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DARWIN-OP HUMANOID ROBOT KINEMATICS

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ABSTRACT

This paper presents kinematics analysis for the DARwIn-OP (Dynamic Anthropomorphic Robot with Intelligence – Open-Platform) robot. This is a 20-dof humanoid walking robot developed by Virginia Tech, Purdue, and the University of Pennsylvania and marketed by Robotis Inc. The robot version analyzed in this paper is 455 mm tall and has a mass of 2.8 kg. Ohio University has two of these units for robot applications research and teaching.

Presented are a description of DARwIn-OP, the Denavit-Hartenberg Parameters for each serial chain (2-dof head pan/tilt, 3-dof arms, and 6-dof legs), specific length parameters, joint angle limits, plus forward pose kinematics equations and partial inverse pose kinematics solutions, with examples.

1. INTRODUCTION

DARwIn-OP is a unique open-platform humanoid robot sponsored by NSF, developed collaboratively by Virginia Tech (www.romela.org), Purdue, and the University of Pennsylvania, and marketed by Robotis Inc. (www.robotis.com). Thanks to NSF support, Ohio University was one of the institutions who received two DARwIn humanoid robots for research and teaching. DARwIn is being used to play robot soccer, but we are investigating other humanoid robot applications.

DARwIn-OP is intended to be open-platform, with a community of users contributing to robot developments following the initial collaborator's developments. The software and control architecture is all open-source and free to the world. Multiple computer platforms and various programming languages are said to be allowed. Programming enhancements should be shared with the DARwIn community. In addition, the design of the humanoid robot is open-source, with complete CAD files and parts lists available for free so one could build the robot from scratch. One could make improvements and innovations to the existing robot design, which should also be shared with the DARwIn community.

This paper is organized as follows. First we describe the 20 degrees-of-freedom (dof) DARwIn-OP robot concept and hardware. Then we present the Denavit-Hartenberg (DH) parameters for each serial chain of the humanoid robot (2-dof head pan/tilt, 3-dof arms, and 6-dof legs). Included in this section are the specific length parameters and joint angle limits for the 455 mm tall DARwIn-OP. Next we present forward pose kinematics equations for each serial chain, with examples. Last we present partial inverse pose kinematics solutions for each serial chain of the robot.

All analyses in this paper are from well-known serial robotics techniques (Craig, 2005). We could not find this kinematics analysis from any source so this will be part of our contribution to the DARwIn community.

2. DARwIn-OP DESCRIPTION

The 20-dof DARwIn Humanoid Robot is shown in Figure 1, in the standard pose with all zero joint angles. In the hardware shown in Figure 1, each arm has three single-dof revolute (R) joints, each leg has six single-dof R joints, and the pan/tilt head has two single-dof R joints. Therefore, the overall robot has 20 dof.

Each arm has a 2-dof (offset-U-joint) shoulder joint and a 1-dof elbow joint, for a total of 3-dof per arm. Each leg has a 3-dof (S-joint with 3 intersecting R joints) hip joint, a 1-dof knee joint, and a 2-dof (U-joint) ankle joint for a total of 6-dof per leg. The last hip joint, the knee joint, and the first ankle joint are all about parallel *Z* axes. The 2-dof head (U-joint) enables pan and tilt for the camera via an azimuth R joint and an elevation R joint.

Each DARwIn-OP robot joint is driven by a Dynamixel MX-28 servomotor controlled by an internal CM-730 Robotis servo-controller. The DARwIn-OP sensors include an HD digital video camera, tri-axis gyroscope and accelerometer, and two microphones. The DARwIn-OP on-board computer is a 1.6 GHz Intel Atom Z530 FitPC2i with 4 GB SSD and the Ubuntu Linux operating system. DARwIn is programmed in C++ and is able to operate both tethered and battery-powered. The LiPO 3CELL 11.1v 1000mAh battery provides up to 30 operation minutes (Merlin Robotics, 2012).

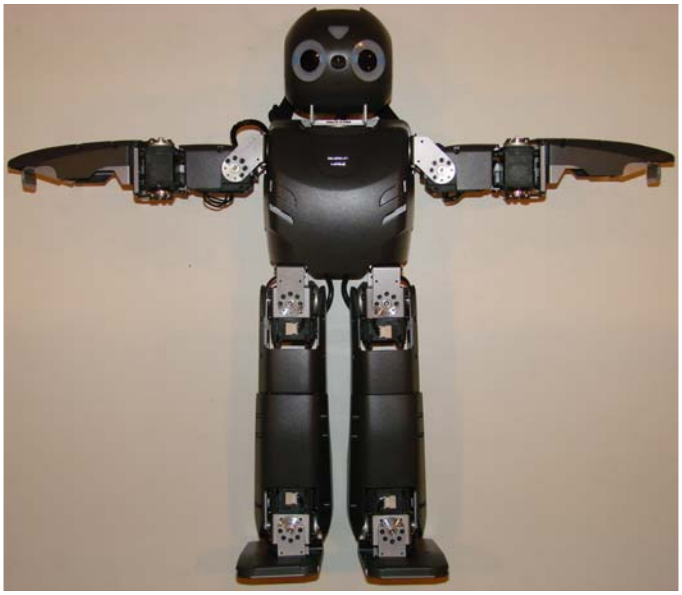

Figure 1. DARwIn-OP Humanoid Robot, Zero Pose

The numbering convention for the 20 joint servomotors is shown in Figure 2. The joint angle values will be denoted by θ_i, $i = 1,2,\ldots,20$, according to the numbering scheme in Figure 2. Again, the pose shown is for all zero joint angles.

For notational simplicity in Cartesian coordinate frame definition, frame numbers {0}, {1}, {2}, etc. will be recycled for the five serial chains. Thus, there will be five Z_0 axes, four Z_3 axes, two Z_6 axes, and so on. This is to avoid the onerous notion of Z_{0RA}, Z_{0LL}, etc. (representing right arm and left leg).

According to Craig (2005) notation for the Denavit-Hartenberg Parameters (1955), joint angle θ_i is supposed to rotate about R-joint axis Z_i. Again for simplicity in notation, this rule will be violated, in order to number the joint angles as shown in Figure 2.

3. DARwIn-OP PARAMETERS

The DARwIn-OP Denavit-Hartenberg (DH, Denavit and Hartenberg, 1955) parameters are presented in this section, for each of the serial chains (head, arm, and leg). DH parameters have been adopted for standard kinematics analysis in serial-chain robots (Craig, 2005). The physical length parameters are given first, followed by the joint angle limits, and then the DH parameters for the DARwIn-OP humanoid robot. We present DH parameters for the 2-dof head, the 3-dof right arm, and the 6-dof right leg (the 3-dof left arm and 6-dof left leg are symmetric to their counterparts, as seen in Figures 1 and 2).

3.1 DARwIn-OP robot lengths

This section presents the various link lengths for the DARwIn-OP Humanoid Robot. These are specific hardware design values associated with the DH Parameters given in Section 3.3. Refer to the lengths definitions in Figure 3 for these parameters.

The basic torso reference frame is shown in red in Figure 3. This {*C*} dextral Cartesian reference frame, stands for Chest. Frame {*C*} is moving and serves as the reference frame for all five serial chains. A fixed world {*Wo*} Cartesian reference frame (not shown) may be used for DARwIn-OP absolute kinematic referencing.

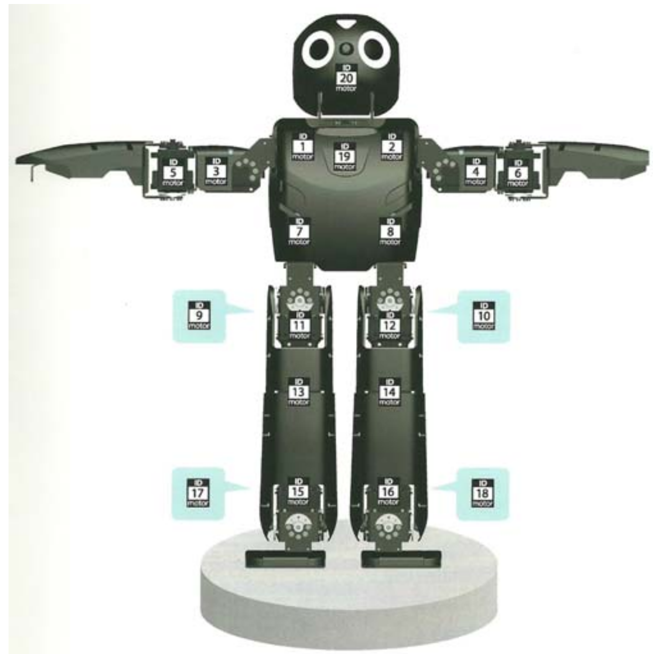

Figure 2. DARwIn-OP Servomotor Numbering (Robotis, 2011)

3.1.1 Torso Lengths

These values are not DH Parameters but will be introduced later in the Forward Pose Kinematics solutions. The subscript *h* for L_h is lower-case and indicates 'head'.

Length	Value (mm)
L_h	50.5
L_0	82.0
L_{TX}	5.0
L_{TY}	122.2
L_{TZ}	37.0

3.1.2 Head Lengths

These values are not DH Parameters but will be introduced later in the Forward Pose Kinematics solutions.

Length	Value (mm)
H_X	33.2
H_Y	34.4
H_Z	22.5

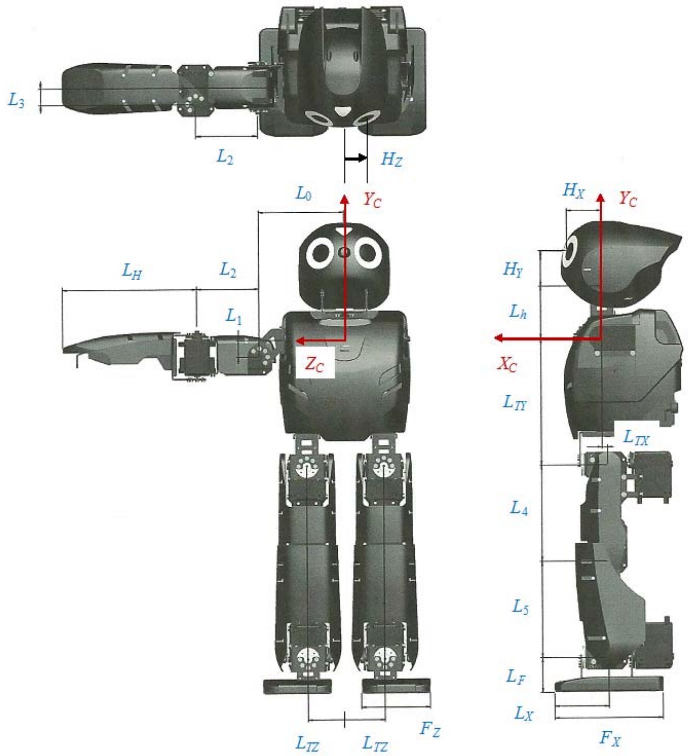

Figure 3. DARwIn Robot Lengths Definitions

3.1.3 Right and Left Arm Lengths

These DH Parameters are shown in Figure 3. L_H is not a DH Parameter but will be used later in the Forward Pose Kinematics solutions. The subscript H for L_H is upper-case and indicates 'hand'.

Length	Value (mm)
L_1	16.0
L_2	60.0
L_3	16.0
L_H	129.0

3.1.4 Right and Left Leg Lengths

These DH Parameters are shown in Figure 3. L_F is not a DH Parameter but will be used later in the Forward Pose Kinematics solutions.

Length	Value (mm)
L_4	93.0
L_5	93.0
L_F	33.5

3.1.5 Right and Left Foot Lengths

DARwIn's feet (see Figure 3) are 104 mm long, 66 mm wide and 15 mm thick (tapering to 10 mm thick at the front edge). The feet are centered with the ankle joint front-to-back (L_X = 52 mm), but not side-to-side (L_Z = 23 mm from the inner edge of each foot to the ankle joint).

Length	Value (mm)
F_X	104.0
F_Y	15.0 (10.0 front)
F_Z	66.0
L_X	52.0
L_Z	23.0

3.2 DARwIn-OP Joint Angle Limits

This section presents the kinematic joint angle limits for the DARwIn-OP Humanoid Robot. These are specific hardware design values associated with the DH Parameters given in the following section. Refer to the joint numbering given in Figure 2. All units in Table I are degrees.

Table I. DARwIn-OP Joint Limits

Joint *i*	Joint name	Axis	θ_{iMIN}	θ_{iMAX}
Right Arm				
1	shoulder pitch	Z_1	–250	250
3	shoulder roll	Z_2	–100	100
5	elbow	Z_3	0	160
Left Arm				
2	shoulder pitch	Z_1	–250	250
4	shoulder roll	Z_2	–100	100
6	elbow	Z_3	–160	0
Right Leg				
7	hip yaw	Z_1	–150	45
11	hip roll	Z_2	0	60
9	hip pitch	Z_3	–100	30
13	knee	Z_4	0	130
17	ankle pitch	Z_5	–60	60
15	ankle roll	Z_6	–30	60
Left Leg				
8	hip yaw	Z_1	–45	150
12	hip roll	Z_2	–60	0
10	hip pitch	Z_3	–30	100
14	knee	Z_4	–130	0
18	ankle pitch	Z_5	–60	60
16	ankle roll	Z_6	–30	60
Head				
19	head pan	Z_1	–150	150
20	head tilt	Z_2	–60	30

3.3 DARwIn-OP DH Parameters

The standardized Denavit-Hartenberg (DH) Parameters (1955) are used to describe the links/joints geometry of a serial-chain robot. In this section the DH Parameters are presented for each of the DARwIn-OP serial chains, i.e. pan/tilt head, right arm, and right legs. Craig (2005) convention is used for the DH Parameters in this paper. All angular units in the DH Parameter tables below are degrees.

The Cartesian reference frame definitions for three DARwIn-OP serial chains (2-dof head, 3-dof right arm, 6-dof right leg) are given in Figures 4, 5, and 6. It is important to note that all of these DH parameters figures are shown in the zero-angle poses. Therefore it looks like many moving coordinate frame axes line up, but this will change as joint angles rotate away from zero in all cases. The 3-dof left arm and 6-dof left leg are symmetric to the right arm and leg, so these cases are not presented here.

3.3.1 Two-dof Pan/Tilt Head

The Cartesian reference frame definitions for the two-dof pan/tilt head are shown in Figure 4. Table II gives the DH parameters for the two-dof pan/tilt head.

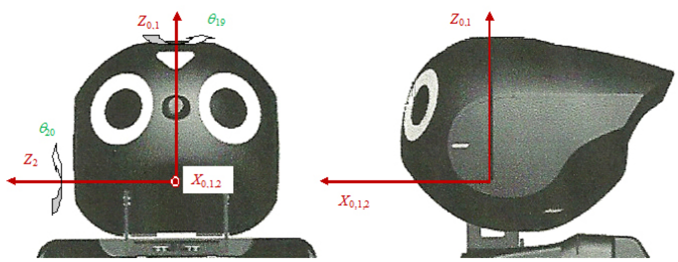

Figure 4. Two-dof Pan/Tilt Head Coordinate Frames

Table II. Two-dof Pan/Tilt Head DH Parameters

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_{19}
2	90	0	0	θ_{20}

3.3.2 Three-dof Right Arm

The Cartesian reference frame definitions for the three-dof right arm are shown in Figure 5. Table III gives the DH parameters for the three-dof right arm.

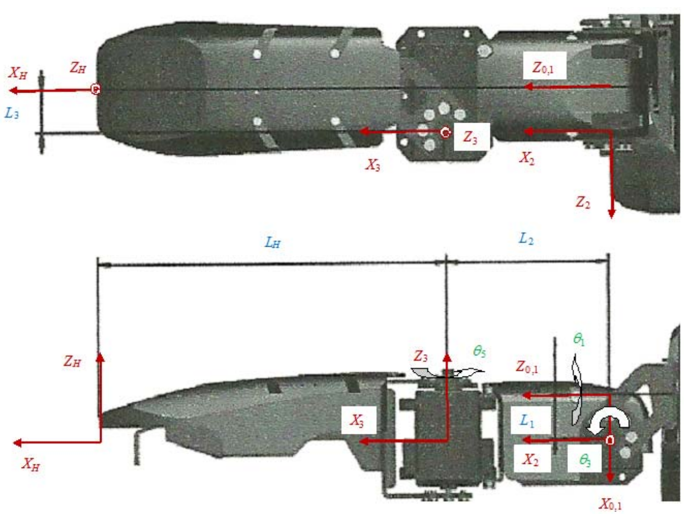

Figure 5. Three-dof Right Arm Coordinate Frames (Top and Front Views)

Table III. Three-dof Right Arm DH Parameters

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	–90	L_1	L_3	θ_3–90
3	90	L_2	0	θ_5

3.3.3 Six-dof Right Leg

The Cartesian reference frame definitions for the six-dof right leg are shown in Figure 6. Table IV gives the DH parameters for the six-dof right leg.

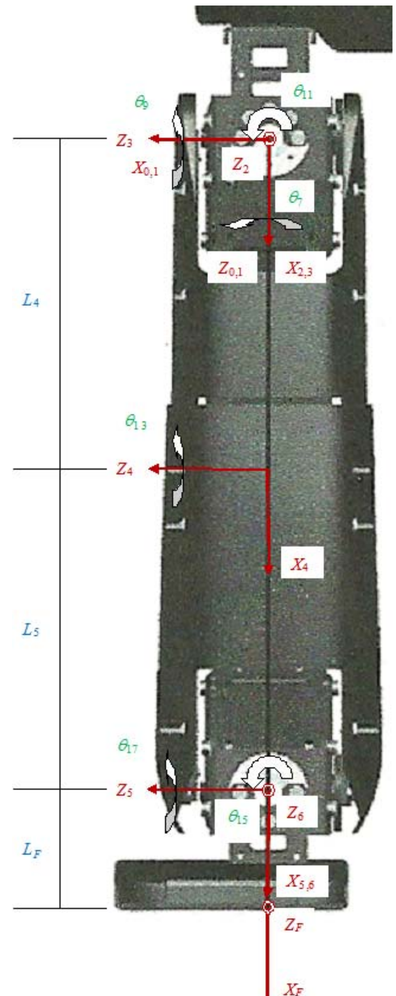

Figure 6. Six-dof Right Leg Coordinate Frames

Table IV. Six-dof Right Leg DH Parameters

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_7
2	90	0	0	θ_{11}+90
3	90	0	0	θ_9
4	0	L_4	0	θ_{13}
5	0	L_5	0	θ_{17}
6	–90	0	0	θ_{15}

4. DARwIn-OP FORWARD POSE KINEMATICS

In general, the Forward Pose Kinematics (FPK) problem for a serial-chain robot is stated: Given the joint values, calculate the pose (position and orientation) of the end frame of interest. For serial-chain robots, the FPK problem set up and solution is straight-forward. It is based on substituting each line of the Denavit-Hartenberg Parameters table into the equation below (Craig, 2005), giving the pose of frame $\{i\}$ with respect to its nearest neighbor frame $\{i–1\}$ back along the serial chain:

$$\left[{}^{i-1}_{\;\;i}T\right]=\begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -d_i s\alpha_{i-1} \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & d_i c\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The equation above represents pose (position and orientation) of frame $\{i\}$ with respect to frame $\{i–1\}$ by using a 4x4 homogeneous transformation matrix. The upper left 3x3 matrix is the rotation matrix giving the orientation and the upper right 3x1 vector is the position vector.

Then transformation equations are used to find the pose of the overall end-frame of interest with respect to the base reference frame, to complete the FPK solution for each serial chain.

4.1 Two-dof Pan/Tilt Head FPK Expressions

The statement of the FPK problem for the two-dof pan/tilt head serial chain of the DARwIn-OP humanoid robot is:

Given $(\theta_{19},\theta_{20})$, calculate $\left[{}^0_2T\right]$ and $\left\{{}^0X_2\right\}$.

where $\left\{{}^0X_2\right\}$ is the pointing vector along the X_2 unit vector direction, expressed in $\{0\}$ coordinates.

Substitute each row of the DH parameters in Table II into the above equation for $\left[{}^{i-1}_{\;\;i}T\right]$ to obtain the two neighboring homogeneous transformation matrices as a function of the joint angles.

$$\left[{}^0_1T\right]=\begin{bmatrix} c_{19} & -s_{19} & 0 & 0 \\ s_{19} & c_{19} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \left[{}^1_2T\right]=\begin{bmatrix} c_{20} & -s_{20} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_{20} & c_{20} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where the following abbreviations were used: $c_i=\cos\theta_i$, $s_i=\sin\theta_i$, for i = 19,20.

Now substitute these two neighboring homogeneous transformation matrices into the following homogeneous transform equation to derive the FPK result.

$$\left[{}^0_2T\right]=\left[{}^0_1T(\theta_{19})\right]\left[{}^1_2T(\theta_{20})\right]=\begin{bmatrix} c_{19}c_{20} & -c_{19}s_{20} & s_{19} & 0 \\ s_{19}c_{20} & -s_{19}s_{20} & -c_{19} & 0 \\ s_{20} & c_{20} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note that the fourth column result, the position vector $\left\{{}^0X_2\right\}$ giving the origin of $\{2\}$ from the origin of $\{0\}$, expressed in $\{0\}$ coordinates, is all zero components since the pan/tilt axes are intersecting. There are no kinematic lengths or length offsets involved.

To derive the second FPK result $\left\{{}^0X_2\right\}$, use the following transform equation, where $\left\{{}^2X_2\right\}=\{1 \quad 0 \quad 0 \quad 1\}^T$.

$$\left\{{}^0X_2\right\}=\left[{}^0_2T\right]\left\{{}^2X_2\right\}=\begin{Bmatrix} c_{19}c_{20} \\ s_{19}c_{20} \\ s_{20} \\ 1 \end{Bmatrix}$$

The desired pointing vector is contained in the first three elements of $\left\{{}^0X_2\right\}$, i.e. the 1 in the fourth location is merely a placeholder for homogeneous transformation matrix multiplication.

The basic pan/tilt head FPK result is $\left[{}^0_2T\right]$. To calculate the pose of each eye frame (*RE* and *LE* for right and left eye, respectively) with respect to the DARwIn chest reference frame $\{C\}$, the following transform equations are used.

$$\left[{}^{C}_{RE}T\right]=\left[{}^C_0T\right]\left[{}^0_2T(\theta_{19},\theta_{20})\right]\left[{}^{2}_{RE}T\right]$$
$$\left[{}^{C}_{LE}T\right]=\left[{}^C_0T\right]\left[{}^0_2T(\theta_{19},\theta_{20})\right]\left[{}^{2}_{LE}T\right]$$

$$\left[{}^C_0T\right]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & L_h \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \left[{}^{2}_{RE}T\right]=\begin{bmatrix} 1 & 0 & 0 & H_X \\ 0 & 1 & 0 & H_Y \\ 0 & 0 & 1 & H_Z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[{}^{2}_{LE}T\right]=\begin{bmatrix} 1 & 0 & 0 & H_X \\ 0 & 1 & 0 & H_Y \\ 0 & 0 & 1 & -H_Z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The overall transform equations above can be evaluated numerically. Constants L_h, H_X, H_Y, and H_Z were given earlier.

Note that $\left[{}^C_0T\right]$, $\left[{}^{2}_{RE}T\right]$, and $\left[{}^{2}_{LE}T\right]$ are not evaluated by any row in the DH parameter table, since there is no variable associated with these fixed homogeneous transformation matrices based on constant lengths and orientation. Instead, they are determined by inspection, using the rotation matrix and position vector components of the homogeneous transformation matrix definition.

In the above expressions for $\left[{}^{2}_{RE}T\right]$ and $\left[{}^{2}_{LE}T\right]$ the orientation of each eye frame $\{RE\}$ and $\{LE\}$ was arbitrarily assigned as identity, i.e. identical to the orientation of $\{2\}$. This can easily be changed as required.

4.2 Three-dof Right Arm FPK Expressions

The statement of the FPK problem for the three-dof right arm serial chain of the DARwIn-OP humanoid robot is:

Given $(\theta_1,\theta_3,\theta_5)$, calculate $\left[{}^{0}_{3}T\right]$ and $\left[{}^{C}_{H}T\right]$.

Where $\{H\}$ is the right-arm end-effector (hand) frame and $\{C\}$ is the DARwIn reference frame in the chest. As stated earlier, for notational simplicity in Cartesian coordinate frame definition, frame numbers $\{0\}$, $\{1\}$, $\{2\}$, etc. will be recycled for the five serial chains (head, right and left arms, right and left legs). Therefore, in this paper many repeated frame numbers are context-dependent, which must be sorted out in programming the overall humanoid robot.

Substitute each row of the DH parameters in Table III into the equation for $\left[{}^{i-1}_{i}T\right]$ to obtain the three neighboring homogeneous transformation matrices as a function of the joint angles.

$$\left[{}^{0}_{1}T\right]=\begin{bmatrix} c_1 & -s_1 & 0 & 0\\ s_1 & c_1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \qquad \left[{}^{1}_{2}T\right]=\begin{bmatrix} s_3 & c_3 & 0 & L_1\\ 0 & 0 & 1 & L_3\\ c_3 & -s_3 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$\left[{}^{2}_{3}T\right]=\begin{bmatrix} c_5 & -s_5 & 0 & L_2\\ 0 & 0 & -1 & 0\\ s_5 & c_5 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Where the following abbreviations were used: $c_i=\cos\theta_i$, $s_i=\sin\theta_i$, for i = 1,3,5.

Now substitute these three neighboring homogeneous transformation matrices into the following homogeneous transform equation to derive the FPK result.

$$\left[{}^{0}_{3}T\right]=\left[{}^{0}_{1}T(\theta_1)\right]\left[{}^{1}_{2}T(\theta_3)\right]\left[{}^{2}_{3}T(\theta_5)\right]$$
$$=\begin{bmatrix} -s_1s_5+c_1s_3c_5 & -s_1c_5-c_1s_3s_5 & -c_1c_3 & L_1c_1+L_2c_1s_3-L_3s_1\\ c_1s_5+s_1s_3c_5 & c_1c_5-s_1s_3s_5 & -s_1c_3 & L_1s_1+L_2s_1s_3+L_3c_1\\ c_3c_5 & -c_3s_5 & s_3 & L_2c_3\\ 0 & 0 & 0 & 1\end{bmatrix}$$

The basic right-arm FPK result is $\left[{}^{0}_{3}T\right]$. To calculate $\left[{}^{C}_{H}T\right]$, the pose of the right-arm end-effector frame $\{H\}$ with respect to the DARwIn chest reference frame $\{C\}$, the following transform equation is used.

$$\left[{}^{C}_{H}T\right]=\left[{}^{C}_{0}T\right]\left[{}^{0}_{3}T(\theta_1,\theta_3,\theta_5)\right]\left[{}^{3}_{H}T\right]$$

$$\left[{}^{C}_{0}T\right]=\begin{bmatrix} 0 & 1 & 0 & 0\\ -1 & 0 & 0 & 0\\ 0 & 0 & 1 & L_0\\ 0 & 0 & 0 & 1\end{bmatrix} \qquad \left[{}^{3}_{H}T\right]=\begin{bmatrix} 1 & 0 & 0 & L_H\\ 0 & 1 & 0 & -L_3\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

The overall transform equation above can be evaluated numerically. Constants L_0, L_3, and L_H were given earlier.

Note that $\left[{}^{C}_{0}T\right]$ and $\left[{}^{3}_{H}T\right]$ are not evaluated by any row in the DH parameter table, since there is no variable associated with these fixed homogeneous transformation matrices based on constant lengths and orientation. Instead, they are determined by inspection, using the rotation matrix and position vector components of the homogeneous transformation matrix definition.

For use in inverse pose kinematics, here is the resulting position vector of the hand tip with respect to the $\{0\}$ frame:

$$\left\{{}^{0}X_H\right\}=\left[{}^{0}_{3}T(\theta_1,\theta_3,\theta_5)\right]\left\{{}^{3}X_H\right\}$$
$$=\begin{Bmatrix} L_1c_1+L_2c_1s_3-L_3(s_1-s_1c_5-c_1s_3s_5)+L_H(-s_1s_5+c_1s_3c_5)\\ L_1s_1+L_2s_1s_3-L_3(-c_1+c_1c_5-s_1s_3s_5)+L_H(c_1s_5+s_1s_3c_5)\\ L_2c_3+L_3c_3s_5+L_Hc_3c_5\\ 1\end{Bmatrix}$$

4.3 Six-dof Right Leg FPK Expressions

The statement of the FPK problem for the six-dof right leg serial chain of the DARwIn-OP humanoid robot is:

Given $(\theta_7,\theta_{11},\theta_9,\theta_{13},\theta_{17},\theta_{15})$, calculate $\left[{}^{0}_{6}T\right]$ and $\left[{}^{C}_{F}T\right]$.

Where $\{F\}$ is the right-leg end-effector (foot) frame and $\{C\}$ is the DARwIn chest reference frame. As stated earlier, for notational simplicity in Cartesian coordinate frame definition, frame numbers $\{0\}$, $\{1\}$, $\{2\}$, etc. will be recycled for the five serial chains (head, right and left arms, right and left legs). Therefore, in this paper many repeated frame numbers are context-dependent, which must be sorted out in programming the overall humanoid robot.

Substitute each row of the DH parameters in Table IV into the equation for $\left[{}^{i-1}_{i}T\right]$ to obtain the six neighboring homogeneous transformation matrices as a function of the joint angles.

$$\left[{}^{0}_{1}T\right]=\begin{bmatrix} c_7 & -s_7 & 0 & 0\\ s_7 & c_7 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \qquad \left[{}^{1}_{2}T\right]=\begin{bmatrix} -s_{11} & -c_{11} & 0 & 0\\ 0 & 0 & -1 & 0\\ c_{11} & -s_{11} & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$\left[{}^{2}_{3}T\right]=\begin{bmatrix} c_9 & -s_9 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_9 & c_9 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left[{}^{3}_{4}T\right]=\begin{bmatrix} c_{13} & -s_{13} & 0 & L_4 \\ s_{13} & c_{13} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[{}^{4}_{5}T\right]=\begin{bmatrix} c_{17} & -s_{17} & 0 & L_5 \\ s_{17} & c_{17} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left[{}^{5}_{6}T\right]=\begin{bmatrix} c_{15} & -s_{15} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{15} & -c_{15} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where the following abbreviations were used: $c_i = \cos\theta_i$, $s_i = \sin\theta_i$, for i = 7,11,9,13,17,15.

Now substitute these three neighboring homogeneous transformation matrices into the following homogeneous transform equation to derive the FPK result.

$$\left[{}^{0}_{6}T\right]=\left[{}^{0}_{1}T(\theta_7)\right]\left[{}^{1}_{2}T(\theta_{11})\right]\left[{}^{2}_{3}T(\theta_9)\right]\left[{}^{3}_{4}T(\theta_{13})\right]\left[{}^{4}_{5}T(\theta_{17})\right]\left[{}^{5}_{6}T(\theta_{15})\right]$$

Since there are three parallel Z axes (3,4,5) in the right leg, we can group the above matrix multiplications as follows for a significant simplification.

$$\left[{}^{0}_{6}T\right]=\left[{}^{0}_{1}T(\theta_7)\right]\left[{}^{1}_{2}T(\theta_{11})\right]\left[{}^{2}_{5}T(\theta_9,\theta_{13},\theta_{17})\right]\left[{}^{5}_{6}T(\theta_{15})\right]$$

where:

$$\left[{}^{2}_{5}T\right]=\begin{bmatrix} c_{abc} & -s_{abc} & 0 & L_4c_9+L_5c_{ab} \\ 0 & 0 & -1 & 0 \\ s_{abc} & c_{abc} & 0 & L_4s_9+L_5s_{ab} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where the abbreviations $c_{ab}=\cos(\theta_9+\theta_{13})$, $s_{ab}=\sin(\theta_9+\theta_{13})$, $c_{abc}=\cos(\theta_9+\theta_{13}+\theta_{17})$, $s_{abc}=\sin(\theta_9+\theta_{13}+\theta_{17})$ were used. The right leg FPK result is:

$$\left[{}^{0}_{6}T\right]=\begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where:

$$\begin{aligned} r_{11} &= \left(s_7s_{abc}-c_7s_{11}c_{abc}\right)c_{15}-c_7c_{11}s_{15} \\ r_{21} &= \left(-c_7s_{abc}-s_7s_{11}c_{abc}\right)c_{15}-s_7c_{11}s_{15} \\ r_{31} &= -s_{11}s_{15}+c_{11}c_{15}c_{abc} \end{aligned}$$

$$\begin{aligned} r_{12} &= -\left(s_7s_{abc}-c_7s_{11}c_{abc}\right)s_{15}-c_7c_{11}c_{15} \\ r_{22} &= -\left(-c_7s_{abc}-s_7s_{11}c_{abc}\right)s_{15}-s_7c_{11}c_{15} \\ r_{32} &= -s_{11}c_{15}-c_{11}s_{15}c_{abc} \end{aligned}$$

$$\begin{aligned} r_{13} &= s_7c_{abc}+c_7s_{11}s_{abc} \\ r_{23} &= -c_7c_{abc}+s_7s_{11}s_{abc} \\ r_{33} &= -c_{11}s_{abc} \end{aligned}$$

$$\begin{aligned} p_x &= \left(L_4s_9+L_5s_{ab}\right)s_7-\left(L_4c_9+L_5c_{ab}\right)c_7s_{11} \\ p_y &= -\left(L_4s_9+L_5s_{ab}\right)c_7-\left(L_4c_9+L_5c_{ab}\right)s_7s_{11} \\ p_z &= \left(L_4c_9+L_5c_{ab}\right)c_{11} \end{aligned}$$

The basic right-leg FPK result is $\left[{}^{0}_{6}T\right]$. To calculate $\left[{}^{C}_{F}T\right]$, the pose of the right-leg end-effector frame {F} with respect to the DARwIn chest reference frame {C}, the following transform equation is used.

$$\left[{}^{C}_{F}T\right]=\left[{}^{C}_{0}T\right]\left[{}^{0}_{6}T(\theta_7,\theta_{11},\theta_9,\theta_{13},\theta_{17},\theta_{15})\right]\left[{}^{6}_{F}T\right]$$

$$\left[{}^{C}_{0}T\right]=\begin{bmatrix} 0 & -1 & 0 & -L_{TX} \\ 0 & 0 & -1 & -L_{TY} \\ 1 & 0 & 0 & L_{TZ} \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \left[{}^{6}_{F}T\right]=\begin{bmatrix} 1 & 0 & 0 & L_F \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The overall transform equation above can be evaluated numerically. Constants L_{TX}, L_{TY}, L_{TZ}, and L_F were given earlier.

Note that $\left[{}^{C}_{0}T\right]$ and $\left[{}^{6}_{F}T\right]$ are not evaluated by any row in the DH parameter table, since there is no variable associated with these fixed homogeneous transformation matrices based on constant lengths and orientation. Instead, they are determined by inspection, using the rotation matrix and position vector components of the basic homogeneous transformation matrix definition.

4.4 FPK Examples

4.4.1 Two-dof Pan/Tilt Head

Given $(\theta_{19},\theta_{20})=(10^\circ,20^\circ)$:

$$\left[{}^{0}_{2}T\right]=\begin{bmatrix} 0.93 & -0.34 & 0.17 & 0 \\ 0.16 & -0.06 & -0.98 & 0 \\ 0.34 & 0.94 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \left\{{}^{0}X_2\right\}=\begin{Bmatrix} 0.93 \\ 0.16 \\ 0.34 \\ 1 \end{Bmatrix}$$

4.4.2 Three-dof Right Arm

Given $(\theta_1,\theta_3,\theta_5)=(10^\circ,20^\circ,30^\circ)$:

$$\left[{}^{C}_{H}T\right]=\begin{bmatrix} 0.54 & 0.82 & -0.16 & 79.1 \\ -0.20 & 0.32 & 0.93 & -64.7 \\ 0.81 & -0.47 & 0.34 & 250.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4.4.3 Six-dof Right Leg

Given $\left(\theta_7,\theta_{11},\theta_9,\theta_{13},\theta_{17},\theta_{15}\right)=\left(10^\circ,-30^\circ,-20^\circ,-35^\circ,-45^\circ,60^\circ\right)$

$$\left[{}^{C}_{F}T\right]=\begin{bmatrix}-0.35 & 0.90 & -0.26 & -135.2\\ -0.36 & -0.38 & -0.85 & -256.1\\ -0.87 & -0.20 & 0.45 & 58.5\\ 0 & 0 & 0 & 1\end{bmatrix}$$

These FPK results were validated by comparing the symbolic formulae derived in this paper with a numerical approach in each case. Figure 7 shows the MATLAB graphical results for the zero configuration (see Figures 1 and 2). Figure 8 shows the MATLAB graphical results for the FPK examples configuration (symmetry is lost since we are using identical, rather than symmetric, joint angles for the right and left arms and legs).

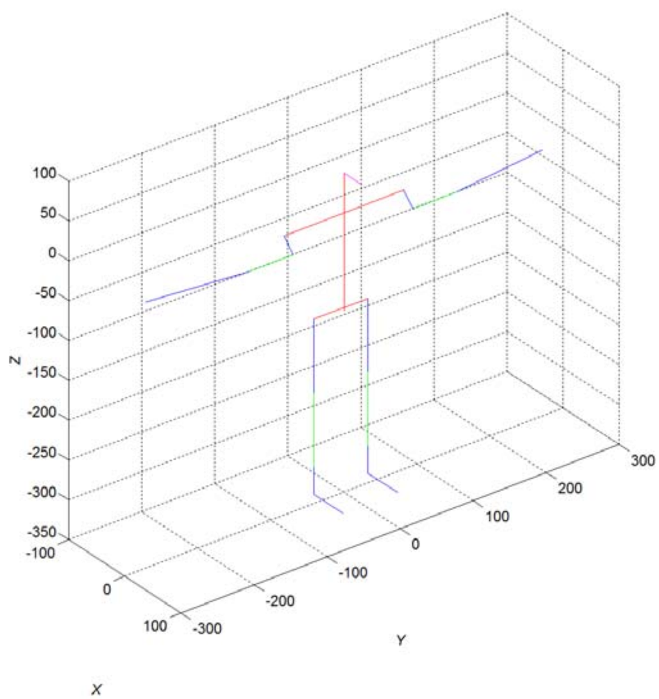

Figure 7. DARwIn-OP MATLAB Simulation, zero configuration

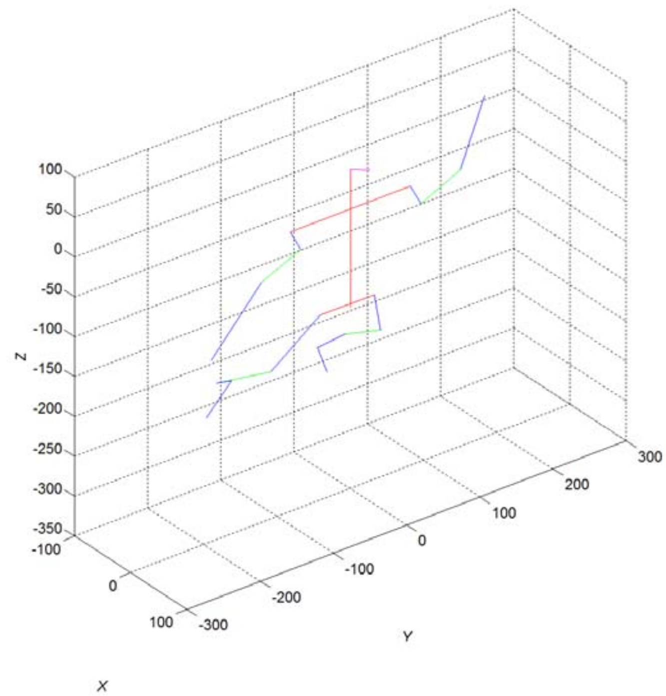

Figure 8. DARwIn-OP MATLAB Simulation, examples configuration

5. DARwIn-OP INVERSE POSE KINEMATICS

In general, the Inverse Pose Kinematics (IPK) problem for a serial-chain robot is stated: Given the pose (position and orientation) of the end frame of interest, calculate the joint values. For serial-chain robots, the IPK solution starts with the FPK equations. The solution of coupled nonlinear algebraic equations is required and multiple solution sets generally result. The head IPK solution is complete but the right arm and right leg IPK solutions are partial below. The FPK examples can also serve as IPK examples when the input is reversed.

5.1 Two-dof Pan/Tilt Head IPK Solution

The statement of the IPK problem for the two-dof pan/tilt head serial chain of the DARwIn-OP humanoid robot is:

$$\text{Given } \left\{{}^{0}X_2\right\}\text{, calculate } \left(\theta_{19},\theta_{20}\right).$$

$\left\{{}^{0}X_2\right\}$ is the pointing vector along the X_2 unit vector direction, expressed in {0} coordinates; it must be entered as a unit vector. Its FPK expression is recalled below:

$$\left\{{}^{0}X_2\right\}=\begin{Bmatrix}c_{19}c_{20}\\ s_{19}c_{20}\\ s_{20}\end{Bmatrix}=\begin{Bmatrix}x\\ y\\ z\end{Bmatrix}$$

where x, y, z are the given components of the desired unit vector $\left\{{}^{0}X_2\right\}$.

The IPK solution for the pan/tilt head is obtained in the following order. First use a ratio of the y to x equations to eliminate c_{20}:

$$\theta_{19} = \text{atan2}(y, x)$$

where atan2 is the quadrant-specific inverse tangent function. Next use a ratio of the z to x equations:

$$\theta_{20} = \text{atan2}(z, x / c_{19})$$

or, equivalently, use a ratio of the z to y equations:

$$\theta_{20} = \text{atan2}(z, y / s_{19})$$

If $\theta_{19} = \pm 90^\circ$ use the y equation solution, and if $\theta_{19} = 0, 180^\circ$ use the x equation solution. If neither of these artificial singularity conditions exist, both solutions will yield the same result for θ_{20}. There is a single solution set, i.e. no multiple solutions in this case.

5.2 Three-dof Right Arm IPK Solution (partial)

The statement of the IPK problem for the three-dof right arm serial chain of the DARwIn-OP humanoid robot is:

Given $\left[{}^{C}_{H}T \right]$, calculate $\left(\theta_1, \theta_3, \theta_5\right)$.

Where $\{H\}$ is the right-arm end-effector hand frame and $\{C\}$ is the DARwIn chest reference frame.

The given pose $\left[{}^{C}_{H}T \right]$ represents 6-dof, while the unknown joint space in this case is only 3-dof. Therefore, the entire $\left[{}^{C}_{H}T \right]$ cannot be given, but only a 3-dof subspace. Let us specify the position vector and calculate the unknown joint angles:

Given $\left\{ {}^{C}X_H \right\}$, calculate $\left(\theta_1, \theta_3, \theta_5\right)$.

From FPK, here are the equations to solve:

$$\left\{ {}^{0}X_H \right\} = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} L_1c_1 + L_2c_1s_3 - L_3(s_1 - s_1c_5 - c_1s_3s_5) + L_H(-s_1s_5 + c_1s_3c_5) \\ L_1s_1 + L_2s_1s_3 - L_3(-c_1 + c_1c_5 - s_1s_3s_5) + L_H(c_1s_5 + s_1s_3c_5) \\ L_2c_3 + L_3c_3s_5 + L_Hc_3c_5 \\ 1 \end{Bmatrix}$$

where x, y, z are the given components of the desired tip position vector $\left\{ {}^{0}X_H \right\}$.

We must first simplify the given $\left\{ {}^{C}X_H \right\}$ to $\left\{ {}^{0}X_H \right\}$ using the following transform equation:

$$\left\{ {}^{0}X_H \right\} = \left[{}^{C}_{0}T \right]^{-1} \left\{ {}^{C}X_H \right\}$$

Where constant transform matrix $\left[{}^{C}_{0}T \right]$ was given in the FPK section for the right arm. The above set of three equations fully coupled in the three unknowns $\left(\theta_1, \theta_3, \theta_5\right)$ is nonlinear (transcendental). In order to simplify this set of equations, use the following transform equation:

$$\left\{ {}^{0}X_H \right\} = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \left[{}^{0}_{1}T(\theta_1) \right] \left[{}^{1}_{2}T(\theta_3) \right] \left[{}^{2}_{3}T(\theta_5) \right] \left\{ {}^{3}X_H \right\}$$

$$\left[{}^{1}_{2}T(\theta_3) \right]^{-1} \left[{}^{0}_{1}T(\theta_1) \right]^{-1} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \left[{}^{2}_{3}T(\theta_5) \right] \left\{ {}^{3}X_H \right\}$$

$$\begin{aligned} xc_1s_3 + ys_1s_3 + zc_3 - L_1s_3 - L_2 &= L_3s_5 + L_Hc_5 \\ xc_1c_3 + ys_1c_3 - zs_3 - L_1c_3 &= 0 \\ -xs_1 + yc_1 - L_3 - L_2 &= -L_3c_5 + L_Hs_5 \end{aligned}$$

This solution has not yet been completed.

5.3 Six-dof Right Leg IPK Solution (partial)

The statement of the IPK problem for the six-dof right leg serial chain of the DARwIn-OP humanoid robot is:

Given $\left[{}^{C}_{F}T \right]$, calculate $\left(\theta_7, \theta_{11}, \theta_9, \theta_{13}, \theta_{17}, \theta_{15}\right)$.

Where $\{F\}$ is the right-leg end-effector foot frame and $\{C\}$ is the DARwIn chest reference frame.

We can first simplify the given $\left[{}^{C}_{F}T \right]$ to $\left[{}^{0}_{6}T \right]$ using the following transform equation:

$$\left[{}^{0}_{6}T \right] = \left[{}^{C}_{0}T \right]^{-1} \left[{}^{C}_{F}T \right] \left[{}^{6}_{F}T \right]^{-1}$$

Where constant transform matrices $\left[{}^{C}_{0}T \right]$ and $\left[{}^{6}_{F}T \right]$ were given in the FPK section for the right leg. Let the following symbols represent the numerical values for $\left[{}^{0}_{6}T \right]$, derived from the originally-given $\left[{}^{C}_{F}T \right]$.

$$\left[{}^{0}_{6}T \right] = \begin{bmatrix} r_{11} & r_{12} & r_{13} & x \\ r_{21} & r_{22} & r_{23} & y \\ r_{31} & r_{32} & r_{33} & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

These numerical values are equated to the FPK solution as a function of the six unknown joint angles:

$$\left[{}^{0}_{6}T \right] = \left[{}^{0}_{1}T(\theta_7) \right] \left[{}^{1}_{2}T(\theta_{11}) \right] \left[{}^{2}_{3}T(\theta_9) \right] \left[{}^{3}_{4}T(\theta_{13}) \right] \left[{}^{4}_{5}T(\theta_{17}) \right] \left[{}^{5}_{6}T(\theta_{15}) \right]$$

The above represents 16 equations (4 trivial) in the six unknown joint angles. The three position vector component equations are all independent, but there are only three independent equations among the 9 equations of the rotation matrix. The equations are coupled and nonlinear (transcendental).

Looking at the FPK terms given earlier, we need some simplification prior to solving these equations. There are three possibilities: 1. Notice that the hip is spherical (i.e. frames {0}, {1}, {2}, {3} share a common origin point); 2. FPK already took advantage of the three parallel Z axes (last hip Z_9, knee Z_{13}, and first ankle Z_{17}); 3. Since the hip is spherical and the 2-dof ankle also rotates about a common point, the distance from the hip center to the ankle center is only a function of one joint angle, θ_{13}. The third simplification will be exploited first.

$$\left\| \left\{ {}^0P_6 \right\} \right\| = \left\| \left\{ {}^3P_5 \right\} \right\| = \sqrt{x^2 + y^2 + z^2}$$

$$\left\{ {}^3P_5 \right\} = \begin{Bmatrix} L_4 + L_5 c_{13} \\ L_5 s_{13} \\ 0 \end{Bmatrix}$$

$$\left\| \left\{ {}^3P_5 \right\} \right\| = \sqrt{L_4^2 + L_5^2 + 2L_4 L_5 c_{13}} = \sqrt{x^2 + y^2 + z^2}$$

$$\theta_{13} = \cos^{-1}\left[\frac{\left\| \left\{ {}^0P_6 \right\} \right\|^2 - L_4^2 - L_5^2}{2L_4 L_5} \right]$$

Note this is equivalent to the Law of Cosines, looking at the plane containing the hip, knee, ankle, and both leg lengths L_4 and L_5.

This solution has not yet been completed.

6. CONCLUSION

This paper has presented kinematic analysis for the 20-dof DARwIn-OP humanoid walking robot, including the Denavit-Hartenberg Parameters for each serial chain (2-dof head pan/tilt, 3-dof arm, and 6-dof leg), specific length parameters, joint angle limits, plus forward pose kinematics equations and partial inverse pose kinematics solutions, with examples. Future work plans include finishing the IPK solutions.

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