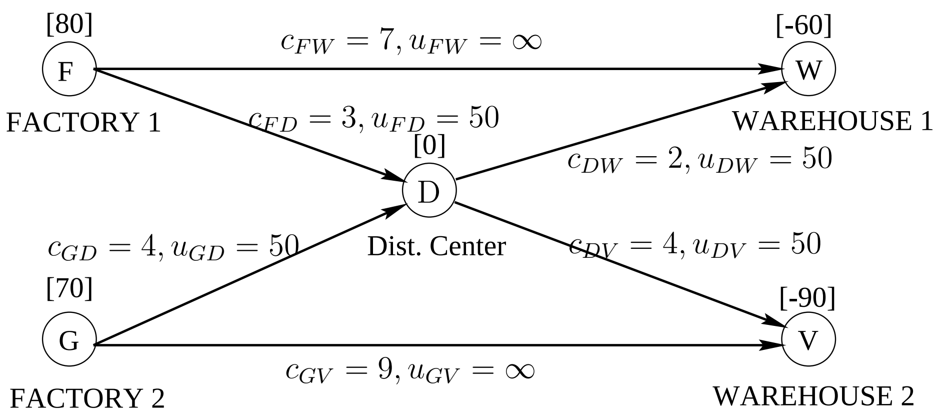
Math 4620/5620

Solution to Min Cost Flow practice problems

1. The min-cost flow problem is represented by the following network.



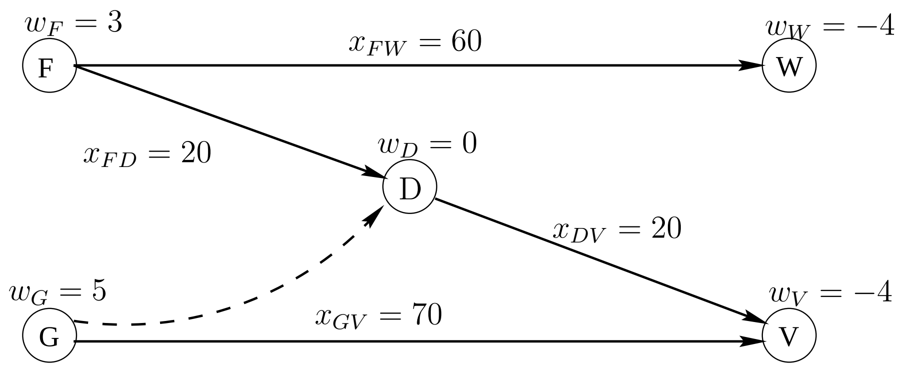
The linear programming formulation:

$$
\begin{array}{lrrrrrrcrl}
\min & 7x_{FW} & +3x_{FD} & +2x_{DW} & +4x_{GD} & +4x_{DV} & +9x_{GV} & & & \\
\text{s. t.} & x_{FW} & +x_{FD} & & & & & = & 80 & \text{(Factory 1)} \\
 & & & & x_{GD} & & +x_{GV} & = & 70 & \text{(Factory 2)} \\
 & & -x_{FD} & +x_{DW} & -x_{GD} & +x_{DV} & & = & 0 & \text{(Distribution Center)} \\
 & -x_{FW} & & -x_{DW} & & & & = & -60 & \text{(Warehouse 1)} \\
 & & & & & -x_{DV} & -x_{GV} & = & -90 & \text{(Warehouse 2)} \\
 & & x_{FD}, & x_{DW}, & x_{GD}, & x_{DV}, & & \leq & 50 & \text{(Capacity)} \\
 & x_{FW}, & x_{FD}, & x_{DW}, & x_{GD}, & x_{DV}, & x_{GV} & \geq & 0 &
\end{array}
$$

2. (a) We assume that there is no capacity on any arc. We obtain a complementary dual solution w to the initial primal feasible solution by solving the following system:

$$
\begin{aligned}
0 &= c_{FW} - w_F + w_W = 7 - w_F + w_W \\
0 &= c_{FD} - w_F + w_D = 3 - w_F + w_D \\
0 &= c_{DV} - w_D + w_V = 4 - w_D + w_V \\
0 &= c_{GV} - w_G + w_V = 9 - w_G + w_V
\end{aligned}
$$

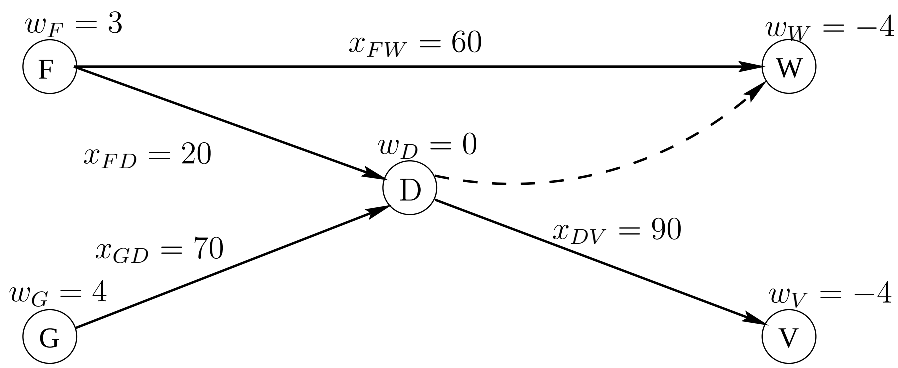
By setting $w_D = 0$, we obtain $(w_F, w_G, w_D, w_W, w_V) = (3, 5, 0, -4, -4)$. The initial primal and dual solutions are given below:



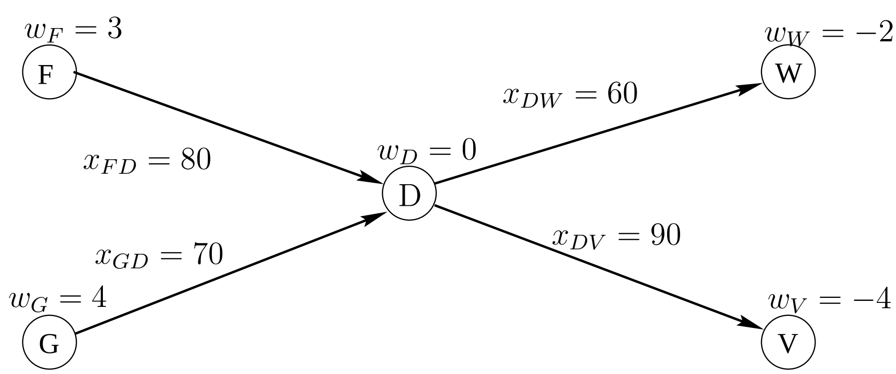
(b) We notice that the arc GD does not satisfy dual feasibility since $c_{GD} - (w_G - w_D) = 4 - (5 - 0)$ is negative. Then arc GD becomes basic and we augment along the cycle DVG. Since GV is the only backward arc in this cycle, it leaves the basis and we can increase th flow on GD by 70 units to obtain a new primal basic feasible solution. To find the dual feasible solution, we solve:

$$
\begin{aligned}
0 &= c_{FW} - w_F + w_W = 7 - w_F + w_W \\
0 &= c_{FD} - w_F + w_D = 3 - w_F + w_D \\
0 &= c_{DV} - w_D + w_V = 4 - w_D + w_V \\
0 &= c_{GD} - w_G + w_D = 4 - w_G + w_V
\end{aligned}
$$

So we obtain a dual solution $(w_F, w_G, w_D, w_W, w_V) = (3, 4, 0, -4, -4)$.



Now, do we have dual feasibility? Not yet. We observe $c_{DW} - (w_D - w_W) = 2 - (0 - (-4))$ is negative, and so arc DW enters the basis. Since FW is the only backward arc in the cycle DWF, we augment 60 units in the cycle, and FW leaves the basis. After some calculation, we get $(w_F, w_G, w_D, w_W, w_V) = (3, 4, 0, -2, -4)$.



Can we find any more entering arc? We check all nonbasic arcs:

$$c_{FW} - (w_F - w_W) = 7 - (3 - (-2)) \geq 0$$
$$c_{GV} - (w_G - w_V) = 9 - (4 - (-4)) \geq 0$$

The current dual solution is feasible and so we found a primal-dual pair of optimal solutions.