Math 4630/5630

Homework 1
(Problems 1 and 2 are due Tuesday, February 4)

Problem 1. Facility Location problem.
A company is considering opening warehouses in four cities: New York, Los Angeles, Chicago, and Atlanta. Each warehouse can ship 100 units per week. The weekly fixed cost of keeping each warehouse open is $400 for New York, $500 for Los Angeles, $300 for Chicago, and $150 for Atlanta. Region 1 of the country requires 80 units per week, region 2 requires 70 units per week, and region 3 requires 40 units per week. The costs of sending 1 unit from a warehouse to a region are shown in the following table:

	To		
from	Region 1	Region 2	Region 3
New York	$20	$40	$50
Los Angeles	$48	$15	$26
Chicago	$26	$35	$18
Atlanta	$24	$50	$35

We wish to meet weekly demands at minimum cost, subject to the preceding information and the following restrictions:

1. If the New York warehouse is opened, then the Los Angeles warehouse must be opened.
2. At most three warehouses can be opened.
3. Either the Atlanta or the Los Angeles warehouse must be opened.

Formulate an integer program that can be used to minimize the weekly costs of meeting demands.

Problem 2. Blending problem.
A farmer is raising pigs for market and wishes to determine the quantities of the available types of feed (corn, tankage, and alfalfa) that should be given to each pig. Since pigs will eat any mix of these feed types, the objective is to determine which mix will meet certain nutritional requirements at a minimum cost. The number of units of each type of basic nutritional ingredient contained within 1 kilogram of each feed type is given in the following table, along with the daily nutritional requirements and feed costs.

Nutritional ingredient	Kg of corn	Kg of tankage	Kg of alfalfa	Min daily requirement
Carbohydrates	90	20	40	200
Protein	30	80	60	180
Vitamins	10	20	60	150
Cost (cent)	84	72	60	

a) Formulate an integer program for this problem.

b) Suppose the farmer wants to have at most 2 feed types in the mix. Modify the model of part (a) to take the new restriction into account.
c) The farmer thinks that satisfying all the nutritional requirements costs him too much. He wants to keep only two of the three requirements. Modify the model of part (a) to minimize the cost while satisfying only two (any two) of the three nutritional requirements.

Problem 3. Generalized knapsack problem.
Suppose that Tom is going on an overnight hike. There are 3 different items Tom is considering taking along the trip. Tom's knapsack can hold up to 20 lbs of items. The weight of item i is w[i]. Tom might take at most L[i] copies of item i. The benefit Tom feels he would get from taking k copies of item i is b[i,k]. The values of w[i], L[i] and b[i,k] are listed in the following table.

item	weight(lbs)	max # of copies	Benefit			
i	w[i]	L[i]	b[i,1]	b[i,2]	b[i,3]	b[i,4]
1	4	3	45	70	90	
2	5	3	20	40	50	
3	3	4	50	70	85	95

Formulate an integer program whose solution will tell Tom how to maximize the total profit.

Hint: Note that b[i,k] is not a linear function on k, and that complicates the problem. To overcome this difficulty, introduce the following decision variables. For each item i (i=1,2,3) and possible number of copies k (k=1,…,L[i]):

$$y_{i,k} = \begin{cases} 1 & \text{if } k \text{ copies of item } i \text{ are taken} \\ 0 & \text{otherwise} \end{cases}$$

Problem 4. Portfolio selection.
National Insurance Associates carries an investment portfolio of a variety of stocks, bonds, and other investment alternatives. Currently $200,000 of funds have become available and must be considered for new investment opportunities. The four stock options National is considering and the relevant financial data are as follows:

	Investment Alternative			
	A	B	C	D
Price per share	$100	$50	$80	$40
Projected annual rate of return	0.12	0.08	0.06	0.10
Risk measure per dollar invested (*Higher values indicate greater risk*)	0.10	0.07	0.05	0.08

The risk measure provided by the firm's top financial advisor, indicates the relative uncertainty associated with the stock regarding the realization of its projected annual rate of return.

National's top management has stipulated the following investment guidelines:

1. The projected annual rate of return for the portfolio must be at least 9%.
2. No one stock can count for more than 50% of the total dollar investment.

(a) For this overall situation, develop a integer programming model to yield an investment portfolio which minimizes total risk.

(b) Revise the model in (a) to ignore risk and maximize projected return on investment.

Problem 5. Linear regression.

A very common and important problem in statistics is linear regression, the problem of fitting a straight line to statistical data. The most commonly employed technique is the method of least squares, but there are other interesting criteria where linear programming can be used to solve for the optimal values of the regression parameters.

Let $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ be data points and a_1 and a_0 be the parameters of the regression line $y=a_1x+a_0$.

(a) Formulate a linear program whose optimal solution minimizes the sum of the absolute deviations of the data from the line, i.e., formulate

$$\min_a \sum_{i=1}^{n} \left| y_i - \left(a_1 x_i + a_0\right) \right|$$

as an LP (the variables can be either integer or continuous).

(b) Formulate the minimization of the maximum absolute deviation as an LP, i.e., formulate

$$\min_a \max_i \left| y_i - (a_1 x_i + a_0) \right|$$

as an LP.

(c) Generalize the model to allow fitting to general polynomials

$y = a_k x^k + a_{k-1} x^{k-1} + \ldots + a_1 x + a_0$.

Hint: The difficulty here is that the absolute value function is *not* linear. To overcome this difficulty you need to introduce a new variable for each absolute value. If you add new constraints, make sure that there are not absolute values there too.