{3} \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$.

23. $\|\mathbf{x} - \mathbf{y}\|^2 = \langle \mathbf{x} - \mathbf{y}, \mathbf{x} - \mathbf{y} \rangle = \|\mathbf{x}\|^2 - 2\langle \mathbf{x}, \mathbf{y} \rangle + \|\mathbf{y}\|^2$.

24. $\|s\mathbf{x} + t\mathbf{y}\|^2 = \langle s\mathbf{x} - t\mathbf{y},\ s\mathbf{x} - t\mathbf{y} \rangle = \|s\mathbf{x}\|^2 - 2st\langle \mathbf{x}, \mathbf{y} \rangle + \|t\mathbf{y}\|^2$.

25. $\mathbf{I}$. **26.** Set $\mathbf{y} = \mathbf{x}$ and use Property (I1) of Section 10.1.

28. Denote the columns of $\mathbf{A}$ as $\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_n$, and the elements of $\mathbf{y}$ as $y_1, y_2, \ldots, y_n$, respectively. Then, $\mathbf{A}\mathbf{y} = \mathbf{A}_1 y_1 + \mathbf{A}_2 y_2 + \cdots + \mathbf{A}_n y_n$ and $\langle \mathbf{A}\mathbf{y}, \mathbf{p} \rangle = y_1\langle \mathbf{A}_1, \mathbf{p} \rangle + y_2\langle \mathbf{A}_2, \mathbf{p} \rangle + \cdots + y_n\langle \mathbf{A}_n, \mathbf{p} \rangle$.

Section 10.3

1. (a) $\theta = 36.9°$, (b) $\begin{bmatrix} 1.6 \\ 0.8 \end{bmatrix}$, (c) $\begin{bmatrix} -0.6 \\ 1.2 \end{bmatrix}$.

2. (a) $\theta = 14.0°$, (b) $\begin{bmatrix} 0.7059 \\ 1.1765 \end{bmatrix}$, (c) $\begin{bmatrix} 0.2941 \\ -0.1765 \end{bmatrix}$.

3. (a) $\theta = 78.7°$, (b) $\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$, (c) $\begin{bmatrix} 2.5 \\ -2.5 \end{bmatrix}$.

4. (a) $\theta = 90°$, (b) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$, (c) $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$.

5. (a) $\theta = 118.5°$, (b) $\begin{bmatrix} -0.7529 \\ -3.3882 \end{bmatrix}$, (c) $\begin{bmatrix} -6.2471 \\ 1.3882 \end{bmatrix}$.

6. (a) $\theta = 50.8°$, (b) $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, (c) $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$.

7. (a) $\theta = 19.5°$, (b) $\begin{bmatrix} 8/9 \\ 8/9 \\ 4/9 \end{bmatrix}$, (c) $\begin{bmatrix} 1/9 \\ 1/9 \\ -4/9 \end{bmatrix}$.

8. (a) $\theta = 17.7°$, (b) $\begin{bmatrix} 1.2963 \\ 3.2407 \\ 3.2407 \end{bmatrix}$, (c) $\begin{bmatrix} -1.2963 \\ -0.2407 \\ 0.7593 \end{bmatrix}$.

9. (a) $\theta = 48.2°$, (b) $\begin{bmatrix} 2/3 \\ 2/3 \\ 2/3 \\ 0 \end{bmatrix}$, (c) $\begin{bmatrix} -2/3 \\ 1/3 \\ 1/3 \\ 1 \end{bmatrix}$.

10. (a) $\theta = 121.4°$, (b) $\begin{bmatrix} -7/6 \\ 7/3 \\ 0 \\ 7/6 \end{bmatrix}$, (c) $\begin{bmatrix} 13/6 \\ -1/3 \\ 3 \\ 17/6 \end{bmatrix}$.

11. $\begin{bmatrix} 0.4472 & 0.8944 \\ 0.8944 & -0.4472 \end{bmatrix} \begin{bmatrix} 2.2361 & 1.7889 \\ 0.0000 & 1.3416 \end{bmatrix}$.

12. $\begin{bmatrix} 0.7071 & -0.7071 \\ 0.7071 & 0.7071 \end{bmatrix} \begin{bmatrix} 1.4142 & 5.6569 \\ 0.0000 & 1.4142 \end{bmatrix}$.

13. $\begin{bmatrix} 0.8321 & 0.5547 \\ -0.5547 & 0.8321 \end{bmatrix} \begin{bmatrix} 3.6056 & 0.8321 \\ 0.0000 & 4.1603 \end{bmatrix}$.

14. $\begin{bmatrix} 0.3333 & 0.8085 \\ 0.6667 & 0.1617 \\ 0.6667 & -0.5659 \end{bmatrix} \begin{bmatrix} 3.0000 & 2.6667 \\ 0.0000 & 1.3744 \end{bmatrix}$.

15. $\begin{bmatrix} 0.3015 & -0.2752 \\ 0.3015 & -0.8808 \\ 0.9045 & 0.3853 \end{bmatrix} \begin{bmatrix} 3.3166 & 4.8242 \\ 0.0000 & 1.6514 \end{bmatrix}$.

16. $\begin{bmatrix} 0.7746 & 0.4034 \\ -0.5164 & 0.5714 \\ 0.2582 & 0.4706 \\ -0.2582 & 0.5378 \end{bmatrix} \begin{bmatrix} 3.8730 & 0.2582 \\ 0.0000 & 1.9833 \end{bmatrix}$.

17. $\begin{bmatrix} 0.8944 & -0.2981 & 0.3333 \\ 0.4472 & 0.5963 & -0.6667 \\ 0.0000 & 0.7454 & 0.6667 \end{bmatrix} \begin{bmatrix} 2.2361 & 0.4472 & 1.7889 \\ 0.0000 & 1.3416 & 0.8944 \\ 0.0000 & 0.0000 & 2.0000 \end{bmatrix}$.

18. $\begin{bmatrix} 0.7071 & 0.5774 & -0.4082 \\ 0.7071 & -0.5774 & 0.4082 \\ 0.0000 & 0.5774 & 0.8165 \end{bmatrix} \begin{bmatrix} 1.4142 & 1.4142 & 2.8284 \\ 0.0000 & 1.7321 & 0.5774 \\ 0.0000 & 0.0000 & 0.8165 \end{bmatrix}$.

19. $\begin{bmatrix} 0.00 & 0.60 & 0.80 \\ 0.60 & 0.64 & -0.48 \\ 0.80 & -0.48 & 0.36 \end{bmatrix} \begin{bmatrix} 5 & 3 & 7 \\ 0 & 5 & 2 \\ 0 & 0 & 1 \end{bmatrix}$.

20. $\begin{bmatrix} 0.0000 & 0.7746 & 0.5071 \\ 0.5774 & -0.5164 & 0.5071 \\ 0.5774 & 0.2582 & -0.6761 \\ 0.5774 & 0.2582 & 0.1690 \end{bmatrix} \begin{bmatrix} 1.7321 & 1.1547 & 1.1547 \\ 0.0000 & 1.2910 & 0.5164 \\ 0.0000 & 0.0000 & 1.1832 \end{bmatrix}$.

21. $\begin{bmatrix} 0.7071 & -0.4082 & 0.5774 \\ 0.7071 & 0.4082 & -0.5774 \\ 0.0000 & -0.8165 & -0.5774 \\ 0.0000 & 0.0000 & 0.0000 \end{bmatrix} \begin{bmatrix} 1.4142 & 0.7071 & 0.7071 \\ 0.0000 & 1.2247 & 0.4082 \\ 0.0000 & 0.0000 & 1.1547 \end{bmatrix}$.

24. $\mathbf{QR} \neq \mathbf{A}$.

Section 10.4

1. $\mathbf{A}_1 = \mathbf{R}_0\mathbf{Q}_0 + 7\mathbf{I}$

$$= \begin{bmatrix} 19.3132 & -1.2945 & 0.0000 \\ 0.0000 & 7.0231 & -0.9967 \\ 0.0000 & 0.0000 & 0.0811 \end{bmatrix} \begin{bmatrix} -0.3624 & 0.0756 & 0.9289 \\ 0.0000 & -0.9967 & 0.0811 \\ 0.9320 & 0.0294 & 0.3613 \end{bmatrix}$$

$$+7\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.0000 & 2.7499 & 17.8357 \\ -0.9289 & -0.0293 & 0.2095 \\ 0.0756 & 0.0024 & 7.0293 \end{bmatrix}.$$

2. $\mathbf{A}_1 = \mathbf{R}_0\mathbf{Q}_0 - 14\mathbf{I}$

$$= \begin{bmatrix} 24.3721 & -17.8483 & 3.8979 \\ 0.0000 & 8.4522 & -4.6650 \\ 0.0000 & 0.0000 & 3.6117 \end{bmatrix} \begin{bmatrix} 0.6565 & -0.6250 & 0.4223 \\ -0.6975 & -0.2898 & 0.6553 \\ 0.2872 & 0.7248 & 0.6262 \end{bmatrix}$$

$$-14 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 15.5690 & -7.2354 & 1.0373 \\ -7.2354 & -19.8307 & 2.6178 \\ 1.0373 & 2.6178 & -11.7383 \end{bmatrix}.$$

3. Shift by 4.

$$\mathbf{R}_0 = \begin{bmatrix} 4.1231 & -0.9701 & 0.0000 & 13.5820 \\ 0.0000 & 4.0073 & -0.9982 & -4.1982 \\ 0.0000 & 0.0000 & 4.0005 & 12.9509 \\ 0.0000 & 0.0000 & 0.0000 & 3.3435 \end{bmatrix},$$

$$\mathbf{Q}_0 = \begin{bmatrix} -0.9701 & -0.2349 & -0.0586 & -0.0151 \\ 0.2425 & -0.9395 & -0.2344 & -0.0605 \\ 0.0000 & 0.2495 & -0.9376 & -0.2421 \\ 0.0000 & 0.0000 & 0.2500 & -0.9683 \end{bmatrix}.$$

$$\mathbf{A}_1 = \mathbf{R}_0\mathbf{Q}_0 + 4\mathbf{I} = \begin{bmatrix} -0.2353 & -0.0570 & 3.3809 & -13.1545 \\ 0.9719 & -0.0138 & -1.0529 & 4.0640 \\ 0.0000 & 0.9983 & 3.4864 & -13.5081 \\ 0.0000 & 0.0000 & 0.8358 & 0.7626 \end{bmatrix}.$$

4. $7.2077,\ -0.1039 \pm 1.5769i$. **5.** $-11,\ -22,\ 17$. **6.** $2, 3, 9$.

7. Method fails. $\mathrm{A}_0 - 7\mathbf{I}$ does not have linearly independent columns, so no **QR**-decomposition is possible.

8. $2, 2, 16$. **9.** $1, 3, 3$. **10.** $2,\ 3 \pm i$. **11.** $1,\ \pm i$.

12. $\pm i,\ 2 \pm 3i$. **13.** $3.1265 \pm 1.2638i,\ -2.6265 \pm 0.7590i$.

14. $0.0102, 0.8431, 3.8581, 30.887$.

Section 10.5

1. $x = 2.225,\ y = 1.464$. **2.** $x = 3.171,\ y = 2.286$.

3. $x = 9.879,\ y = 18.398$. **4.** $x = -1.174,\ y = 8.105$.

5. $x = 1.512,\ y = 0.639,\ z = 0.945$. **6.** $x = 7.845,\ y = 1.548,\ z = 5.190$.

7. $x = 81.003,\ y = 50.870,\ z = 38.801$. **8.** $x = 2.818,\ y = -0.364,\ z = -1.364$.

9. 2 and 4. **10.** (b) $y = 2.3x + 8.1$, (c) 21.9.

11. (b) $y = -2.6x + 54.4$, (c) 31 in week 9, 28 in week 10.

12. (b) $y = 0.27x + 10.24$, (c) 12.4.

13. $$m = \frac{N \sum_{i=1}^{N} x_i y_i - \sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i}{N \sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)^2}, \quad c = \frac{\sum_{i=1}^{N} y_i \sum_{i=1}^{N} x_i^2 - \sum_{i=1}^{N} x_i \sum_{i=1}^{N} x_i y_i}{N \sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)}.$$

If $N \sum_{i=1}^{N} x_i^2$ is near $\left(\sum_{i=1}^{N} x_i\right)^2$, then the denominator is near zero.

14. $\sum_{i=1}^{N} x_i' = 0$, so the denominator for m and c as suggested in Problem 13 is simply $N \sum_{i=1}^{N} \left(x_i'\right)^2$.

15. $y = 2.3x' + 15$.

16. $y = -2.6x' + 42.9$.

17. (a) $y = -0.198x' + 21.18$, (b) Year 2000 is coded as $x' = 30$; $y(30) = 15.2$.

23. $\mathbf{E} = \begin{bmatrix} 0.841 \\ 0.210 \\ -2.312 \end{bmatrix}$.

24. $\mathbf{E} = \begin{bmatrix} 0.160 \\ 0.069 \\ -0.042 \\ -0.173 \end{bmatrix}$.

This page intentionally left blank