Math 4620/5620

Homework 3

(Problems 2 and 3(b,c) are due Thursday, March 21)

1. Recall the problem 4 from Homework 2. We are given the following linear program:

$$\begin{array}{rrcl} \min & 2x_1 + x_2 + 3x_3 & & \\ \text{s. t.} & 5x_1 + 2x_2 + 7x_3 & = & 420 \\ & 3x_1 + 2x_2 + 5x_3 & \geq & 280 \\ & x_1, x_2, x_3 & \geq & 0 \end{array}$$

   After applying Phase 1 we determine that the basis $\{x_4, x_3\}$ gives a basic feasible solution for the problem (x_4 is the surplus variable of the second constraint). Do Phase II with the Revised Simplex.

2. It's late Thursday night, you're sitting in a smoky pub with loud music and you see your roommate doing the latest Math assignment that's due on the next day. You walk over, stumble and spill a glass of good Sam Adams Ale on a sheet with solutions. Besides being sorry for the waste of beer, you feel even more sorry for yourself when your roommate demands the (correct!) solution to the problem from you.

$$\begin{array}{rrcrcrcr} \max & \multicolumn{7}{l}{6x_1 + x_2 + 2x_3} \\ \text{s. t.} & 2x_1 & + & 2x_2 & + & \frac{1}{2}x_3 & \leq & 2 \\ & -4x_1 & - & 2x_2 & - & \frac{3}{2}x_3 & \leq & 3 \\ & x_1 & + & 2x_2 & + & \frac{1}{2}x_3 & \leq & 1 \\ & \multicolumn{7}{l}{x_1, x_2, x_3 \geq 0} \end{array}$$

   After a short and futile argument you give up and sit down to see what can be salvaged from both the sheet and your friendship. You can barely decipher a fraction of the optimal tableau:

	x_1	x_2	x_3	x_4	x_5	x_6	
z	?	?	?	2	0	2	?
x_5	?	?	?	1	1	2	?
x_3	?	?	?	-2	0	4	?
x_1	?	?	?	1	0	-1	?

   Use your fundamental insight of the simplex method to identify the missing numbers in the final simplex tableau.

3. Consider the following linear programming problem:

$$\begin{array}{rrl} \max & 3x_1 + x_2 + 2x_3 & \\ \text{s. t.} & x_1 - x_2 + 2x_3 & \leq 20 \\ & 2x_1 + x_2 - x_3 & \leq 10 \\ & x & \geq 0 \end{array}$$

If we apply the simplex method, the final tableau is

	x_1	x_2	x_3	x_4	x_5	
	8	0	0	3	4	100
x_3	3	0	1	1	1	30
x_2	5	1	0	1	2	40

(a) Perform sensitivity analysis to determine which of c_j's and b_i's are sensitive parameters in the sense that *any* change in just that parameter's value will change the optimal solution or the optimal value. You do not have to do any computations, simply indicate whether a change in parameter will affect the solution/value and why.

(b) Find the allowable range to stay optimal for each c_j

(c) Find the allowable range to stay feasible for each b_i