Math 4620/5620

Examples of Modeling Real-life Situations as Linear Programs

1. Linear regression.

A very common and important problem in statistics is linear regression, the problem of fitting a straight line to statistical data. The most commonly employed technique is the method of least squares, but there are other interesting criteria where linear programming can be used to solve for the optimal values of the regression parameters.

Let $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ be data points and a_1 and a_0 be the parameters of the regression line $y=a_1x+a_0$.

(a) Formulate a linear program whose optimal solution minimizes the sum of the absolute deviations of the data from the line, i.e., formulate

$$\min_a \sum_{i=1}^{n} \left| y_i - \left(a_1 x_i + a_0\right) \right|$$

as an LP.

(b) Formulate the minimization of the maximum absolute deviation as an LP, i.e., formulate

$$\min_a \max_i \left| y_i - (a_1 x_i + a_0) \right|$$

as an LP.

(c) Generalize the model to allow fitting to general polynomials $y = a_k x^k + a_{k-1} x^{k-1} + \ldots + a_1 x + a_0$.

2. Multi-period Production Scheduling.

(a) A manufacturer wishes to schedule production for six months in advance to meet known monthly demands for a given product. The demand for month i is D_i. In month i, the manufacturer can produce at most C_i items at cost $\$p_i$ per item. The demand of the current month can be satisfied by the items produced in earlier production periods. The cost of holding an item in inventory from period i to period i+1 is $\$h_i$. Suppose there is no inventory at the beginning of the first month.

Formulate an LP which will minimize the total cost while satisfying the demands.

(b) Note that the demand of the current month can be totally satisfied by the inventory carried from the previous month. Suppose there is a setup cost $\$s_i$ for month i if the manufacturer decides to have any production for that month (and there is no setup cost if there is no production). Change your model to take the setup costs into account.