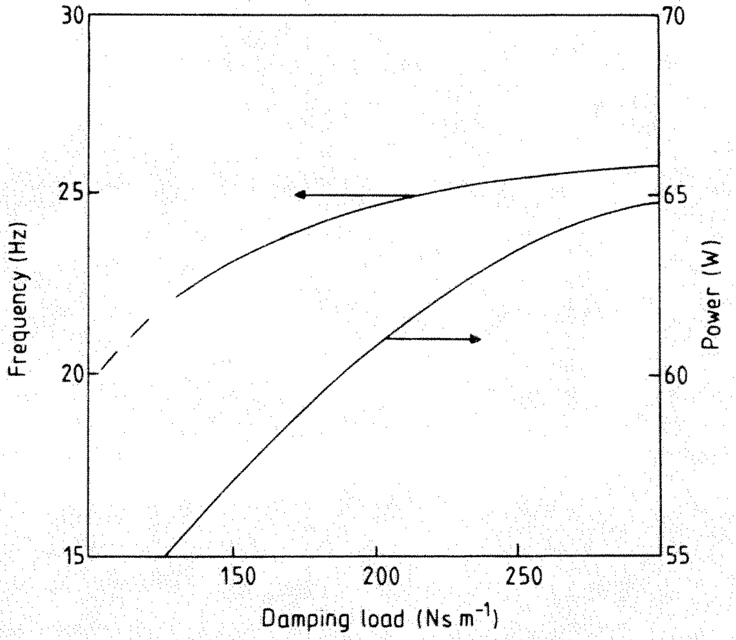

Figure 3.12 Frequency and power versus load for the Sunpower M100 free-cylinder engine.

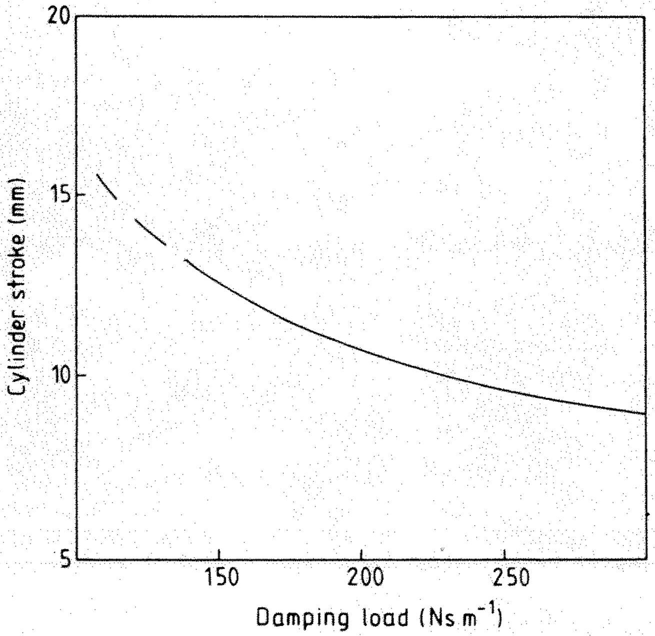

Figure 3.13 Stroke versus load for the Sunpower M100 free-cylinder engine.

3.6 The Harwell Thermo-Mechanical Generator (TMG)

The TMG is a unique free-piston machine in many respects. Instead of a piston, this engine has a diaphragm which flexes to effect the change of working gas volume. In addition, the stiffness of the diaphragm acts as a mechanical spring.

The displacer, too, is supported on a mechanical spring, which is connected to the casing (figure 3.14). The motion of the displacer is not a direct result of the pressure forces but is a result of feedback from the casing motions. Fortunately, it is still possible to neglect the casing motions for the purposes of the linear analysis since the required casing amplitude is considerably smaller than both the diaphragm and displacer amplitudes. A disadvantage of this system is that it is necessary to limit the mechanical stress to below the fatigue limit for the diaphragm material, in order that infinite operating life be obtained. Thus, the amplitude may be typically only of the order of 1 mm or less. This usually requires that the machine operates at high frequencies to achieve any reasonable power output. However, since there is no sliding contact, the reciprocating elements do not suffer any static friction at rest and consequently the machine will self-start reliably. In many applications this feature may be the most important one. Another important feature is that there is no wear, so there is essentially no limitation to operating life.

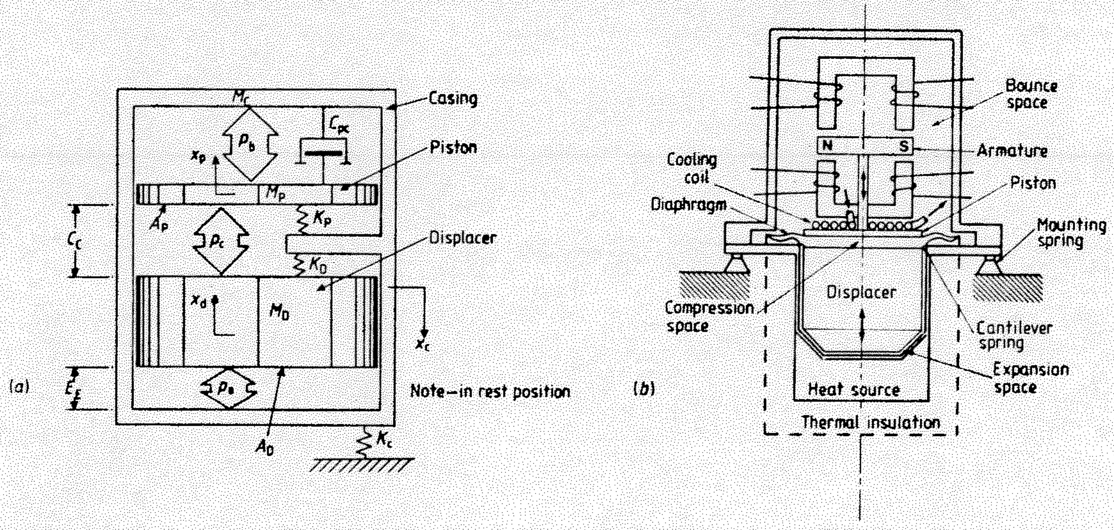

Figure 3.14 The Harwell Thermo-Mechanical Generator: (*a*) Notation; (*b*) Schematic.

A schematic view of the TMG is shown in figure 3.15.

In writing the equations of motion, the diaphragm will be indicated by the subscript p, since it is essentially the piston of the device. Thus, referring to figure 3.14, the equations of motion are

$$M_P \ddot{x}_p = A_P(p_c - p_b) - K_P(x_p + x_c) - C_{pc}(\dot{x}_p + \dot{x}_c) \tag{3.119}$$

$$M_D \ddot{x}_d = -K_D(x_d + x_c) + A_D \Delta p \tag{3.120}$$

$$M_C \ddot{x}_c = A_P(p_c + \Delta p - p_b) - K_P(x_c + x_p) - K_D(x_c + x_d) - K_C x_c - C_{pc}(\dot{x}_c + \dot{x}_p). \tag{3.121}$$

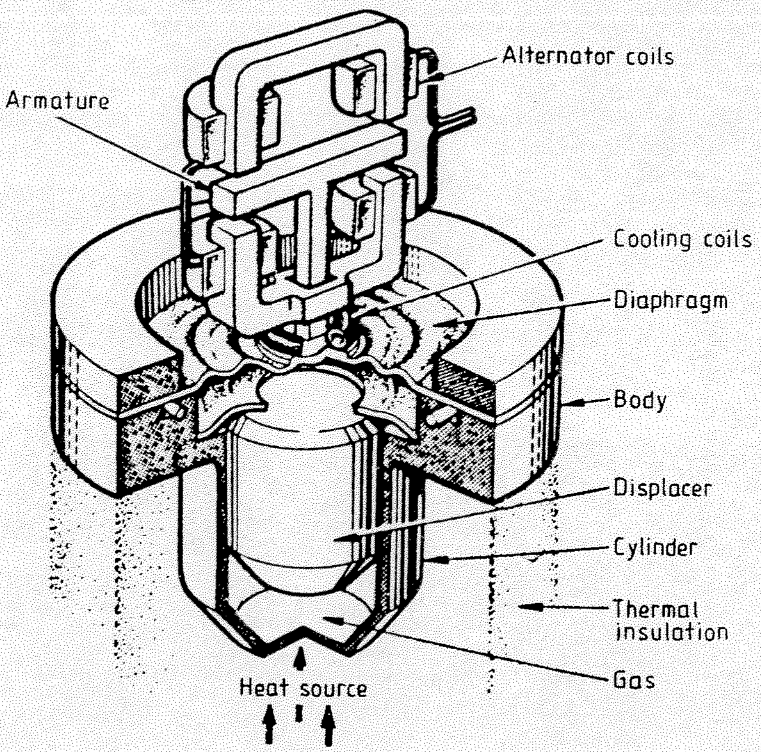

Figure 3.15 The Harwell Thermo-Mechanical Generator—schematic view (courtesy *The Chartered Mechanical Engineer*).

Owing to the generally large bounce space volume compared with the diaphragm (piston) stroke volume, the bounce space pressure may be taken as constant and the gas spring hysteresis neglected. The linear coefficients for this case are therefore:

$$S_{\mathrm{pp}} = -\frac{1}{M_{\mathrm{P}}}\left(\frac{p_{\mathrm{mean}}A_{\mathrm{P}}^{2}}{T_{\mathrm{k}}S} + K_{\mathrm{P}}\right) \qquad S_{\mathrm{pd}} = p_{\mathrm{mean}}\frac{A_{\mathrm{P}}A_{\mathrm{D}}}{M_{\mathrm{P}}S}\left(\frac{1}{T_{\mathrm{k}}} - \frac{1}{T_{\mathrm{h}}}\right)$$

$$D_{\mathrm{pp}} = -C_{\mathrm{pc}}/M_{\mathrm{P}} \qquad D_{\mathrm{pd}} = 0$$

$$S_{\mathrm{dp}} = 0 \qquad S_{\mathrm{dd}} = -K_{\mathrm{D}}/M_{\mathrm{D}}$$

$$D_{\mathrm{dp}} = C_{\mathrm{p}}/M_{\mathrm{D}} \qquad D_{\mathrm{dd}} = C_{\mathrm{d}}/M_{\mathrm{D}}$$

$$S_{\mathrm{cc}} = -\frac{1}{M_{\mathrm{C}}}\left(\frac{p_{\mathrm{mean}}A_{\mathrm{P}}A_{\mathrm{D}}}{T_{\mathrm{h}}S} + K_{\mathrm{P}} + K_{\mathrm{D}} + K_{\mathrm{C}}\right).$$

The frequency equation (from table 3.1) becomes

$$\omega^{2} = (D_{\mathrm{dp}}S_{\mathrm{pd}} - D_{\mathrm{dd}}S_{\mathrm{pp}} - S_{\mathrm{dd}}D_{\mathrm{pp}})/(D_{\mathrm{dd}} + D_{\mathrm{pp}}). \tag{3.122}$$

Typically, the regenerator on the TMG consists of a series of annular spaces connecting the hot and cold ends. The hot and cold heat exchangers are simply the actual working spaces of the machine. These types of heat exchangers tend to exhibit extremely small pressure drops. It is generally possible, therefore, to neglect the effect of the heat exchanger pressure drop on the reciprocating elements. Thus we may set

$$D_{\mathrm{dp}} \simeq 0 \text{ and } D_{\mathrm{dd}} \simeq 0. \tag{3.123}$$

The frequency equation now becomes

$$\omega^2 = -S_{\mathrm{dd}} \tag{3.124}$$

which is simply the natural frequency of the displacer, and is independent of the load.

From the geometric constraint equation it can be seen that the following must also hold

$$S_{\mathrm{pp}} = S_{\mathrm{dd}}. \tag{3.125}$$

Again from table 3.1, the phase and the amplitude ratio are simply

$$\phi = -90^\circ \tag{3.126}$$

$$r = \omega D_{\mathrm{pp}}/S_{\mathrm{pd}}. \tag{3.127}$$

Unfortunately, data for this engine are not available in the open literature. These data have, however, been made available to the authors on a confidential basis and may not, therefore, be published here. The performance characteristics indicated in figure 3.16 are generated from the data supplied. Actual

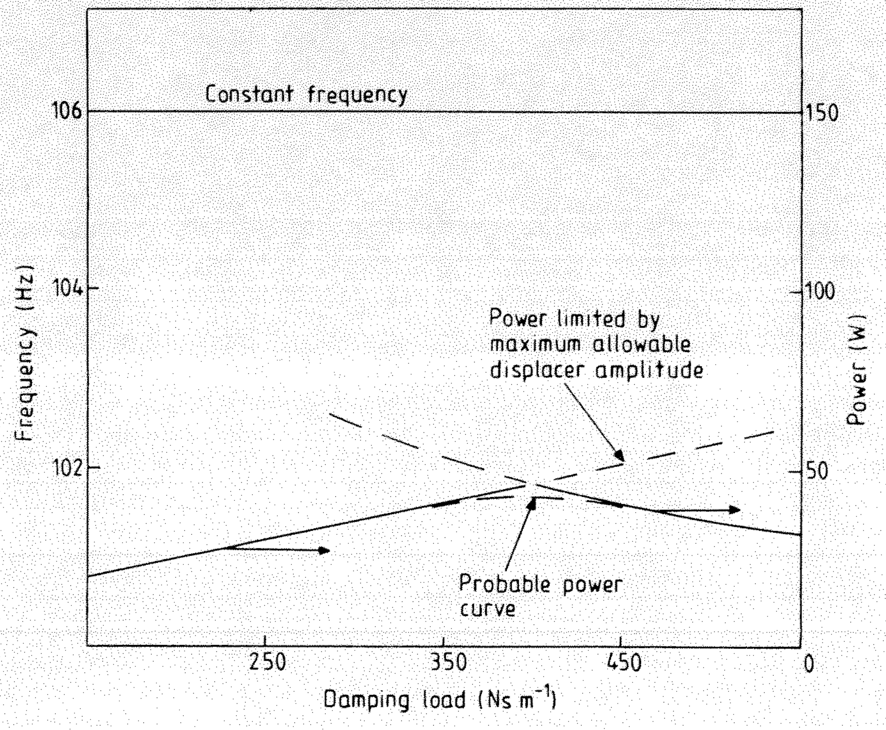

Figure 3.16 Frequency and power versus load for the Harwell Thermo-Mechanical Generator engine.

frequency and power output of the TMG are 106 Hz and 25 W electrical power respectively. Again, it can be seen that the linear analysis agrees with actual results to within an acceptable margin. Note that in this case the increase of power with load is limited at around 400 Ns m^{-1}, when the growth of the displacer amplitude becomes constrained by the geometry of the engine. At this point the displacer amplitude remains fixed at its maximum value and the piston amplitude begins to drop off with further load, thus reducing the power.

References

Anvoner S 1970 *Solution of Problems in Mechanics of Machines* vol 1 (London: Pitman)

Beale W T 1969 Free-piston Stirling Engines—Some Model Tests and Simulations *SAE Paper No 690230*

—— 1971 Stirling Cycle Type Thermal Device *US Patent 3 552 120*

—— 1979 A Free Cylinder Stirling Engine Solar Powered Water Pump *Int. Solar Energy Soc. Int. Congress*

Beale W T, Rankin C F and Wood J G 1975 *Free-piston Stirlilg engine–Inertia compressor prototype gas fired air conditioner* (American Gas Association)

Beale W T, Rauch J, Lewis R and Mulej D 1971 Free Cylinder Stirling Engines for Solar-Powered Water Pumps *ASME Paper No 71-WA/Sol-11*

Benson G M 1973 Thermal Oscillators *Proc. 8th IECEC, Philadelphia, August 13–17 1973* Paper No. 739076 pp 182–29

—— 1977 Thermal Oscillator *US Patent 4,044,558*

Berchowitz D M and Wyatt-Mair G F 1979 Closed-Form Polutions for a Coupled Ideal Analysis of Free-Piston Stirling Engines. *Proc. 14th IECEC, Boston*, pp 1114–9

Cooke-Yarborough E H 1967 A Proposal for a Heat-Powered Nonrotating Electrical Alternator *Harwell Memorandum AERE-M881 UK AERE*

—— 1970 Heat Engines *US Patent 3,548,589*

Cooke-Yarborough E H, Franklin E, Geisow J, Howlett R and West C D 1974 Harwell Thermo-Mechanical Generator *Proc. 9th IECEC, San Francisco August 26–30 1974* Paper No 749156 pp 1132–6

Gedeon D R 1978 The Optimisation of Stirling Cycle Machines, *Proc, 13th IECEC San Diego August 20–25 1978* Paper No. 789193 pp 1784–90

Goldberg L F 1979 *A Computer Simulation and Experimental Development of Liquid Piston Stirling Cycle Engines* MSc Dissertation, University of the Witwatersrand, South Africa

Goldberg L F and Rallis C H 1979 Prototype Liquid-Piston Free-Displacer Stirling Engine *Proc. 14th IECEC, Washington DC* Paper No 799239 pp 1103–7

Goldwater B and Morrow R B 1977 Demonstration of a Free-Piston Stirling Linear Alternator Power Conversion System. *Proc. 12th IECEC, Washington DC* Paper No 779249 pp 1488–95

Gupta S C and Hasdorff L 1970 *Fundamentals of Automatic Control* (Chichester: Wiley)

Martini W R 1975 The Free-Displacer, Free-Piston Stirling engine—Potential Energy Conserver, *Proc. 10th IECEC* Paper No 759149 pp 995–1002

Reader G T 1979 Modes of Operation of a Jet-stream Fluidyne *Proc. 14th IECEC, Boston* Paper No 799238 pp 1098–102

Walker G 1980 *Stirling Engines* (Oxford: Oxford University Press)

Wood J G 1980 *A Program for Predicting the Dynamics of Free Piston Stirling Engines ME Thesis* Mech. Eng. Dept. Ohio University