We want to analyze the problem

$$(1)\qquad \begin{array}{rl} \min & 16x_1 + 17x_2 + 20x_3 \\ & 40x_1 + 30x_2 + 25x_3 \geq 100 \\ & 20x_1 + 40x_2 + 50x_3 \geq 100, \\ \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

First we write it in standard form, getting

$$(P)\qquad \begin{array}{rl} \max Z = & -16x_1 - 17x_2 - 20x_3 \\ & -40x_1 - 30x_2 - 25x_3 \leq -100 \\ & -20x_1 - 40x_2 - 50x_3 \leq -100, \\ \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

a) Add slack variables x_4 and x_5, and write the tableau corresponding to the basic solution with x_4 and x_5 basic.

b) Is the basic solution in (a) feasible? How would you most efficiently find an optimal solution to (P) starting from this tableau?

The final tableau is

Basic Variable	Eqn.	Z	x_1	x_2	x_3	x_4	x_5	RHS
Z	0	1			$\frac{5}{2}$	$\frac{3}{10}$	$\frac{2}{10}$	-50
x_1	1		1		$-\frac{1}{2}$	$-\frac{4}{100}$	$\frac{3}{100}$	1
x_2	2			1	$\frac{3}{2}$	$\frac{2}{100}$	$-\frac{4}{100}$	2

c) By inspection, write down the optimal basis B and its inverse.

d) By inspection, write down the optimal solution to the dual of (P).

e) For what range of values for b_1 is the current basis still optimal (the values of the basic variables might change)? What are the optimal solution and optimal value if b_1 changes to -150.

f) For what range of values for c_1, currently -16, is the current basic solution optimal? If c_1 changes to -10, how does the final tableau change? How would you re-optimize (don't perform any pivots)?

g) The original problem (1) represented a diet problem: you want to decide how much of each of three cereals to eat each day to satisfy nutritional requirements at minimum cost. Suppose a new cereal, Coco Fistula, becomes available, providing 50% of the daily requirement of the first vitamin and 20% of the daily requirement of the second vitamin. At what level of its cost would you consider changing your diet by adding the new cereal?