

## Sensitivity Analysis (cont.)

### Variable deletion (optional)

Case 1: If  $x_j$  is to be deleted from the final tableau and it is nonbasic, simply drop  $x_j$  and tableau remains optimal.

Case 2:  $x_j$  to be deleted and it is basic.

Can't do simple deletion because the basic sol-n will be lost.

Example: Final tableau:

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	RHS
Z	0	0	1	$\frac{1}{2}$	1	4
$x_1$	1	0	-1	1	-2	1
$x_2$	0	1	1	2	0	3

Idea: Get  $x_j$  out of basis while keeping dual feasibility.

Subcase 2.1: Row of  $x_j$  has a negative entry. Do min-ratio test: choose as entering variable that with the ratio  $(\text{row 0 coeff}) / (\text{coeff. in pivot row})$

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closest to zero, considering only nonbasic variables with (coeff. in pivot row) negative.  
 (Recall that this was the normal min-ratio test in dual simplex).

Ex. (cont.)

Suppose  $x_1$  is to be deleted. Then min-ratio test:  $\min\left(\frac{1}{-1}, \frac{1}{-2}\right) = \frac{1}{2}$ .

Thus,  $x_1$  leaves the basis and  $x_5$  enters.  
 New tableau:

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	RHS
$z$	$\frac{1}{2}$	0	$\frac{1}{2}$	1	0	4.5
$x_5$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	1	$-\frac{1}{2}$
$x_2$	0	1	1	2	0	3

Now can delete column of  $x_1$ ; new basis:  $\{5, 2\}$   
 dual feasibility maintained  $\rightarrow$  reoptimize by dual simplex.

Subcase 2.2: Row of  $x_j$  has no negative entry. Again min-ratio test: choose entering variable with (row 0 coef.)/(coeff. in pivot row) closest to zero, considering only nonbasic variables with (coeff. in pivot row) positive.

## Ex. (cont.)

Suppose  $x_2$  is to be deleted. Then

$$\text{min-ratio test: } \min\left(\frac{x_3}{1}, \frac{x_4}{\frac{1}{2}}\right) = \frac{1}{4}$$

Thus,  $x_2$  leaves the basis and  $x_4$  enters.

New tableau:

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	RHS
$z$	0	$-\frac{1}{4}$	$\frac{3}{4}$	0	1	3.25
$x_1$	1	$-\frac{1}{2}$	$-\frac{3}{2}$	0	-2	-0.5
$x_4$	0	$\frac{1}{2}$	$\frac{1}{2}$	1	0	1.5

Now can delete column of  $x_2$ ; new basis:  $\{1, 4\}$   
dual feasibility maintained  $\rightarrow$  reoptimize by dual simplex.

## Changing coefficients of basic variables

- Current simplex tableau:

B.v.	$x_B$	$x_N$	RHS
$z$	0	$C_B^T B^{-1} N - C_N^T$	$C_B^T B^{-1} b$
$x_B$	I	$B^{-1} N$	$B^{-1} b$

- In this case,  $B^{-1}$  changed  $\rightarrow$  all the entries in tableau are changed  $\rightarrow$  reoptimize using primal or dual simplex (whichever necessary)