Math 4630/5630

Homework 3 Solution

1. The initial tableau is the following:

	x_1	x_2	x_3	x_4	x_5	
z	-1	-1	0	0	0	0
x_3	1^*	0	1	0	0	5
x_4	1	1	0	1	0	6
x_5	$-\frac{1}{2}$	1	0	0	1	6

Entering x_1 and performing the min-ratio test yields that x_3 will leave the basis. And so, after one pivot, we get the following tableau:

	x_1	x_2	x_3	x_4	x_5	
z	0	-1	1	0	0	5
x_1	1	0	1	0	0	5
x_4	0	1^*	-1	1	0	1
x_5	0	1	$\frac{1}{2}$	0	1	$\frac{17}{2}$

The corresponding solution now is $x = (5, 0, 0, 1, 17/2)^T$, the current value is $z = 5$. After one more pivot step (in which x_2 enters the basis and x_4 leaves), we get the following tableau:

	x_1	x_2	x_3	x_4	x_5	
z	0	0	0	1	0	6
x_1	1	0	1^*	0	0	5
x_2	0	1	-1	1	0	1
x_5	0	0	$\frac{3}{2}$	-1	1	$\frac{15}{2}$

Since all coefficients in row 0 are nonnegative this tableau is an optimal one and we have found an optimal solution $x^* = (5, 1, 0, 0, 7.5)$ with optimal value $z^* = 6$. In terms of the original problem variables, the optimal solution is $(x_1, x_2) = (5, 1)$.

Looking at this last tableau a bit more carefully we see that there is a non-basic variable x_3 with an objective function coefficient 0 in row 0. This means that if we were to increase x_3 it would have no effect on the objective value, and we would get another solution with the same value as the current one! Now, if we perform the min-ratio test we see that there is a tie between x_1 and x_5 to leave the basis, the ratio for both is 5. This corresponds to the fact that both $x_1 = 0$ and $x_5 = 0$ (as well as $x_4 = 0$) intersect at the point $x = (0, 6)$. We can break this tie arbitrarily; in this solution we choose x_1 to leave the basis and arrive at the following tableau:

	x_1	x_2	x_3	x_4	x_5	
z	0	0	0	1	0	6
x_3	1	0	1	0	0	5
x_2	1	1	0	1	0	6
x_5	$-\frac{3}{2}$	0	0	-1	1	0

Here we obtain another optimal solution $(x_1, x_2) = (0, 6)$ with the same optimal value 6. Then all the points on the line segment connecting $(0, 6)$ and $(5, 1)$ are feasible and their value is also 6 (the optimal value).

2. The starting tableau should look like this:

	x_1	x_2	x_3	x_4	x_5	x_6	
z	-2	-3	1	0	0	0	0
x_4	2	2^*	-1	1	0	0	10
x_5	3	-2	1	0	1	0	10
x_6	1	-3	1	0	0	1	10

The corresponding basis is $\{4, 5, 6\}$, the basic feasible solution $x = (0, 0, 0, 10, 10, 10)$ with objective function value $z = 0$. Now when we pivot, x_2 enters the basis and x_4 leaves:

	x_1	x_2	x_3	x_4	x_5	x_6	
z	1	0	$-\frac{1}{2}$	$\frac{3}{2}$	0	0	15
x_2	1	1	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	5
x_5	5	0	0	1	1	0	20
x_6	4	0	$-\frac{1}{2}$	$\frac{3}{2}$	0	1	25

From here we see immediately that this problem is unbounded: we can increase x_3 as much as we want as long as we increase x_2 and x_6 correspondingly. More specifically, the ray

$$\left\{ \begin{pmatrix} 0 \\ 5 \\ 0 \\ 0 \\ 20 \\ 25 \end{pmatrix} + \alpha \begin{pmatrix} 0 \\ \frac{1}{2} \\ 1 \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix} : \alpha \geq 0 \right\},$$

which we can obtain immediately from the last tableau, is a feasible ray on which the objective function increases infinitely. The objective function value on this ray is $z = 15 + \frac{1}{2}\alpha$. To construct a solution with objective function value of at least 1717 we simply solve for α:

$$\begin{aligned} 15 + \frac{1}{2}\alpha &\geq 1717 \\ \iff \qquad \alpha &\geq 3404 \end{aligned}$$

and one feasible point with objective function value of exactly 1717 is $(x_1, x_2, x_3) = (0, 1707, 3404)$.