Math 4620/5620 – Linear and nonlinear optimization

Homework 2

(Problems 2 and 3 are due Tuesday, February 20)

1. Consider a Linear Programming Problem in Standard Form. Label each of the following statements as true or false and then justify your answers (either by supplying a counterexample or by proving the statement):
   (a) If a feasible solution is optimal, then it must be a CPF solution.
   (b) The number of CPF solutions is at least
   $$\frac{(m+n)!}{m!n!}$$
   (c) If a feasible solution is optimal but not a CPF solution, then infinitely many optimal solutions exist.
   (d) In a feasible LP problem, the best CPF solution is always an optimal solution.

2. Consider the following Linear Programming Problem
$$\begin{array}{rrcl} \max & x_1 + x_2 & & \\ \text{s. t.} & x_1 & \leq & 5 \\ & x_1 + x_2 & \leq & 6 \\ & x_2 & \leq & 4 \\ & x_1, x_2 & \geq & 0 \end{array}$$
   Solve it by the Simplex Method (choose x_1 as entering variable in the first tableau). How many optimal solutions are there? How does this reflect in the final simplex tableau? How can you iterate to another optimal basic solution (do it!)?

3. Apply the Simplex Method to the following problem:
$$\begin{array}{rrcl} \max & x_1 + 2x_2 - x_3 & & \\ \text{s. t.} & 2x_1 + 2x_2 - 2x_3 & \leq & 10 \\ & 3x_1 - 2x_2 + 2x_3 & \leq & 5 \\ & x_1 - 4x_2 + x_3 & \leq & 10 \\ & x_1, x_2, x_3 & \geq & 0 \end{array}$$
   What is your conclusion about this linear program? Use the final tableau to construct a feasible solution with an objective function value of at least 2024.

4. Consider the following problem:
$$\begin{array}{rrcl} \min & 2x_1 + x_2 + 3x_3 & & \\ \text{s. t.} & 5x_1 + 2x_2 + 7x_3 & = & 420 \\ & 3x_1 + 2x_2 + 5x_3 & \geq & 280 \\ & x_1, x_2, x_3 & \geq & 0 \end{array}$$
   Using the two-phase method, solve by the simplex method.

5. Solve the following linear program:

$$\begin{array}{rrcl} & \max \quad 2x_2 + 3x_3 & & \\ \text{s. t.} & 3x_1 + 2x_2 - x_3 & \leq & -3 \\ & -x_1 - x_2 + 2x_3 & \leq & -1 \\ & x_1, x_2, x_3 & \geq & 0 \end{array}$$

What is your conclusion?

6. Determine whether the following statements about Linear Programming are true or false. Justify your answers.

(a) The Phase I problem of the Simplex Method can be unbounded.

(b) The Phase I problem of the Simplex Method can be infeasible.

(c) The Phase I problem of the Simplex Method can have multiple optimal solutions.

(d) The Simplex Method's minimum ratio rule for choosing the leaving basic variable is used because another choice with a larger ratio would yield a basic solution that is not feasible.

(e) If an LP has multiple optimal solutions, then it must have a bounded feasible region.