Math 4620/5620

Handout 1 Solutions

Problem 1. Portfolio selection.

National Insurance Associates carries an investment portfolio of a variety of stocks, bonds, and other investment alternatives. Currently $200,000 of funds have become available and must be considered for new investment opportunities. The four stock options National is considering and the relevant financial data are as follows:

	Investment Alternative			
	A	B	C	D
Price per share	$100	$50	$80	$40
Projected annual rate of return	0.12	0.08	0.06	0.10
Risk measure per dollar invested (*Higher values indicate greater risk*)	0.10	0.07	0.05	0.08

The risk measure provided by the firm's top financial advisor, indicates the relative uncertainty associated with the stock regarding the realization of its projected annual rate of return.

National's top management has stipulated the following investment guidelines:

1. The projected annual rate of return for the portfolio must be at least 9%.
2. No one stock can count for more than 50% of the total dollar investment.

(a) For this overall situation, develop a linear programming model to yield an investment portfolio which minimizes total risk.

(b) Revise the model in (a) to ignore risk and maximize projected return on investment.

Solution

Define the following decision variables. For j=A,B,C,D, let
X_j=number of shares invested in alternative j.

Then the total investment can't exceed $200,000:

$$100X_A + 50X_B + 80X_C + 40X_D \le 200{,}000$$

The constraint providing that the annual rate of return must be at least 9% is:

$$0.12*100X_A + 0.08*50X_B + 0.06*80X_C + 0.1*40X_D \ge 0.09*200{,}000$$

No one stock can count for more than 50% of the total dollar investment. The corresponding constraints are:

$$100X_A \le 0.5*(100X_A + 50X_B + 80X_C + 40X_D)$$
$$50X_B \le 0.5*(100X_A + 50X_B + 80X_C + 40X_D)$$
$$80X_C \le 0.5*(100X_A + 50X_B + 80X_C + 40X_D)$$
$$40X_D \le 0.5*(100X_A + 50X_B + 80X_C + 40X_D)$$

a) The total risk is $0.1*100X_A + 0.07*50X_B + 0.05*80X_C + 0.08*40X_D$.
Thus, the LP model for this case is:

Minimize $0.1*100X_A + 0.07*50X_B + 0.05*80X_C + 0.08*40X_D$
Subject to $100X_A + 50X_B + 80X_C + 40X_D \le 200{,}000$
$0.12*100X_A + 0.08*50X_B + 0.06*80X_C + 0.1*40X_D \ge 0.09*200{,}000$
$100X_A \le 0.5*(100X_A + 50X_B + 80X_C + 40X_D)$
$50X_B \le 0.5*(100X_A + 50X_B + 80X_C + 40X_D)$
$80X_C \le 0.5*(100X_A + 50X_B + 80X_C + 40X_D)$
$40X_D \le 0.5*(100X_A + 50X_B + 80X_C + 40X_D)$
$X_A , X_B , X_C , X_D \ge 0$

b) The projected return on investment is
$0.12*100X_A + 0.08*50X_B + 0.06*80X_C + 0.1*40X_D$.
Thus, the IP model for this case is:

Maximize $0.12*100X_A + 0.08*50X_B + 0.06*80X_C + 0.1*40X_D$
Subject to $100X_A + 50X_B + 80X_C + 40X_D \le 200{,}000$
$0.12*100X_A + 0.08*50X_B + 0.06*80X_C + 0.1*40X_D \ge 0.09*200{,}000$
$100X_A \le 0.5*(100X_A + 50X_B + 80X_C + 40X_D)$
$50X_B \le 0.5*(100X_A + 50X_B + 80X_C + 40X_D)$
$80X_C \le 0.5*(100X_A + 50X_B + 80X_C + 40X_D)$
$40X_D \le 0.5*(100X_A + 50X_B + 80X_C + 40X_D)$
$X_A , X_B , X_C , X_D \ge 0$