Handout 3 Solution

Problem 1. Facility Location problem.

- Define the decision variables in the following way.

  For i = NY, LA, Ch, At, let

$$W_i = \begin{cases} 1 & \textit{if a warehouse is opened in city } i \\ 0 & \textit{otherwise} \end{cases}$$

  For i=NY,LA,Ch,At, and j=1,2,3, let

  X_{ij} = number of units sent from warehouse i to region j .

- Then the total cost is

  $400W_{NY} + 500W_{LA} + 300W_{Ch} + 150W_{At}$ (fixed costs)
  $+ 20X_{NY,1} + 40X_{NY,2} + 50X_{NY,3}$ (shipping costs)
  $+ 48X_{LA,1} + 15X_{LA,2} + 26X_{LA,3}$
  $+ 26X_{Ch,1} + 35X_{Ch,2} + 18X_{Ch,3}$
  $+ 24X_{At,1} + 50X_{At,2} + 35X_{At,3}$

We need the following constraints.

- We can't send units from a warehouse unless it is open:

  $X_{NY,1} + X_{NY,2} + X_{NY,3} \le 100\, W_{NY}$
  $X_{LA,1} + X_{LA,2} + X_{LA,3} \le 100\, W_{LA}$
  $X_{Ch,1} + X_{Ch,2} + X_{Ch,3} \le 100\, W_{Ch}$
  $X_{At,1} + X_{At,2} + X_{At,3} \le 100\, W_{At}$

  Note that, by choosing the big M number equal to 100, we take care of warehouse capacity constraints too.

- The demands of the three regions should be satisfied:

  $X_{NY,1} + X_{LA,1} + X_{Ch,1} + X_{At,1} \ge 80$ (region 1)
  $X_{NY,2} + X_{LA,2} + X_{Ch,2} + X_{At,2} \ge 70$ (region 2)
  $X_{NY,3} + X_{LA,3} + X_{Ch,3} + X_{At,3} \ge 40$ (region 3)

- The three restrictions:
  1. If the New York warehouse is opened, then the Los Angeles warehouse must be opened:
  $$W_{LA} \ge W_{NY}$$
  2. At most three warehouses can be opened:
  $$W_{NY} + W_{LA} + W_{Ch} + W_{At} \le 3$$
  3. Either the Atlanta or the Los Angeles warehouse must be opened:
  $$W_{LA} + W_{At} \ge 1$$

Summarizing, we have the following IP model:

Minimize $400W_{NY} + 500W_{LA} + 300W_{Ch} + 150W_{At}$
$+ 20X_{NY,1} + 40X_{NY,2} + 50X_{NY,3}$
$+ 48X_{LA,1} + 15X_{LA,2} + 26X_{LA,3}$
$+ 26X_{Ch,1} + 35X_{Ch,2} + 18X_{Ch,3}$
$+ 24X_{At,1} + 50X_{At,2} + 35X_{At,3}$

subject to

Constraints (i) relating W_i's and X_i's, (ii) giving capacity limits.

$X_{NY,1} + X_{NY,2} + X_{NY,3} \leq 100\ W_{NY}$

$X_{LA,1} + X_{LA,2} + X_{LA,3} \leq 100\ W_{LA}$

$X_{Ch,1} + X_{Ch,2} + X_{Ch,3} \leq 100\ W_{Ch}$

$X_{At,1} + X_{At,2} + X_{At,3} \leq 100\ W_{At}$

Demand constraints:

$X_{NY,1} + X_{LA,1} + X_{Ch,1} + X_{At,1} \geq 80$ (region 1)

$X_{NY,2} + X_{LA,2} + X_{Ch,2} + X_{At,2} \geq 80$ (region 2)

$X_{NY,3} + X_{LA,3} + X_{Ch,3} + X_{At,3} \geq 80$ (region 3)

The extra restrictions:

$W_{LA} \geq W_{NY}$

$W_{NY} + W_{LA} + W_{Ch} + W_{At} \leq 3$

$W_{LA} + W_{At} \geq 1$

Set constraints:

W_i binary for i=NY,LA,Ch,At

X_j integer for j=1,2,3.