Last time

- Revised Simplex
- Abstract view of Simplex

Let's continue with the abstract view.

Question from last time: Can we get the Simplex tableau corresponding to the current basis without doing simplex iterations.

Answer: Yes → see the tableau

Basic var.	X_B	X_N	RHS
Z	0	$C_B^T B^{-1} N - C_N^T$	$C_B^T B^{-1} b$
X_B	I	$B^{-1} N$	$B^{-1} b$

$(*)$

In this tableau, the order of columns corresponding to basic and nonbasic variables is changed from the original system.

Another Question: Suppose we have a current tableau as the result of several Simplex iterations (the order of columns is the same as in the original system). Can we read B^{-1} directly from current tableau?

- The original system (without changing column order):

$$\begin{aligned} \max\ & c^T x \\ \text{st}\ & Ax \le b \\ & x \ge 0 \end{aligned} \quad \longleftrightarrow \quad \begin{aligned} \max\ & c^T x \\ \text{st}\ & Ax + x_s = b \\ & x, x_s \ge 0 \end{aligned}$$

Initial tableau:

	x	x_s	RHS
Z	$-c^T$	0	0
x_s	A	I	b

- To get the current tableau, we need to premultiply the functional constraints by B^{-1} (as we did in last lecture).
- Then the current tableau (except row 0):

Bas. var.	x	x_s	RHS
x_B	$B^{-1}A$	B^{-1}	$B^{-1}b$

(**)

Answer to the Question: The columns of slack variables in the current tableau reveal B^{-1} for the current basis.

("fundamental insight" from H&L)

Example:

$$\begin{aligned} \max \quad & 4x_1 + 3x_2 + x_3 + 2x_4 \\ \text{st} \quad & 4x_1 + 2x_2 + x_3 + x_4 \le 5 \\ & 3x_1 + x_2 + 2x_3 + x_4 \le 4 \\ & x_1, x_2, x_3, x_4 \ge 0 \end{aligned}$$

- Suppose the current basis is $\{x_2, x_4\}$.
- First get the current tableau (except row 0) as in $(*)$:

$$B = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}, \quad B^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$B^{-1}N = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 4 & 1 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 & -1 \\ 2 & 3 & -1 & 2 \end{pmatrix}$$

$$B^{-1}b = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 5 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

	x_B		x_N				
	x_2	x_4	x_1	x_3	x_5	x_6	RHS
Z	–	–	–	–	–	–	–
x_2	1	0	1	−1	1	−1	1
x_4	0	1	2	3	−1	2	3

$(*)$

- After rearranging the columns to get the original order:

	X				X_s		
	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	–	–	–	–	–	–	–
x_2	1	1	-1	0	1	-1	1
x_4	2	0	3	1	-1	2	3

(**)

is B^{-1}

Example: max $x_1 - x_2 + 2x_3$

$$
\begin{aligned}
\text{st}\quad & 2x_1 - 2x_2 + 3x_3 \le 5 \\
& x_1 + x_2 - x_3 \le 3 \\
& x_1 - x_2 + x_3 \le 2 \\
& x_1, x_2, x_3 \ge 0
\end{aligned}
$$

A portion of final tableau given:

	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z				1	1	0	
x_2				1	3	0	
x_6				0	1	1	
x_3				1	2	0	

- Recover the rest of the tableau.

Ex. (cont.)

- Based on fundamental insight,

$$B^{-1} = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}$$

- Use the expressions in (*).

$$A_1^{cur.} = B^{-1} \cdot A_1^{in.} = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix}$$

column of x_1 in current tableau

column of x_1 in initial tableau

$$A_2^{cur.} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad A_3^{cur.} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$ (basic variables)

RHS: $$B^{-1} b = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 14 \\ 5 \\ 11 \end{pmatrix}$$

Obj. f-n value: $$C_B^T B^{-1} b = \begin{pmatrix} -1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 14 \\ 5 \\ 11 \end{pmatrix} = 8$$

coefficients in row 0

$$C_1^{cur.} = \begin{pmatrix} -1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix} - 1 = 2$$

$$C_2^{cur.} = C_3^{cur.} = 0$$ (basic variables)

▼ Summarizing, the recovered tableau is:

	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	2	0	0	1	1	0	8
x_2	5	1	0	1	3	0	14
x_6	2	0	0	0	1	1	5
x_3	4	0	1	1	2	0	11