

Math 4620/5620

Review Problems on Modeling

1. Consider the following facility location problem. You have to decide where to locate a fire-station in a new city. Suppose there are n important objects in the city, and you should provide they are quickly reachable from the fire-station. The locations of those n objects are given by Euclidean coordinates: $(a_1, b_1), (a_2, b_2), \dots, (a_n, b_n)$. The time to get from any point (x, y) to any point (u, v) is $|x - u| + |y - v|$ (that is, we are given rectilinear distances). You should decide where to locate the fire-station so that the sum of the times to get to n important objects is minimized. Give a linear program which solves this problem.

2. Oilco has oil fields in San Diego and Los Angeles. The San Diego field can produce 500,000 barrels per day, and the Los Angeles field can produce 400,000 barrels per day. Oil is sent from the field to a refinery, in either Dallas or Houston (assume each refinery has unlimited capacity). To refine 100,000 barrels costs \$700 at Dallas and \$900 at Houston. Refined oil is shipped to customers in Chicago and New York. Chicago customers require 400,000 barrels per day, and New York customers require 300,000 barrels per day. The costs of shipping 100,000 barrels of oil (refined or unrefined) between cities are shown in the table:

| From | To (\$) | | | |
|-------------|---------|---------|----------|---------|
| | Dallas | Houston | New York | Chicago |
| Los Angeles | 300 | 110 | | |
| San Diego | 420 | 100 | | |
| Dallas | | | 450 | 550 |
| Houston | | | 470 | 530 |

- a) Formulate a Min-Cost Flow problem that can be used to determine how to minimize the total cost of meeting all demands.
- b) If each refinery had a capacity of 500,000 barrels per day, how would the part (a) answer be modified.

3. You are given the following payoff table for a two-player game.

| | | Column player | | |
|------------|----|---------------|----|----|
| | | C1 | C2 | C3 |
| Row player | R1 | 90 | 70 | 20 |
| | R2 | 40 | 50 | 80 |
| | R3 | 70 | 80 | 30 |

Formulate the problem of finding optimal strategies as a pair of linear programs.