# Math 4620/5620

## Homework 4
## Subproblems 3(a,d,f) are due Tuesday, April 2

1. Suppose that a primal problem has a *degenerate* BF solution (one or more basic variables equal to zero) as its optimal solution. What does this degeneracy imply about the dual problem? Why? Is the converse also true?

2. Consider the following linear program:

$$
\begin{aligned}
\max \quad & 5x_1 + 2x_2 + 5x_3 \\
\text{s. t.} \quad & 2x_1 + 3x_2 + x_3 \leq 4 \\
& x_1 + 2x_2 + 3x_3 \leq 7 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

   (a) What is the corresponding dual problem?

   (b) Solve the dual graphically.

   (c) Use complementary slackness to solve the primal problem.

3. Consider the problem of finding a minimal cost diet subject to minimum daily requirements for two nutrients (vitamins A and K). There are 5 food types for you to choose from, and the requirements are as follows:

| Nutrient | Nutrient units/unit food | | | | | MDR for nutrient |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | |
| vitamin A | 1 | 0 | 1 | 1 | 2 | 21 |
| vitamin K | 0 | 1 | 2 | 1 | 1 | 12 |
| cost (cents/unit) | 20 | 20 | 31 | 11 | 12 | |

   (a) Formulate the problem of finding a minimum cost diet meeting the requirements as an LP, and find an optimal solution for it using the Dual Simplex Method. What is $B^{-1}$ for the optimal solution?

   (b) Identify the complementary dual solutions in each of the tableaus in the previous part.

   (c) A local pharmacist is selling vitamin K pills at a cost of 12 cents/unit of vitamin K content. Is this price competitive with the available foods in meeting this vitamin requirement? **Hint:** Use shadow prices. (**optional**)

   (d) A delicious new food containing 3 (2) units of vitamin A (K) per unit has become available at a price of 28 cents/unit. Would you include this food into the optimal diet?

   (e) Your favorite grocery story went out of business and you cannot find food 5 anywhere else. What is the optimal diet now? (**optional**)

   (f) You became concerned with the saturated fat content of your diet. Foods 1 to 5 contain 1,1,0,1 and 2 units of saturated fat, and according to your doctor you are not supposed to consume more than 20 units of fat a day. How can you incorporate this assumption into your problem formulation? Is the optimal solution still feasible? What will be your new optimal diet?

4. For any linear programming problem in our standard form and its dual problem, label each of the following statements as true or false and then justify your answer.

(a) The sum of the number of functional constraints and the number of variables (before augmenting) is the same for both the primal and the dual problems.

(b) At each iteration, the simplex method simultaneously identifies a basic *feasible* solution for the primal problem and a basic *feasible* solution for the dual problem such that their objective function values are the same.

(c) If the primal problem has an unbounded objective function, then the optimal value of the objective function for the dual problem must be zero.