

## Applications of Linear Programming

### 1. Fair representation in electoral systems.

Suppose that a state sends  $R$  persons to the U.S. House of Representatives. There are  $D$  counties in the state ( $D > R$ ), and the state legislature wants to group these counties into  $R$  distinct electoral districts, each of which sends a delegate to Congress. The total population of the state is  $P$ , and the legislature wants to form districts whose population approximates  $p = P/R$ . Suppose that the appropriate legislative committee studying the electoral districting problem generates a long list of  $N$  candidates to be districts ( $N > R$ ). Each of these candidates contains contiguous counties and a total population  $p_j$  ( $j = 1, 2, \dots, N$ ) that is acceptably close to  $p$ . Define  $c_j = |p_j - p|$ . Each county  $i$  ( $i = 1, 2, \dots, D$ ) is included in at least one candidate and typically will be included in a considerable number of candidates (in order to provide many feasible ways of selecting a set of  $R$  candidates that includes each county exactly once). Define

$$a_{ij} = \begin{cases} 1 & \text{if county } i \text{ is included in candidate } j, \\ 0 & \text{if not.} \end{cases}$$

Given the values of the  $c_j$  and the  $a_{ij}$ , the objective is to select  $R$  of these  $N$  possible districts such that each county is contained in a single district and such that the largest of the associated  $c_j$  is as small as possible.

Formulate an LP model for this problem (integer variables are allowed).