Problem 1 (Team Selection)

A basketball coach is trying to choose the starting lineup for his team. The team consists of seven players who have been rated (on a scale of 1=poor to 3=excellent) according to their ball-handling, shooting, rebounding, and defensive abilities. The positions that each player is allowed to play and the player's abilities are listed in the following table.

Player	Position	Ball-handling	Shooting	Rebounding	Defense
1	G	3	3	1	3
2	C	2	1	3	2
3	G-F	2	3	2	2
4	F-C	1	3	3	1
5	G-F	3	3	3	3
6	F-C	3	1	2	3
7	G-F	3	2	2	1

The five-player starting lineup must satisfy the following restrictions.

1. At least 3 members must be able to play guard, at least two members must be able to play forward, and at least 1 member must be able to play center.
2. The average ball-handling, shooting, and rebounding level of the starting lineup must be at least 2.
3. If player 3 starts, then player 6 cannot start.
4. If player 1 starts, then players 4 and 5 must both start.
5. Either player 2 or player 3 must start.

Given these constraints, the coach wants to maximize the total defensive ability of the starting team. Formulate an IP that will help him choose his starting lineup.

Problem 2 (Pollution Control)

Because of excessive pollution on a river flowing through a state, the state is going to build pollution control stations. Three sites (1, 2, and 3) are under consideration. The state is interested in controlling the pollution levels of two pollutants (1 and 2). The state legislature requires that at least 80,000 tons of pollutant 1 and at least 50,000 tons of pollutant 2 be removed from the river. The relevant data for this problem are shown in the following table.

Site	Cost of building station ($)	Cost of treating 1 ton water ($)	Amount removed per ton of water	
			Pollutant 1	Pollutant 2
1	100,000	20	0.4	0.3
2	60,000	30	0.25	0.2
3	40,000	40	0.2	0.25

Formulate an IP to minimize the cost of meeting the state legislature's goals.

Problem 3 (Class Scheduling)

To graduate from a university with a major in Operations Research, a student must complete at least two Math courses, at least two OR courses, and at least two CS courses. Some courses can be used to fulfill more than one requirement:

- Calculus can fulfill the Math requirement;
- Discrete Optimization can fulfill the Math and OR requirements;
- Data Structures can fulfill the Math and CS requirements;
- Business Statistics can fulfill the Math and OR requirements;
- Computer Simulation can fulfill the OR and CS requirements;
- Introduction to Computer Programming can fulfill the CS requirement;
- Forecasting can fulfill the Math and OR requirements.

Some courses are prerequisites for others:

- Calculus is a prerequisite Business Statistics;
- Introduction to Computer Programming is a prerequisite for Computer Simulation and for Data Structures.
- Business Statistics is a prerequisite for Forecasting.

Formulate an IP that minimizes the number of courses needed to satisfy the major requirements.

Problem 4 (Production Planning)

Ford has four automobile plants. Each is capable of producing the Taurus, Lincoln, or Escort, but it can only produce one of these cars. The fixed cost of operating each plant for a year and the variable cost of producing a car of each type at each plant are given in the following table.

Plant	Fixed Cost ($)	Variable Cost ($)		
		Taurus	Lincoln	Escort
1	7 billion	12,000	16,000	9,000
2	6 billion	15,000	18,000	11,000
3	4 billion	17,000	19,000	12,000
4	2 billion	19,000	22,000	14,000

Ford faces the following restrictions.

a) Each plant can produce only one type of car.
b) The total production of each type of car must be at a single plant; that is, for example, if any Tauruses are made at plant 1, then all Tauruses must be made there.
c) If plants 3 and 4 are used, then plant 1 must also be used.

Each year, Ford must produce 500,000 of each type of car. Formulate an IP whose solution will tell Ford how to minimize the annual cost of producing cars.

Problem 5 (Resource Allocation)

A mall has 10,000 sq ft of space to rent and wants to determine the types of stores that should occupy the mall. The minimum number and the maximum number of each type of store (along with the square footage of each type) is given in the following table.

Store type	Square Footage	Minimum	Maximum
Jewelry	500	1	3
Shoe	600	1	3
Department	1,500	1	3
Book	700	0	3
Clothing	900	1	3

The annual profit made by each type of store will, of course, depend on how many stores of that type are in the mall. This dependence is given in the following table (all profits are in units of $10,000).

Store type	Number of Stores		
	1	**2**	**3**
Jewelry	9	8	7
Shoe	10	9	5
Department	27	21	20
Book	16	9	7
Clothing	17	13	10

Thus, if there are two department stores in the mall, each department store earns $210,000 profit per year. Each store pays 5% of its annual profit as rent to the mall. Formulate an IP whose solution will tell the mall how to maximize its rental income.

Problem 6 (Work Scheduling)

A post office requires different numbers of full-time employees on different days of the week. The number of full-time employees required on each day is given in the table below.

	Number of full-time employees required
Day 1=Monday	17
Day 2=Tuesday	13
Day 3=Wednesday	15
Day 4=Thursday	19
Day 5=Friday	14
Day 6=Saturday	16
Day 7=Sunday	11

(a) Union rules state that each full-time employee must work five consecutive days and then receive two days off. For example, an employee who works Thursday to Monday must be off on Tuesday and Wednesday. The post office wants to meet its daily requirements using only full-time employees. Formulate an LP that the post office can use to minimize the number of full-time employees that must be hired.

(b) Suppose that the post office can force employees to work one day of overtime each week. For example, an employee whose regular shift is Monday to Friday can also be required to work on Saturday. Each employee is paid $50 a day for each of the first five days worked during a week and $62 for the overtime day (if any). Formulate an LP whose solution will enable the post office to minimize the cost of meeting its weekly work requirements.