Math 4620/5620 – Linear and nonlinear optimization

Homework 1

(problems 5 and 6 are due Thursday, February 1)

1. (adapted from Bradley, Hax, Magnanti: Applied Math. Programming, p37) A strategic planner for an airline that flies to four different cities from its Ithaca, NY hub owns 10 Boeing 737's, 15 Airbus 321's and two ATR-70's. Assuming constant flying conditions and passenger use, the following data is available (data refers to round trips):

	City	Cost (USD)	Revenue (USD)	Flying time
B 737	Boston	6,000	5,000	1
	DC	7,000	7,000	2
	Miami	8,000	10,000	4
	L.A.	10,000	18,000	6
AB 321	Boston	4,000	3,000	1
	DC	3,500	5,500	2
	Miami	6,000	8,000	5
	L.A.	10,000	14,000	8
ATR-70	Boston	1,000	3,000	2
	DC	2,000	4,000	4
	Miami	N/A	N/A	N/A
	L.A.	N/A	N/A	N/A

Formulate linear constraints to take the following into account:

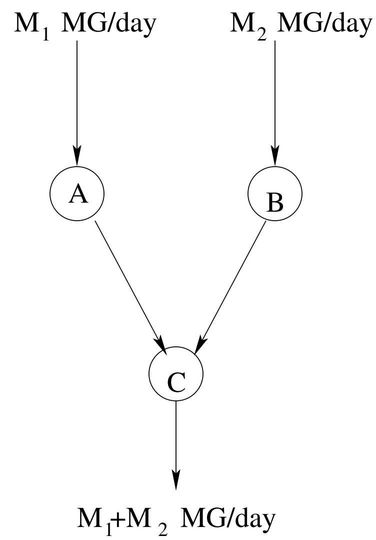
1. L.A. must be served twice daily, all other cities must be served at least four times daily
2. ATR-type planes can fly for at most 18 hours a day, all other planes can fly for at most 15 hours a day.

Formulate linear objective functions for

1. cost minimization
2. profit maximization
3. fleet flying time minimization

Remember to define your variables properly.

2. The Environmental Protection Agency (EPA) wants to restrict the amount of pollutants added by a company to the river water. The company has three plants A, B, and C in a river network as shown below.



The west branch of the river where plant A is located has a flow of M_1 MG/day (million gallons per day) and the east branch has a flow of M_2 MG/day. Plant C is located at the place where these branches meet, so the river has a flow of $M_1 + M_2$ MG/day at that point. The river can be clean enough for (1) swimming, or (2) to support bio-life (but not clean enough for swimming). Maximum permissible concentrations of pollutants (in lbs./MG) for both cases are given in the table below:

	Swimming	*Biological Life*
Phenol	P_S	P_B
Nitrogen	N_S	N_B

The company diverts at each of its plants a portion of the river water, adds the pollutants to it and sends the water back to the river. The company has four possible ways of treating the water it uses before returning it to the river at each plant. The characteristics of each treatment are given in the following table which shows the amount of pollutants (in pounds) added to one MG of water after treatment.

Treatment	1	2	3	4
Phenol	P_1	P_2	P_3	P_4
Nitrogen	N_1	N_2	N_3	N_4
Cost/MG	c_1	c_2	c_3	c_4

Assume that A, B, and C must process at least K_A, K_B, and K_C MG/day respectively. It is necessary that the waters in the river sections between A and C and after C be clean enough for swimming and the water in section between B and C is clean enough to support bio-life.
Further assume: (i) that the river is free of pollutants upstream from plants A and B; and (ii) that adding pollutants does not affect the amount of water flow. Formulate a linear program that minimizes the total treatment costs.

3. A farmer is raising pigs for market and wishes to determine the quantities of the available types of feed (corn, tankage, and alfalfa) that should be given to each pig. Since pigs will eat any mix of these feed types, the objective is to determine which mix will meet certain nutritional requirements at a minimum cost. The number of units of each type of basic nutritional ingredient contained within 1 kilogram of each feed type is given in the following table, along with the daily nutritional requirements and feed costs. Formulate an LP for this problem.

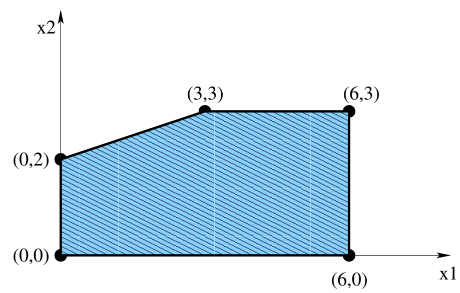
Nutritional ingredient	Kg of corn	Kg of tankage	Kg of alfalfa	Min daily requirement
Carbohydrates	90	20	40	200
Protein	30	80	60	180
Vitamins	10	20	60	150
Cost(cent)	84	72	60	

4. The shaded area in the following graph represents the feasible region of a linear programming problem whose objective function is to be maximized. Label each of the following statements as true or false, and then justify your answer based on the graphical method. In each case, give an example of an objective function that illustrates your answer.

(a) If (3,3) produces a larger value of the objective function than (0,2) and (6,3), then (3,3) must be an optimal solution.

(b) If (3,3) is an optimal solution and multiple optimal solutions exist, then either (0,2) or (6,3) must also be an optimal solution.

(c) The point (0,0) cannot be an optimal solution.



5. A company manufactures two products. Each unit of product 1 can be sold for \$16, and each unit of product 2 for \$20. Each product requires raw material and two types of labor (skilled and unskilled) as shown in the following table:

	product 1	product 2
skilled labor	3 hours	4 hours
unskilled labor	4 hours	2 hours
raw material	2 units	1 unit

At present, the company has available 12 hours of skilled labor, 8 hours of unskilled labor, and 16 units of raw material. Because of marketing considerations at least 2 units of product 2 must be produced.

(a) The company's goal is to maximize revenue. Assuming that everything it produces is sold, formulate a linear program whose solution fulfills that goal.

(b) Solve the problem graphically. Give both the optimal solution and the optimal value of the linear program.

6. A manufacturer of small electronic calculators is working on setting up production plans for the next six months. One product is particularly puzzling to him. The orders on hand for the coming season are:

Month	Jan.	Feb.	March	Apr.	May	June
Orders	100	150	200	100	200	150

The product will be discontinued after satisfying the June demand, so there is no need to keep any inventory after June. There is no inventory on hand now. The production cost using regular manpower is $10 per unit; producing the calculator on overtime costs an additional $2 per unit. The inventory-carrying cost is $0.50 per unit per month. The regular shift production is limited to 100 units per month, while overtime production is limited to an additional 75 units per month. The goal is to satisfy demand at minimum cost. Formulate the problem as an LP.