Math 4630/5630

Homework 4
(Problems 2 and 3 are due Wednesday, April 5 at class time)

Problem 1. Solving IP.
Consider the following IP problem.

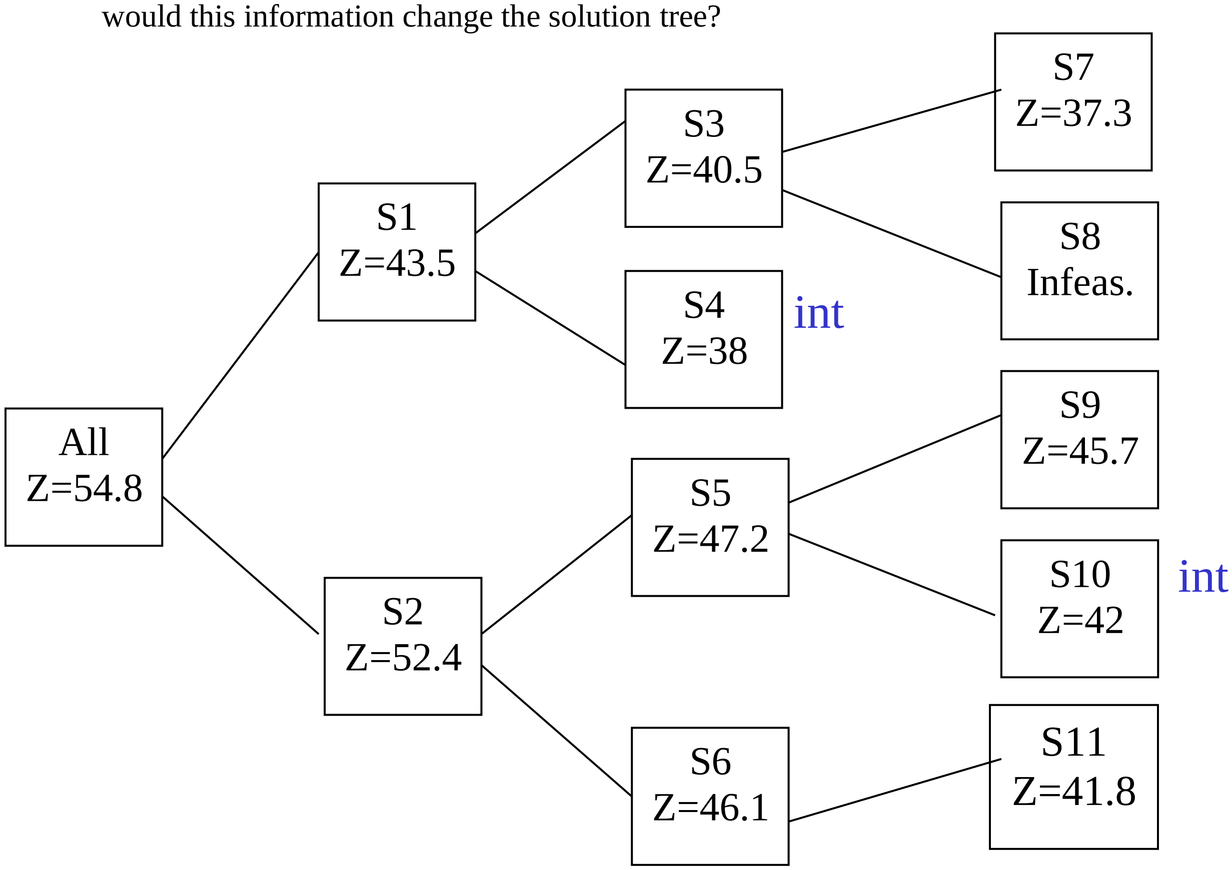
$$
\begin{aligned}
\text{Maximize} \quad & Z = 11x_1 + 4x_2 \\
\text{s.t.} \quad & 5x_1 + 2x_2 \leq 16 \\
& 2x_1 - x_2 \leq 4 \\
& -x_1 + 2x_2 \leq 4 \\
& x_1, x_2 \geq 0 \text{ integer}
\end{aligned}
$$

a) Solve the integer program graphically.
b) Solve the LP relaxation graphically. Round this solution to the *nearest* integer solution and check whether it is feasible. Then enumerate *all* the rounded solutions by rounding this solution for the LP relaxation in all possible ways (i.e., by rounding each noninteger value both up and down). For each rounded solution, check for feasibility and, if feasible, calculate Z. Are any of these feasible rounded solutions optimal for the IP problem?
c) Use the branch-and-bound method to solve the IP. You can solve each subproblem either graphically or by simplex method.
d) Find a cutting plane which makes all CPF solutions of the LP-relaxation integral. What can you say about the new optimal solution of the LP-relaxation?

Problem 2. Branch-and-bound.

You are given the following (incomplete) solution tree which is obtained by applying the branch-and-bound method to an integer program (maximization problem). The subproblems were solved in the increasing order of their indexes. Subproblem S8 is infeasible; the solutions of subproblems S4 and S10 are integral; all other subproblems have fractional solutions. Answer the following questions.

a) What is the current incumbent? What is its value?
b) Give as tight as possible lower and upper bounds on the IP optimal value.
c) Which nodes are fathomed? Why?
d) Suppose before starting the branch-and-bound we have applied a fast heuristic to the problem. The heuristic has returned an integer solution with Z=41. How would this information change the solution tree?



Problem 3. Cutting planes for knapsack problem.
You are given the following knapsack problem:

item	1	2	3	4	5
size	31	28	45	18	25
benefit	43	35	50	24	30

The knapsack size is 60. Multiple copies of the same item can be taken.

a) Give an integer programming formulation for this problem.

b) Recall that for $k \geq 1$ and $S = \{ i \mid w_i > W / k \}$, we have the following cutting plane:

$$\boxed{\sum_{i \in S} x_i \leq k - 1}$$

Add cutting planes for k=2, 3, 4 to the formulation of part (a).

Problem 4 (optional). Applications of Graph Coloring.
Suppose that in a particular quarter there are students taking each of the following combinations of courses:

- *Math, English, Biology*
- *Math, French, Computer Science*
- *Biology, Computer Science, History*
- *Biology, Psychology*

What is the minimum number of examination periods required for the exams in the seven courses specified so that students taking any of the given combinations of courses have no conflicts? Find a schedule that uses this minimum number of periods.