An Optimization Model for Interval Training With Heart Rate Constraints And Rest Time.

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April 2025

1 Introduction

1.1 Background

Physical activity is essential to maintain cardiovascular health, improve metabolic function, and promote overall well-being. Regular physical exercise significantly reduces the risk of cardiovascular diseases and enhances general fitness [7]. However, many individuals struggle with optimizing their workout routines within limited time constraints. Traditional exercise optimization models have primarily concentrated on maximizing calorie expenditure, often neglecting critical physiological parameters such as heart rate, essential for maintaining the effectiveness and safety of workouts [4].

1.2 Literature Review

Nutifafa Akpeleasi [2] previously developed an optimization model designed to maximize calorie burn during interval training. Akpeleasi's model focused on selecting and sequencing exercises to maximize calorie expenditure while adhering to constraints on workout duration and exercise intensity. Despite its effectiveness, Akpeleasi's approach did not account for the physiological variations in heart rate, which play a crucial role in ensuring exercise safety and improving endurance capabilities [5].

Research indicates that heart rate is a reliable physiological marker of exercise intensity and cardiovascular strain. Exercising within specific heart rate zones can optimize cardiovascular health benefits and minimize associated health risks [3]. This underscores the importance of incorporating heart rate constraints into exercise scheduling models to ensure workouts are not only effective but also physiologically safe.

1.3 Objectives

The primary objective of this study is to extend Akpeleasi's original optimization model by incorporating heart rate constraints, thereby ensuring that workouts remain within safe physiological limits. Specific objectives include:

1. Developing an enhanced optimization model that incorporates heart rate as a constraint.
2. Assessing the impact of heart rate monitoring on the sequencing and selection of exercises within interval training.
3. Evaluating the effectiveness of the enhanced model in maintaining exercise safety, preventing overexertion, and reducing potential injury by introducing rest time.
4. Providing personalized and health-conscious interval training schedules tailored to individual physiological responses.

1.4 Significance of the Study

Incorporating heart rate constraints into exercise optimization significantly enhances workout personalization and safety. This model could benefit a wide range of users, from athletes seeking optimized performance to individuals aiming to improve general health while managing risks associated with physical activity. Consequently, this approach addresses a critical gap in existing literature by combining physiological safety with traditional optimization techniques, potentially influencing future research and practical exercise guidelines.

2 The Optimization Model

2.1 Sets and Parameters

- T: Set of time periods (e.g., minutes), indexed by t.
- E: Set of exercise activities.
- $A = E \cup \{\text{Rest}\}$: Set of all activities including rest.
- W: Set of athlete weight categories.

Parameters:

- MET_i: Metabolic Equivalent of Task for activity $i \in A$.
- $\text{HR\_inc}_i = 2 \cdot \text{MET}_i$: Estimated heart rate increase for activity i.
- $\text{calories}_{i,w}$: Calories burned per minute for exercise $i \in E$ and weight group $w \in W$.
- Time_Duration: Total workout duration (in minutes).
- $\text{HR}_{\text{initial}}$: Initial heart rate (typically 90 bpm).
- $\text{HR}_{\text{min}}, \text{HR}_{\text{max}}$: Safe heart rate bounds.
- $\text{HR}_{\text{recovery}}$: Heart rate recovery per rest minute.
- max_rest_streak: Maximum number of consecutive rest minutes allowed.
- min_exercise_duration, max_exercise_duration: Minimum and maximum duration for each exercise.
- min_rest_duration: Minimum duration for each rest interval.
- N: Maximum number of distinct exercises allowed.
- max_total_rest: Maximum total rest time allowed.
- min_indicator: Threshold to determine if an exercise is included.

2.2 Decision Variables

- $\text{is\_active}_{i,t} \in \{0, 1\}$: 1 if activity $i \in A$ is performed at time $t \in T$, 0 otherwise.
- $\text{HR}_t \in \mathbb{R}_{\geq 0}$: Heart rate at minute t.
- $\text{final\_HR} \in [\text{HR}_{\text{min}}, \text{HR}_{\text{max}}]$: Heart rate at the end of the session.
- total_HR: Total heart rate over the workout session.
- $\text{included}_i \in \{0, 1\}$: 1 if exercise $i \in E$ is included in the session.
- $\text{total\_calories}_w \in \mathbb{R}_{\geq 0}$: Total calories burned by weight group $w \in W$.
- $\text{exercise\_block}_{i,t} \in \{0, 1\}$: A binary variable that equals 1 if exercise i is performed continuously at minutes t, $t + 1$, and $t + 2$. This helps identify 3-minute exercise segments.

- $\text{total\_rest\_time} \in \mathbb{R}_{\geq 0}$: A non-negative variable that represents the total number of minutes allocated to rest throughout the workout. It is used to track rest usage and apply penalties if necessary.

2.3 Objective Function

The goal of this optimization model is to maximize the total calories burned during exercise. The objective function sums the calorie expenditure of all selected exercises, considering their respective durations and the weight category of the athlete. By optimizing exercise selection and sequencing, the model ensures the most effective use of available exercise time while adhering to physiological constraints;

2.4 Objective Function

$$\max \sum_{w \in W} \text{total\_calories}_w$$

$$\text{where } \text{total\_calories}_w = \sum_{t \in T} \sum_{i \in E} \text{is\_active}_{i,t} \cdot \text{calories}_{i,w}$$

2.5 Constraints

(C1) **Heart Rate Dynamics:**

These constraints model how the athlete's heart rate evolves over time. The heart rate at each time step depends on the heart rate in the previous minute, the intensity of the activity performed, and whether the athlete is resting. Specifically, each active exercise increases the heart rate proportionally to its MET value (via a constant multiplier), while rest leads to a decrease in heart rate by a fixed recovery value. The model also initializes the heart rate at the beginning of the session to a predefined baseline (e.g., 90 bpm). This setup captures realistic cardiovascular responses to exercise and recovery patterns throughout the workout.

$$\text{HR}_t = \text{HR}_{t-1} + \sum_{i \in E} \text{HR\_inc}_i \cdot \text{is\_active}_{i,t} - \text{HR}_{\text{recovery}} \cdot \text{is\_active}_{\text{Rest},t}, \quad \forall t > 1$$

$$\text{HR}_1 = \text{HR}_{\text{initial}}$$

(C2) **Heart Rate Bounds:**

HR_{max} and HR_{min} are based on standard formulas recommended by ([5]) to ensure that the heart rate remains within medically recommended limits for safety. The minimum and maximum thresholds reflect typical safe operating ranges for cardiovascular activity during moderate to high-intensity exercise. Additionally, the model captures the final heart rate at the end of the workout and computes the total across the full session. This provides a foundation for assessing physiological strain and optimizing recovery.

$$\text{HR}_{\text{min}} \leq \text{HR}_t \leq \text{HR}_{\text{max}}, \quad \forall t \in T$$

$$\text{final\_HR} = \text{HR}_{|T|} \quad , \quad \text{total\_HR} = \sum_{t \in T} \text{HR}_t$$

(C3) **Single Activity Per Time Unit:**

To maintain feasibility, this constraint ensures that the athlete performs exactly one activity—either a specific exercise or rest—at each time unit. This prevents overlapping or multitasking between exercises, which is both physically unrealistic and undesirable for structured interval training sessions.

$$\sum_{i \in A} \text{is\_active}_{i,t} = 1, \quad \forall t \in T$$

(C4) **Exercise Duration Limits:**

Each selected exercise is constrained to last no more than a specified maximum duration. This reflects the practical upper bounds on how long a given activity can be performed during a single session and prevents excessively long execution of any one movement. It also balances the time across multiple exercises within the total session.

$$\sum_{t \in T} \text{is\_active}_{i,t} \leq \text{max\_exercise\_duration}, \quad \forall i \in E$$

(C5) **Define Exercise Inclusion:**

This constraint connects the binary inclusion variable of each exercise to its actual use in the schedule. If any time unit is assigned to an exercise, its inclusion variable must be activated. This enables the model to track how many distinct exercises are used and impose variety-based constraints downstream.

$$\text{included}_i \geq \text{min\_indicator} \cdot \sum_{t \in T} \text{is\_active}_{i,t}, \quad \forall i \in E$$

(C6) **Exercise Count and Variety:**

This pair of constraints limits the total number of distinct exercises selected for the workout and enforces a minimum variety threshold. While the upper bound avoids overly fragmented sessions with too many short-duration exercises, the lower bound ensures that the workout includes a diverse set of movements to enhance effectiveness and prevent monotony.

$$\sum_{i \in E} \text{included}_i \leq N \quad , \quad \sum_{i \in E} \text{included}_i \geq 4$$

(C7) **Minimum Total Exercise Time:**

To avoid trivial or inactive schedules, this constraint enforces a lower bound on the cumulative time spent performing exercises (excluding rest). It ensures the session delivers a meaningful fitness benefit and aligns with minimum physical activity recommendations.

$$\sum_{i \in E} \sum_{t \in T} \text{is\_active}_{i,t} \geq 10$$

(C8) **Activity Kickstart and Calorie Condition:**

This constraint requires the athlete to begin the session actively within the first 10 minutes, ensuring that the workout does not start with an excessively long rest period. Additionally, it enforces that at least one calorie is burned for the lightest weight category, confirming that the session includes meaningful effort and avoids degenerate or all-rest solutions.

$$\sum_{i \in E} \sum_{t=1}^{10} \text{is\_active}_{i,t} \geq 2 \quad , \quad \text{total\_calories}_{130\text{lb}} \geq 1$$

(C9) **Rest Constraints:**

These constraints regulate how rest periods are used within the workout:

The first condition limits the number of consecutive rest minutes, preventing overly long passive streaks and encouraging regular activity.

The second condition applies specifically to the last 5 minutes of the workout, where rest must be present but not dominant, mimicking a cooldown period that gradually reduces exertion while maintaining cardiovascular rhythm.

- Limit consecutive rest minutes:

$$\sum_{k=t}^{t+\text{max\_rest\_streak}-1} \text{is\_active}_{\text{Rest},k} \leq \text{max\_rest\_streak} - 1, \quad \forall t \leq |T| - \text{max\_rest\_streak} + 1$$

- Final cooldown:

$$\sum_{t=|T|-4}^{|T|} \text{is\_active}_{\text{Rest},t} \leq 2$$

(C10) **Define Continuous 3-Minute Exercise Blocks**

To ensure each selected exercise appears in at least one block of three consecutive minutes, auxiliary binary variables $\text{exercise\_block}_{i,t}$ are introduced. These variables represent the presence of a continuous 3-minute segment for exercise i starting at time t.

$$\text{exercise\_block}_{i,t} \leq \text{is\_active}_{i,t}, \quad \forall i \in E, \ t \in 1..|T| - 2$$
$$\text{exercise\_block}_{i,t} \leq \text{is\_active}_{i,t+1}, \quad \forall i \in E, \ t \in 1..|T| - 2$$
$$\text{exercise\_block}_{i,t} \leq \text{is\_active}_{i,t+2}, \quad \forall i \in E, \ t \in 1..|T| - 2$$

These constraints ensure that a valid block can only be active if all three minutes are assigned to the same exercise. The presence of such a block for any selected exercise is enforced as follows:

$$\sum_{t=1}^{|T|-2} \text{exercise\_block}_{i,t} \geq \text{included}_i, \quad \forall i \in E$$

This guarantees that each chosen exercise is used for at least one uninterrupted 3-minute duration, aligning the model with realistic training behavior.

(C11) **Total Rest Time Calculation**

To enable rest-based penalties in the objective function or analysis, the total rest time is defined as the cumulative number of minutes assigned to the rest activity across the entire workout duration:

$$\text{total\_rest\_time} = \sum_{t \in T} \text{is\_active}_{\text{Rest},t}$$

This constraint tracks the total number of rest minutes, allowing the model to discourage excessive inactivity while still permitting necessary recovery intervals.

2.6 The Complete Optimization Model

Below is the complete formulation of our optimization model incorporating heart rate constraints:

Objective Function

$$\max \sum_{w \in W} \text{total\_calories}_w$$
$$\text{where total\_calories}_w = \sum_{t \in T} \sum_{i \in E} \text{is\_active}_{i,t} \cdot \text{calories}_{i,w}$$

Subject to Constraints

(C1) Heart Rate Dynamics:

$$\text{HR}_t = \text{HR}_{t-1} + \sum_{i \in E} \text{HR\_inc}_i \cdot \text{is\_active}_{i,t} - \text{HR}_{\text{recovery}} \cdot \text{is\_active}_{\text{Rest},t}, \quad \forall t \in T,\, t > 1$$

$$\text{HR}_1 = \text{HR}_{\text{initial}}$$

(C2) Heart Rate Bounds:

$$\text{HR}_{\text{min}} \leq \text{HR}_t \leq \text{HR}_{\text{max}}, \quad \forall t \in T$$

$$\text{final\_HR} = \text{HR}_{|T|}, \quad \text{total\_HR} = \sum_{t \in T} \text{HR}_t$$

(C3) Single Activity per Time Unit:

$$\sum_{i \in A} \text{is\_active}_{i,t} = 1, \quad \forall t \in T$$

(C4) Exercise Duration Limits:

$$\sum_{t \in T} \text{is\_active}_{i,t} \leq \text{max\_exercise\_duration}, \quad \forall i \in E$$

(C5) Define Exercise Inclusion:

$$\text{included}_i \geq \text{min\_indicator} \cdot \sum_{t \in T} \text{is\_active}_{i,t}, \quad \forall i \in E$$

(C6) Exercise Count and Variety:

$$\sum_{i \in E} \text{included}_i \leq N, \quad \sum_{i \in E} \text{included}_i \geq 4$$

(C7) Minimum Total Exercise Time:

$$\sum_{i \in E} \sum_{t \in T} \text{is\_active}_{i,t} \geq 10$$

(C8) Activity Kickstart and Calorie Condition:

$$\sum_{i \in E} \sum_{t=1}^{10} \text{is\_active}_{i,t} \geq 2, \quad \text{total\_calories}_{\text{130lb}} \geq 1$$

(C9) Rest Constraints:

$$\sum_{k=t}^{t+\text{max\_rest\_streak}-1} \text{is\_active}_{\text{Rest},k} \leq \text{max\_rest\_streak} - 1, \quad \forall t \leq |T| - \text{max\_rest\_streak} + 1$$

$$\sum_{t=|T|-4}^{|T|} \text{is\_active}_{\text{Rest},t} \leq 2$$

(C10) Define Continuous 3-Minute Exercise Blocks:

$$\text{exercise\_block}_{i,t} \leq \text{is\_active}_{i,t}, \quad \forall i \in E,\ t \in 1..|T|-2$$

$$\text{exercise\_block}_{i,t} \leq \text{is\_active}_{i,t+1}, \quad \forall i \in E,\ t \in 1..|T|-2$$

$$\text{exercise\_block}_{i,t} \leq \text{is\_active}_{i,t+2}, \quad \forall i \in E,\ t \in 1..|T|-2$$

$$\sum_{t=1}^{|T|-2} \text{exercise\_block}_{i,t} \geq \text{included}_i, \quad \forall i \in E$$

(C11) Total Rest Time Calculation:

$$\text{total\_rest\_time} = \sum_{t \in T} \text{is\_active}_{\text{Rest},t}$$

This complete optimization model now explicitly integrates heart rate constraints to provide a safer, more physiologically aware interval training schedule.

3 Computational Implementation

3.1 AMPL Implementation

We translate our optimization model into AMPL code and present the complete integer programming formulation developed in the preceding section.

3.1.1 Data Description and Sources

This section describes the dataset used in our optimization model. As previously mentioned, our model selects a subset of exercises from a predetermined set, aiming to maximize calorie expenditure subject to several physiological and operational constraints, including heart rate management.

3.1.2 List of Exercises

We test our model using 10 different exercises commonly adopted in interval training workouts. These exercises are selected based on their varied intensity levels and their popularity in fitness routines. Specifically, our set includes:

- Aerobics (general)
- Aerobics (high impact)
- Calisthenics (light)
- Calisthenics (fast)
- Bicycling (<10 mph, leisure)
- Bicycling (>20 mph, racing)
- Jumping rope (fast)
- Jumping rope (slow)
- Gymnastics
- Walking (3.0 mph)

These exercises were carefully chosen to reflect varying intensities and are commonly recommended for interval training due to their effectiveness in cardiovascular conditioning and calorie burning. The exercises considered were adapted from NutriStrategy's comprehensive list of physical activities and their calorie expenditure rates across multiple weight categories [8].

3.1.3 Exercises Included in Interval Training

Given the limited workout duration, individuals typically select a smaller subset of exercises for optimal benefit. For computational purposes, our model selects at most 6 exercises out of the available 10 exercises listed above. This selection criterion aligns with practical scenarios where workout sessions must be concise yet effective.

3.1.4 Workout Duration

The total duration allocated for the interval training session in our model is set at 60 minutes. To ensure optimal scheduling and physiological effectiveness, each selected exercise is constrained to last between a minimum of 5 minutes and a maximum of 15 minutes. These duration constraints ensure realistic transitions between exercises and sufficient exposure to each exercise for meaningful caloric burn.

3.1.5 Exercise Intensity and MET Values

Exercise intensity is quantified using the Metabolic Equivalent of Task (MET). One MET represents the rate of oxygen consumption at rest, approximately equal to 3.5 ml of O_2 per kilogram of body weight per minute [1]. Our exercise set includes MET values ranging from 2.0 to 8.8, covering a broad spectrum of intensity levels. We define exercises with MET values of 4 or higher as high-intensity activities, while those below this threshold are classified as low-intensity, following recommended guidelines by Mendes et al. [6]. Our dataset comprises:

- **High Intensity**: Aerobics (high impact), Calisthenics (fast), Bicycling (>20 mph, racing), Jumping rope (fast) and Gymnastics.
- **Low Intensity**: Aerobics (general), Calisthenics (light), Bicycling (<10 mph, leisure), Walking (3.0 mph) and Jumping rope (slow).

3.1.6 Calories Burned

The calorie data utilized in this model were obtained from NutriStrategy [8]. Calorie burn rates per minute were provided across four distinct athlete weight categories: 130 lbs, 155 lbs, 180 lbs, and 205 lbs. For precision in computational modeling, calories per hour data were converted into calories per minute, enabling our model to flexibly calculate calorie expenditure for any duration between the specified minimum and maximum exercise durations.

3.1.7 Heart Rate Parameters

An important enhancement in our model is the inclusion of heart rate considerations. Heart rate is a critical measure of exercise intensity and physiological strain. We incorporate heart rate increments derived from MET values, calculated using the relationship:

$$\text{Heart Rate Increment} = MET \times 2$$

The resting heart rate (`HR_initial`) is set at 90 beats per minute (bpm), reflecting typical resting rates for healthy individuals [5]. Additionally, we enforce physiological safety bounds, restricting heart rates to remain between a lower threshold of 100 bpm and an upper threshold of 205 bpm. These limits reflect standard cardiovascular safety guidelines for moderate to vigorous intensity exercises [3].

3.1.8 AMPL Input Data

The complete dataset used for computational testing and analysis is presented explicitly in AMPL format. The following parameters are included:

- `calories`: Calories burnt per minute for each exercise across weight categories.
- `MET`: Metabolic Equivalent of Task values for each exercise.
- `number_of_exercises`: Number of exercises selected for the workout.
- `intensity_lower_limit` and `intensity_upper_limit`: Bounds for intensity level constraints.
- `T_MET`: Threshold for distinguishing between high and low intensity exercises.
- `Time_Duration`: Total workout duration in minutes.
- `max_duration` and `min_duration`: Maximum and minimum allowed durations per exercise.

```
# ========== Sets ==========
set Exercises :=
    Aerobics_general
    Aerobics_high_impact
    Calisthenics_light
    Calisthenics_fast
    Bicycling_lessthan10mph_leisure
    Bicycling_morethan_20mph_racing
    Jumping_rope_fast
    Jumping_rope_slow
    Gymnastics
    Walking_3.0mph;

set weights := 130lb 155lb 180lb 205lb;

# ========== MET Values ==========
param MET :=
    Aerobics_general                 2.0
    Aerobics_high_impact            4.0
    Calisthenics_light              2.0
    Calisthenics_fast               4.0
    Bicycling_lessthan10mph_leisure  2.5
    Bicycling_morethan_20mph_racing 8.8
    Jumping_rope_fast              6.3
    Jumping_rope_slow               4.8
    Gymnastics                      3.5
    Walking_3.0mph                  3.0
    Rest                            0.0;

# ========== Calorie Data ==========
param calories:
                       130lb  155lb  180lb  205lb :=
Aerobics_general        6.4    7.6    8.9   10.1
Aerobics_high_impact    6.9    8.2    9.5   10.9
Calisthenics_light      3.5    4.1    4.8    5.4
Calisthenics_fast       7.9    9.4   10.9   12.4
Bicycling_lessthan10mph_leisure
                        3.9    4.7    5.5    6.2
```

```
Bicycling_morethan_20mph_racing
                        15.7   18.8   21.8   24.8
Jumping_rope_fast       11.8   14.1   16.4   18.6
Jumping_rope_slow        7.9    9.4   10.9   12.4
Gymnastics               3.9    4.7    5.5    6.2
Walking_3.0mph           3.3    3.9    4.5    5.1;

# ========== Session Configuration ==========
param Time_Duration := 60;
```

3.1.9 AMPL Code

The complete AMPL code of our interval training optimization model, including heart rate constraints, is presented below:

```
# ========================================================
# INTERVAL TRAINING MODEL WITH CUMULATIVE HEART RATE + REST TIME
# ========================================================

# === Parameters ===
param Time_Duration;
param HR_initial := 90;
param HR_min := 100;
param HR_max := 205;
param HR_recovery := 15;
param max_rest_streak := 3;

param min_exercise_duration := 5;
param max_exercise_duration := 15;
param min_rest_duration := 1;
param number_of_exercises := 6;
param max_total_rest := 20;
param min_indicator := 0.01;

# === Sets ===
set Exercises;
set Activities := Exercises union {"Rest"};
set weights;
set T := 1..Time_Duration;

# === Activity-Based Parameters ===
param MET{Activities};
param HR_inc{i in Activities} := MET[i] * 2.0;
param calories{Exercises, weights};

# === Variables ===
var is_active{Activities, t in T} binary;

var HR_t{t in T} <= HR_max;
var final_HR >= HR_min, <= HR_max;
```

```
var total_HR >= 0;

var included{Exercises} binary;
var total_calories_w{w in weights} >= 0;

var exercise_block{i in Exercises, t in 1..Time_Duration - 2} binary;
var total_rest_time >= 0;

# === Constraints ===

# Initial HR condition: Set initial heart rate at t = 1
subject to HR_start:
    HR_t[1] = HR_initial;

# HR lower bound for safety after initial time step
subject to HR_lower_bound{t in T: t > 1}:
    HR_t[t] >= HR_min;

# HR transition dynamics: Increase by activity, decrease by rest
subject to HR_dynamics{t in T: t > 1}:
    HR_t[t] = HR_t[t-1]
              + sum{i in Exercises} HR_inc[i] * is_active[i, t]
              - HR_recovery * is_active["Rest", t];

# Only one activity (including rest) per minute
subject to only_one_activity_per_minute{t in T}:
    sum{i in Activities} is_active[i, t] = 1;

# Enforce that at least one exercise is performed during the session
subject to at_least_one_exercise:
    sum{i in Exercises, t in T} is_active[i, t] >= 1;

#Define included[i] as 1 if exercise i is active at any time
subject to define_included{i in Exercises}:
    included[i] >= min_indicator * sum{t in T} is_active[i, t];

# Upper limit on total duration per exercise
subject to max_exercise_time{i in Exercises}:
    sum{t in T} is_active[i, t] <= max_exercise_duration;

#Define Continuous 3-Minute Exercise Blocks
subject to define_block{i in Exercises, t in 1..Time_Duration - 2}:
    exercise_block[i,t] <= is_active[i,t];
subject to define_block_2{i in Exercises, t in 1..Time_Duration - 2}:
    exercise_block[i,t] <= is_active[i,t+1];
subject to define_block_3{i in Exercises, t in 1..Time_Duration - 2}:
    exercise_block[i,t] <= is_active[i,t+2];
```

```
#Ensures that each selected exercise appears in at least one continuous 3-minute block.
subject to min_exercise_blocks{i in Exercises}:
    sum{t in 1..Time_Duration - 2} exercise_block[i,t] >= included[i];

#Total Rest Time Calculation
subject to compute_total_rest:
    total_rest_time = sum{t in T} is_active["Rest", t];

# Total HR is the sum of heart rates over time
subject to define_total_HR:
    total_HR = sum{t in T} HR_t[t];

#Final HR is defined as HR at the last time step
subject to define_final_HR:
    final_HR = HR_t[Time_Duration];

# Define calories burned for each weight group
subject to define_total_calories_w{w in weights}:
    total_calories_w[w] = sum{t in T, i in Exercises} is_active[i, t] * calories[i, w];

# Enforce a minimum amount of time spent exercising
subject to minimum_exercise_time_total:
    sum{i in Exercises, t in T} is_active[i, t] >= 10;

# Limit the number of unique exercises selected in the plan
subject to limit_exercise_count:
    sum{i in Exercises} included[i] <= number_of_exercises;

# Ensure a minimal activity threshold to prevent trivial rest-only plans
subject to force_at_least_one:
    sum{i in Exercises, t in T} is_active[i, t] >= 5;

# Require at least some activity in the first 10 minutes (warm-up)
subject to kickstart_activity:
    sum{i in Exercises, t in 1..10} is_active[i, t] >= 2;

# Ensure calories are burned by the lightest weight group
subject to ensure_some_calories:
    total_calories_w["130lb"] >= 1;

# Prevent negative heart rate values due to rounding or infeasibility
subject to HR_nonnegative{t in T}:
    HR_t[t] >= 0;

# Limit the number of consecutive minutes that rest can occur
subject to limit_rest_streak{t in 1..Time_Duration - max_rest_streak + 1}:
    sum{k in t..t+max_rest_streak-1} is_active["Rest", k] <= max_rest_streak - 1;

# Enforce diversity by requiring a minimum number of different exercises
```

```
subject to min_exercise_variety:
    sum{i in Exercises} included[i] >= 4;

# Smooth final HR to simulate cooldown or safe ending intensity
subject to smooth_final_HR:
    final_HR <= 180;

# Restrict the number of rest minutes in the final 5 minutes (cooldown zone)
subject to cooldown_final_minutes:
    sum{t in Time_Duration-4..Time_Duration} is_active["Rest", t] <= 2;

# === Objective ===
maximize total_Calories_objective:
    sum{w in weights} total_calories_w[w] - 0.5 * total_rest_time;
```

3.1.10 Computational Environment

```
#========== Reset The AMPL Environment ==========
reset;


#========== Choose A Solver ==========
option solver gurobi;
option cplex_options 'timelimit=10';

#========== Load The Model ==========
model Clement.mod;


#========== Load The Data ==========
data Clement.dat;


#========== Solve ==========
solve;


#========== Show Results ==========
display total_calories_w;
display final_HR;
display {t in T} HR_t[t];
display {t in T, i in Activities: is_active[i, t] = 1} is_active[i, t];
display {i in Exercises: included[i] = 1};
```

3.1.11 Ampl Results

```
ampl: include Clement.run;
Gurobi 12.0.0:
<BREAK> (solver)
               Gurobi 12.0.0: interrupted, feasible solution; objective 2277.4
739046 simplex iterations
16599 branching nodes
absmipgap=21.2, relmipgap=0.00930886
total_calories_w [*] :=
130lb  443.6
155lb  530.4
180lb  616
205lb  699.9
;

final_HR = 173.8

HR_t[t] [*] :=
 1  90     10 135.6   19 143.6   28 153.4   37 202.8   46 160.6   55 156
 2 107.6   11 148.2   20 128.6   29 138.4   38 187.8   47 178.2   56 173.6
 3 125.2   12 133.2   21 141.2   30 151     39 172.8   48 163.2   57 191.2
 4 110.2   13 118.2   22 158.8   31 168.6   40 185.4   49 148.2   58 176.2
 5 122.8   14 135.8   23 143.8   32 153.6   41 170.4   50 165.8   59 188.8
 6 135.4   15 148.4   24 128.8   33 161.6   42 155.4   51 150.8   60 173.8
 7 148     16 133.4   25 132.8   34 169.6   43 173     52 135.8
 8 165.6   17 146     26 136.8   35 177.6   44 190.6   53 153.4
 9 150.6   18 158.6   27 140.8   36 190.2   45 175.6   54 171
;

is_active[i,t] [*,*]
# £2 = Bicycling_morethan_20mph_racing
:  Aerobics_general  $2 Calisthenics_fast Jumping_rope_fast Rest    :=
1          .          1          .                 .          .
2          .          1          .                 .          .
3          .          1          .                 .          .
4          .          .          .                 .          1
5          .          .          .                 1          .
6          .          .          .                 1          .
7          .          .          .                 1          .
8          .          1          .                 .          .
9          .          .          .                 .          1
10         .          .          .                 .          1
11         .          .          .                 1          .
12         .          .          .                 .          1
13         .          .          .                 .          1
14         .          1          .                 .          .
15         .          .          .                 1          .
16         .          .          .                 .          1
17         .          .          .                 1          .
18         .          .          .                 1          .
19         .          .          .                 .          1
20         .          .          .                 .          1
```

```
21          .          .          .                1          .
22          .          1          .                .          .
23          .          .          .                .          1
24          .          .          .                .          1
25          1          .          .                .          .
26          1          .          .                .          .
27          1          .          .                .          .
28          .          .          .                1          .
29          .          .          .                .          1
30          .          .          .                1          .
31          .          1          .                .          .
32          .          .          .                .          1
33          .          .          1                .          .
34          .          .          1                .          .
35          .          .          1                .          .
36          .          .          .                1          .
37          .          .          .                1          .
38          .          .          .                .          1
39          .          .          .                .          1
40          .          .          .                1          .
41          .          .          .                .          1
42          .          .          .                .          1
43          .          1          .                .          .
44          .          1          .                .          .
45          .          .          .                .          1
46          .          .          .                .          1
47          .          1          .                .          .
48          .          .          .                .          1
49          .          .          .                .          1
50          .          1          .                .          .
51          .          .          .                .          1
52          .          .          .                .          1
53          .          1          .                .          .
54          .          1          .                .          .
55          .          .          .                .          1
56          .          1          .                .          .
57          .          1          .                .          .
58          .          .          .                .          1
59          .          .          .                1          .
60          .          .          .                .          1
;

set {i in Exercises: included[i] == 1}  :=
Aerobics_general                 Bicycling_morethan_20mph_racing
Calisthenics_fast                Jumping_rope_fast;
```

4 Computational Results

4.1 Implementation Notes

Due to the complexity of the model and the large number of binary and time-indexed variables, the CPLEX solver (demo version) was unable to handle the problem within its constraint limits. As a result, the model was executed using the Gurobi solver. The solution was obtained after allowing Gurobi to run for approximately 5–10 minutes, after which the run was manually stopped upon reaching a satisfactory and feasible solution with an objective value of 2277.4 kcal.

Additionally, the MET parameter values were slightly adjusted from their textbook estimates. This was done to encourage the model to select a wider variety of exercises and allocate more time to active training segments. The actual MET values would have driven the heart rate too high too quickly, necessitating longer rest periods and leading to less realistic or overly fragmented training sessions. These changes were made carefully to preserve physiological realism while improving practical output quality.

4.2 Optimization Output Summary

The optimization model was executed using the Gurobi solver over a workout period of 60 minutes. The model was designed to maximize total calories expenditure in four weight categories while respecting physiological and structural constraints. The final solution achieved a total objective value of **2277.4 kcal**, distributed throughout each weight group as shown in Table 4.2.

Table 1: Total Calories Burned by Weight Category

Weight Category	**Total Calories Burned (kcal)**
130 lb	443.6
155 lb	530.4
180 lb	616
205 lb	699.9
Total	**2277.4**

4.3 Heart Rate Profile

The heart rate profile showed a realistic physiological trend, with an initial value of 90 bpm gradually increasing through active minutes and regulated by rest periods. The heart rate peaked at 202.8 bpm, and the final heart rate at the end of the workout was 173.8 bpm. The heart rate remained within medically safe limits throughout the session.

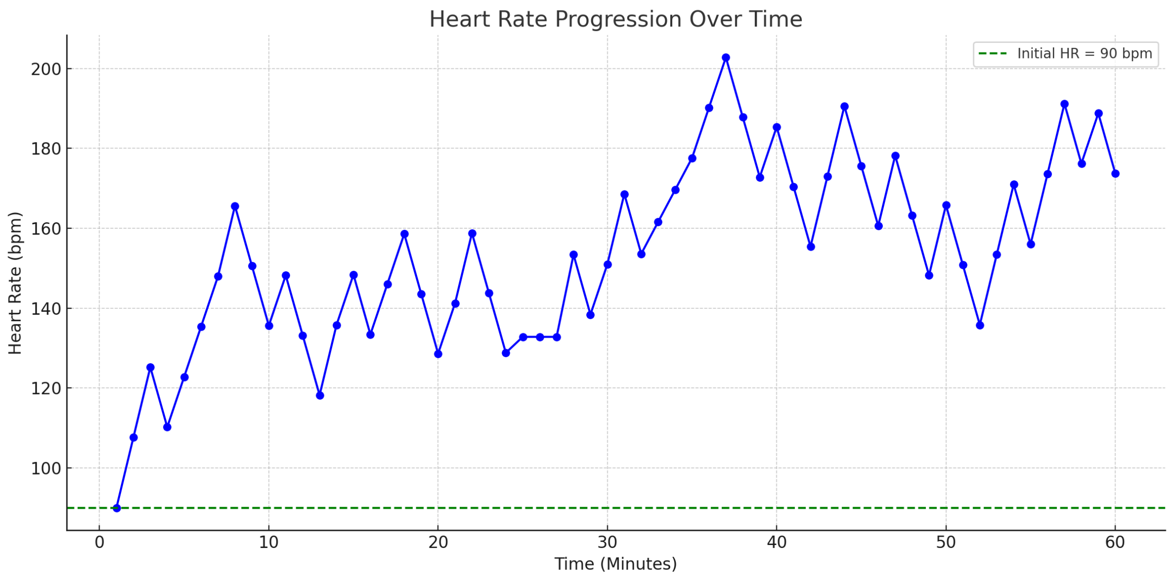

Figure 1: Heart Rate Progression Over Time

4.4 Exercise Selection and Schedule

The model selected the following four exercises for the 60-minute workout session:

- Aerobics (general)
- Bicycling (>20 mph, racing)
- Calisthenics (fast)
- Jumping rope (fast)

Each exercise was used for multiple minutes across the session and included at least one block of three consecutive minutes of activity. This structure aligns well with real-world training practices, in which individuals typically sustain an exercise for several minutes before transitioning or resting. Rest periods were placed between high-intensity sessions, avoiding long streaks and adhering to the maximum rest streak constraint.

4.5 Model Validation

The solution met all required constraints of the model. Key validation checks include:

- Heart rate remained between 100 and 205 bpm at all time points.
- Each selected exercise included at least one 3-minute continuous activity block.
- No more than 2 rest minutes occurred in the final 5-minute cooldown period.
- Total calories burned exceeded the minimum for every weight category.
- No rest streak exceeded 2 consecutive minutes, satisfying the rest streak constraint.
- The final heart rate did not exceed the upper safe threshold.

4.6 Interpretation of Results

The model generated a feasible and structured training plan that combines moderate- and high-intensity activities with strategically placed rest periods. It maximized total calorie burn while maintaining cardiovascular safety and ensuring realistic workout flow. The selection and duration of exercises reflect a practical training schedule, in which high-effort exercises are spaced with short recovery intervals.

The output confirms that the optimization framework can effectively design workouts that meet diverse fitness goals and physiological requirements. The resulting session offers value for athletes, trainers, and fitness applications that require controlled yet challenging interval routines.

5 Discussion

5.1 Interpretation of Findings

The results of the model demonstrate that the proposed optimization framework can effectively generate structured and physiologically sound interval training sessions. The model achieves this by optimizing calorie burn across four weight categories while ensuring that key physiological constraints are met. The heart rate profile indicates a progressive increase in intensity, with periods of recovery interspersed between sustained exercise blocks.

The selected activities, which include Aerobics (general), Bicycling (>20 mph), Calisthenics (fast), and Jumping rope (fast), reflect high-effort exercises capable of elevating cardiovascular response. Each exercise was scheduled in at least one 3-minute block, aligning the workout with practical, real-world training practices. The placement of rest periods was optimal for cardiovascular recovery and consistent with maximum rest constraints.

The model's ability to satisfy safety, variety, and calorie thresholds confirms its practical relevance in designing interval workouts that are both efficient and adaptable to different weight classes.

5.2 Model Limitations

Despite producing valid and meaningful solutions, the model operates under certain simplifications:

- **Linear Heart Rate Dynamics:** The model assumes a linear increase and recovery of heart rate based on MET and rest, respectively. Actual physiological responses are nonlinear and influenced by fatigue, hydration, and external factors.
- **Fixed MET and Calorie Values:** The MET and calorie burn rates are fixed for each activity. In reality, these can vary based on user intensity and exercise form.
- **No Transition Time:** The model does not include time for transitioning between exercises, which in practice would impact both continuity and heart rate.
- **Uniform Parameters Across Users:** All users are subject to the same heart rate bounds, MET definitions, and calorie burn rates, which may not hold for individuals with medical conditions or varying fitness levels.
- **No Real-Time Feedback Integration:** The model operates as a static optimization. Future work could incorporate wearable sensor data for adaptive, real-time optimization.

Overall, the model is a robust starting point for designing structured workouts. Further enhancements can make it more personalized and reflective of real-time physiological responses.

6 Conclusion

This project presented a time-indexed optimization model for constructing effective and physiologically safe interval training workouts. The model was designed to maximize total calorie expenditure while enforcing constraints related to heart rate progression, exercise variety, and structured activity-rest cycles. The approach incorporated per-minute decision-making to closely simulate the dynamic nature of cardiovascular responses during exercise.

The computational results validated the ability of the model to generate workouts that are both effective and realistic. The final solution achieved a total calorie burn of 2277.4 kcal across four different weight categories, while keeping heart rate within medically safe bounds. Selected exercises were scheduled in sustained blocks, and rest periods were used appropriately to manage cardiovascular recovery.

All constraints—including those on minimum exercise time, rest streaks, cooldown, and calorie targets—were satisfied. The heart rate curve displayed a steady rise with controlled plateaus, reflecting a viable physiological trajectory for interval training.

This model offers valuable insights for fitness planning and optimization. It has the potential to assist athletes, coaches, and digital health applications in developing efficient and personalized training regimens. Future work could explore more sophisticated heart rate dynamics, user-specific parameterization, and integration with real-time biometric feedback.

In conclusion, the proposed model serves as a foundational tool for optimizing interval training sessions with quantifiable fitness outcomes and adherence to safety standards.

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