

**Practice Problem on Cutting Planes.**

Consider the following integer program:

$$\begin{aligned} \text{Maximize} \quad & Z = x_1 + 3x_2 + 2x_3 + 4x_4 \\ \text{s.t.} \quad & 2x_1 + 2x_2 + x_3 + 2x_4 \leq 5 \quad (1) \\ & 5x_1 + 3x_2 + 5x_3 + 2x_4 \leq 9 \quad (2) \\ & 2x_1 + 3x_2 + 3x_4 \geq 5 \quad (3) \\ & x_1, x_2, x_3, x_4 \geq 0 \text{ integer} \end{aligned}$$

Parts (a) - (c) below are independent of each other. You are given three fractional solutions which are feasible for the LP-relaxation of the problem. For each of the fractional points, give a cutting plane that will cut off the fractional solution.

a)  $(1, 0, 1/3, 1)$

$$x_1 + x_3 \leq 1 \quad \text{based on constraint (2)}$$

b)  $(0, 0, 0, 2.5)$

$$x_4 \leq 2 \quad \text{based on constraint (1)}$$

c)  $(0.25, 1.5, 0, 0)$

$$x_1 + x_2 + x_4 \geq 2 \quad \text{based on constraint (3)}$$