Math 4630/5630

Homework 2

(Problems 2 and 3 are due Monday, February 20 at class time)

1. A manufacturer of small electronic calculators is working on setting up his production plans for the next six months. One product is particularly puzzling to him. The orders on hand for the coming season are:

Month:	Jan	Feb	Mar	Apr	May	June
Orders:	100	150	200	100	200	150

The product will be discontinued after satisfying the June demand, so there is no need to keep any inventory after June. There is no inventory on hand now. The production cost using regular manpower is \$10 per unit; producing the calculator on overtime costs an additional \$2 per unit. The inventory-carrying cost is \$0.50 per unit per month. The regular shift production is limited to 100 units per month, while overtime production is limited to additional 75 units per month. The goal is to satisfy demand at minimum cost. Formulate an integer program for this problem.

2. Below are seven functions of integer variable n. Sort these functions in increasing order of their running times.

(a) 1.2^n
(b) $\frac{n}{\log n}$
(c) $n^{1/2} + \frac{1}{8}n!$
(d) $n \log n + n^9$
(e) $n^{1/3} \log n$
(f) $11(\log n)^2$
(g) $5n^2 + 2\log n$

3. A company manufactures two products. Each unit of product 1 can be sold for \$12, and each unit of product 2 for \$18. Each product requires raw material and two types of labor (skilled and unskilled) as shown in the following table:

	product 1	product 2
skilled labor	6 hours	6 hours
unskilled labor	3 hours	6 hours
raw material	10 units	4 unit

At present, the company has available 24 hours of skilled labor, 18 hours of unskilled labor, and 20 units of raw material. Because of marketing considerations at least 2 units of product 2 must be produced.

(a) The company's goal is to maximize revenue. Assume that everything it produces is sold. Assuming that it is possible to have fractional number of units, formulate a linear program for this problem.

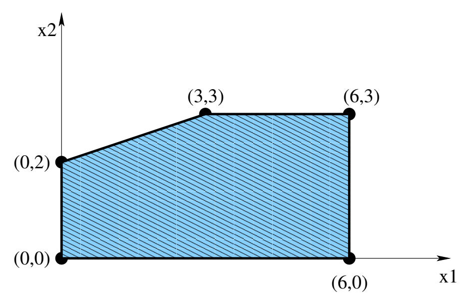
(b) Solve the problem graphically. Give both the optimal solution and the optimal value of the linear program.

4. The shaded area in the following graph represents the feasible region of a linear programming problem whose objective function is to be maximized. Label each of the following statements as true or false, and then justify your answer based on the graphical method. In each case, give an example of an objective function that illustrates your answer.

(a) If (3,3) produces a larger value of the objective function than (0,2) and (6,3), then (3,3) must be an optimal solution.

(b) If (3,3) is an optimal solution and multiple optimal solutions exist, then either (0,2) or (6,3) must also be an optimal solution.

(c) The point (0,0) cannot be an optimal solution.



5. Consider a Linear Programming Problem in Standard Form. Label each of the following statements as true or false and then justify your answers (either by supplying a counterexample or by proving the statement):

(a) If a feasible solution is optimal, then it must be a CPF solution.

(b) The number of CPF solutions is at least

$$\frac{(m+n)!}{m!n!}$$

(c) If a feasible solution is optimal but not a CPF solution, then infinitely many optimal solutions exist.

(d) In a feasible LP problem, the best CPF solution is always an optimal solution.