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NEW SIMPLIFIED THREE-SPHERES INTERSECTION ALGORITHM FOR THE FORWARD POSE KINEMATICS OF CABLE-SUSPENDED ROBOTS

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ABSTRACT

A novel simplified analytical three-spheres intersection algorithm is presented for use with forward pose kinematics solutions of a four-cable-suspended robot (the method is applicable to various other cable-suspended robots with equal pole heights and three cables intersecting in one point). It is required that the vertical center heights of all three spheres are equal (otherwise one can use the existing more-complicated algorithm). We derive this new algorithm and show that the multiple solutions, algorithmic singularity, and imaginary solutions do not cause any trouble in practical implementation. The algorithmic singularity of the original three-spheres intersection algorithm regarding equal Z heights is eliminated with the new algorithm. The new algorithm requires significantly less computation compared with the original algorithm. Examples are presented to demonstrate the new three-spheres intersection algorithm for a 4-cable robot.

KEYWORDS

Cable-suspended robot, tendon-driven robot, wire-driven manipulator, forward pose kinematics, three-spheres intersection algorithm, multiple solutions, algorithmic singularity, imaginary solutions.

1. INTRODUCTION

Cable robots have been used for a variety of applications, including material handling (Albus et al., 1993; Kawamura et al., 1993; Gorman et al., 2001), haptics (Bonivento et al., 1997; Williams, 1998), International Space Station (Campbell et al., 1995), and large outdoor construction (Bosscher et al., 2007).

One of the better known cable robots is the Skycam, which is a cable robot that dynamically positions a video camera for use in stadiums and indoor arenas (Cone, 1985).

Cable-suspended robots (or tendon-driven robots or wire-driven robots), referred to here as cable robots, are a type of robotic manipulator that has attracted interest for large workspace manipulation tasks. Cable robots are relatively simple in form, with multiple cables attached to a mobile platform or end-effector. The end-effector is manipulated by motors that can extend or retract the cables. In addition to large workspaces, cable robots are relatively inexpensive and are easy to transport, disassemble and reassemble.

Based on the degree to which the cable lengths alone determine the pose (position and orientation) of the manipulator, cable robots can be classified into two categories: fully-constrained and underconstrained. In the fully-constrained case the pose of the end-effector can be completely determined given the current lengths of the cables. An example of a fully-constrained cable robot is the FALCON-7 (Kawamura et al., 1995), a small-scale, seven-cable, high-speed manipulator able to achieve accelerations up to $43g$. In underconstrained cable robots, statics and/or dynamics constraints are required in addition to kinematics constraints to solve the forward pose kinematics problem. Carricato and Merlet (2013) present this complex and interesting problem.

Forward pose kinematics (FPK, given the cable lengths calculate the position and orientation of the end-effector platform) can be a very challenging problem for cable robots. The NIST RoboCrane (Albus et al., 1993) has the geometry of an inverted Stewart/Gough Platform and the attendant complexity of FPK, even with symmetry in design. Therefore, Williams, Albus, and Bostelman (2004a and 2004b) designed,

analyzed, and implemented cable robot systems wherein the FPK can be solved easily analytically by using a series of three cables meeting in one point and repeated applications of the presented three-spheres intersection algorithm. Unfortunately, that algorithm has an artificial singularity in the nominal case of many cable robots, i.e. where the cable support poles are all at equal heights in the vertical Z axis. Bosscher and Williams et al. (2007) applied this three-spheres intersection algorithm to translation-only cable robots whose cables do not intersect in one point, introducing the virtual cables concept.

The current paper presents a simplified three-spheres intersection algorithm suitable for easy analytical FPK solutions of cable robots whose groups of three intersecting spheres have the same Z-height centers, i.e. at least three fixed cable support points lie in a horizontal plane above the ground. The ground support points need not lie on a plane. Presented are the simplified three-spheres intersection algorithm plus analysis of the multiple solutions, algorithmic singularity, and imaginary solution cases, and computation requirement (compared with the general three-spheres intersection algorithm) followed by some examples, including a trajectory example.

2. EXAMPLE FOUR-CABLE-SUSPENDED ROBOT

This section presents a four-cable-suspended robot to demonstrate the new algorithm developed in this paper. It must be emphasized that many other practical cable-suspended robot designs can also benefit from this new three-spheres intersection algorithm. The three main assumptions are: 1. At least three cables meet in a single point (for complicated robots it is necessary to solve multiple stages of three cables meeting in a single point); 2. All support cable heights are at the same Z coordinate; 3. All cables remain in tension for all motion.

The fourth cable is redundant since only 3-dof (XYZ) of the end-effector point are controlled. It is included for tension-optimization purposes, to better avoid slack cables.

Figure 1 shows the four-cable-suspended robot kinematic diagram. The base Cartesian reference frame is $\{A\}$ in the center of the base rectangle and the moving Cartesian control point is P. Each tensioning torque motor/cable reel is fixed to the ground. Each of the four active drive cables runs from fixed cable reel point A_i over pulleys at cable support points P_i to moving Cartesian control point P. All four cables meet in a single point P and hence there is no rotation possible in this problem. For the new FPK algorithm of this paper, the support tower heights h_i are such that cable support points P_i lie in the same horizontal plane, i.e. all P_i have the same Z coordinate.

As seen in Figure 1, the active cable lengths are $\mathbf{L} = \left\{ L_1 \quad L_2 \quad L_3 \quad L_4 \right\}^T$. The vector $^A\mathbf{P}_P = \left\{ x \quad y \quad z \right\}^T$ (not shown) gives the position of moving Cartesian control point of

interest P with respect to the $\{A\}$ origin, expressed in $\{A\}$ coordinates.

The fixed-base cable connection points B_i are constant in the base frame $\{A\}$:

$$^A\mathbf{B}_1 = \left\{ -L/2 \quad -W/2 \quad h_1 \right\}^T$$
$$^A\mathbf{B}_2 = \left\{ -L/2 \quad W/2 \quad h_2 \right\}^T$$
$$^A\mathbf{B}_3 = \left\{ L/2 \quad W/2 \quad h_3 \right\}^T$$
$$^A\mathbf{B}_4 = \left\{ L/2 \quad -W/2 \quad h_4 \right\}^T$$

where L and W are the rectangular dimensions (length and width) of the desired workspace footprint, and h_i are the telescoping support pole heights (all equal in this paper, assuming a planar arrangement of base locations).

3. FORWARD POSE KINEMATICS SOLUTION

In general, the inverse pose kinematics (IPK) problem (given the desired end-effector pose, calculate the active cable lengths) is straight-forward for cable robots, even without common cable intersection points, so this solution is not presented here.

Again, the new FPK algorithm of the current paper is applicable to a variety of cable-suspended robots subject to the basic three assumptions mentioned earlier. The forward pose kinematics (FPK) problem for the specific example robot in this paper is stated: Given the four active cable lengths $\mathbf{L} = \left\{ L_1 \quad L_2 \quad L_3 \quad L_4 \right\}^T$, calculate the Cartesian position of the end-effector control point P.

The FPK solution for cable-suspended robots and other parallel robots is generally very difficult. It requires the solution of multiple coupled nonlinear (transcendental) algebraic equations, from the vector loop-closure equations. Multiple valid solutions generally result. However, for cable-suspended robots meeting the three assumptions of this paper, the FPK is straightforward.

In this paper, the generally-complicated coupled transcendental FPK solution simplifies to that of finding the intersection point of three given spheres, all with known centers and given radii (cable lengths). Finding the intersection point of three given spheres may be solved analytically, with two possible solutions.

The approach for the specific robot example in Figure 1 is to use any three of the four given spheres and find their intersection point P. It does not matter which three spheres are chosen. The fourth given sphere not used in the FPK calculation may be used to check the FPK solution, i.e. using

IPK on cable 4 to ensure that its correct cable length L_4 results after the FPK solution is complete.

The original three-spheres intersection algorithm was presented in Williams, Albus, and Bostelman (2004a and 2004b) for general spheres. For support poles yielding equal Z heights for three spheres, this previous algorithm is unnecessarily complicated and results in an artificial singularity. To overcome these issues, next we present the new simplified three-spheres intersection algorithm, assuming all three sphere centers are at the same vertical height.

4. SIMPLIFIED THREE-SPHERES INTERSECTION ALGORITHM

We now derive the equations and solution for the intersection point of three given spheres (see Figure 2), assuming all three spheres have identical vertical center heights. Assume that the three given spheres are ($\mathbf{c}_1$, r_1), ($\mathbf{c}_2$, r_2), and ($\mathbf{c}_3$, r_3). That is, center vectors $\mathbf{c}_1 = \{x_1 \quad y_1 \quad z_1\}^T$, $\mathbf{c}_2 = \{x_2 \quad y_2 \quad z_2\}^T$, $\mathbf{c}_3 = \{x_3 \quad y_3 \quad z_3\}^T$, and radii r_1, r_2, and r_3 are known. The three sphere center vectors must be expressed in the same frame, {A} here, and the answer will be in the same coordinate frame. The equations of the three spheres to intersect are (choosing the first three spheres):

$$(x-x_1)^2 + (y-y_1)^2 + (z-z_n)^2 = r_1^2$$
$$(x-x_2)^2 + (y-y_2)^2 + (z-z_n)^2 = r_2^2 \qquad (1\text{-}3)$$
$$(x-x_3)^2 + (y-y_3)^2 + (z-z_n)^2 = r_3^2$$

Since all Z sphere-center heights are the same, we have $z_1 = z_2 = z_3 = z_n$. (in Figure 1 terms, $h_1 = h_2 = h_3 = h_4 = z_n$, assuming a planar support ground). The unknown three-spheres intersection point is $^A\mathbf{P}_P = \{x \quad y \quad z\}^T$. Expanding (1-3) yields:

$$x^2 - 2x_1x + x_1^2 + y^2 - 2y_1y + y_1^2 + z^2 - 2z_nz + z_n^2 = r_1^2$$
$$x^2 - 2x_2x + x_2^2 + y^2 - 2y_2y + y_2^2 + z^2 - 2z_nz + z_n^2 = r_2^2 \qquad (4\text{-}6)$$
$$x^2 - 2x_3x + x_3^2 + y^2 - 2y_3y + y_3^2 + z^2 - 2z_nz + z_n^2 = r_3^2$$

Subtracting (6) from (4) and (6) from (5) yields:

$$2(x_3-x_1)x + 2(y_3-y_1)y + x_1^2 + y_1^2 - x_3^2 - y_3^2 = r_1^2 - r_3^2$$
$$2(x_3-x_2)x + 2(y_3-y_2)y + x_2^2 + y_2^2 - x_3^2 - y_3^2 = r_2^2 - r_3^2 \qquad (7\text{-}8)$$

All non-linear terms of the unknowns x and y cancelled out in the subtractions above. Also, all z-related terms cancelled out in the above subtractions since all sphere-center Z heights are identical. Equations (7-8) are two linear equations in the two unknowns x, y, given in the following form.

$$\begin{bmatrix} a & b \\ d & e \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} c \\ f \end{Bmatrix} \qquad (9)$$

where:

$$a = 2(x_3 - x_1)$$
$$b = 2(y_3 - y_1)$$
$$c = r_1^2 - r_3^2 - x_1^2 - y_1^2 + x_3^2 + y_3^2$$
$$d = 2(x_3 - x_2)$$
$$e = 2(y_3 - y_2)$$
$$f = r_2^2 - r_3^2 - x_2^2 - y_2^2 + x_3^2 + y_3^2$$

The unique solution for two of the unknowns x, y is:

$$x = \frac{ce - bf}{ae - bd}$$
$$\qquad (10)$$
$$y = \frac{af - cd}{ae - bd}$$

Returning to (1) to solve for the remaining unknown z:

$$Az^2 + Bz + C = 0 \qquad (11)$$

where:

$$A = 1$$
$$B = -2z_n$$
$$C = z_n^2 - r_1^2 + (x - x_1)^2 + (y - y_1)^2$$

Knowing the unique values x and y, the two possible solutions for the unknown z are found from the quadratic formula:

$$z_{p,m} = \frac{-B \pm \sqrt{B^2 - 4C}}{2} \qquad (12)$$

For the cable-suspended robot, ALWAYS choose z_m, i.e. the z height solution with the negative sign since that is the physically-admissible solution, below the plane of the four z_n support pole heights. The z_p solution is physically-impossible since the end-effector point P would be above the plane of the four z_n support pole heights, requiring impossible pushing forces on all four cables.

This simplified three-spheres intersection algorithm solution for x, y, z fails in two cases:

i)   When the determinant of the coefficient matrix in the x, y, linear solution (10) is zero.

$$ae - bd = 2(x_3 - x_1)2(y_3 - y_2) - 2(y_3 - y_1)2(x_3 - x_2) = 0 \quad (13)$$

This is an algorithmic singularity whose condition can be simplified as follows. (13) becomes:

$$(x_3 - x_1)(y_3 - y_2) = (y_3 - y_1)(x_3 - x_2) \quad (14)$$

If (14) is satisfied there will be an algorithmic singularity. Note that the algorithmic singularity condition (14) is only a function of constant terms. Therefore, this singularity can be avoided by design, i.e. proper placement of the support pole locations in the XY plane. For instance, for a rectangular placement of the support poles with a centrally-located $\{A\}$ frame, we have:

$$
\begin{aligned}
x_1 &= x_2 \\
x_1 &= -x_3 \\
y_1 &= -y_2 \\
y_2 &= y_3
\end{aligned}
\quad (15)
$$

and (14) simplifies to:

$$x_3 y_3 = 0 \quad (16)$$

which is only satisfied if $x_3 = 0$ and/or $y_3 = 0$. This is impossible for a rectangular pole arrangement with the base frame $\{A\}$ in the middle, and so in that design this algorithmic singularity does not exist. Similarly, for non-rectangular base designs, one can avoid this algorithmic singularity by design.

ii) When the discriminant in (12) is negative, the solution for z will be imaginary. The condition $B^2 - 4C < 0$ yields:

$$(x - x_1)^2 + (y - y_1)^2 > r_1^2 \quad (17)$$

When this inequality is satisfied, the solution for z will be imaginary, which means that the cable-suspended robot will not assemble for that configuration. Note that (17) is an inequality for a circle. That is, consider cable 1 from a top view. If (17) is satisfied, this means that cable length r_1 is too short to assemble at that configuration. This will NEVER occur if valid inputs are given for the FPK problem, i.e. assuming the cable-length sensing is adequate.

5.   SIMPLIFIED THREE-SPHERES INTERSECTION COMPUTATIONAL REQUIREMENT

Not only does the new three-spheres intersection algorithm for equal z-heights avoid the artificial singularity present in the original three-spheres intersection algorithm, but the computation requirement is significantly reduced too, as shown in the table below.

Computational Requirement for the Two Algorithms

Algorithm	Add/ Subtract	Multiply/ Divide	Square Root
Original	48	62	1
New	25	31	1
Reduction	48%	50%	0%

6.   SIMPLIFIED THREE-SPHERES INTERSECTION EXAMPLES

This section presents examples (two snapshot and one trajectory) to demonstrate the new simplified three-spheres intersection algorithm for equal-height spheres. The following constants are used for this simulation: 1 acre desired workspace with a rectangular shape using the golden ratio, with $W = 50.0$ m and $L = 80.9$ m. An acre is about one American football field without endzones, that is, somewhat smaller than a standard soccer pitch. The four support poles each have the same height $h_i = 7.62$ m.

a.   Given three spheres ($\mathbf{c}$,r), the new three-spheres intersection algorithm yields one valid solution.

$$(\{-40.46 \quad -25 \quad 7.62\}^T, 48)$$

$$(\{-40.46 \quad 25 \quad 7.62\}^T, 48)$$

$$(\{ 40.46 \quad -25 \quad 7.62\}^T, 48)$$

$$\{^A\mathbf{P}_P\} = \{x \quad y \quad z\}^T = \{0 \quad 0 \quad 1.16\}^T$$

For checking purposes, the fourth cable length is identical, $L_4 = 48$ m.

b.   Given three spheres ($\mathbf{c}$,r), the new three-spheres intersection algorithm yields one valid solution.

$$(\{-40.46 \quad -25 \quad 7.62\}^T, 38)$$

$$(\{-40.46 \quad 25 \quad 7.62\}^T, 48)$$

$$(\{ 40.46 \quad -25 \quad 7.62\}^T, 58)$$

$$\left\{{}^{A}\mathbf{P}_{P}\right\} = \left\{x \quad y \quad z\right\}^{T} = \left\{-6.55 \quad -8.60 \quad 2.63\right\}^{T}$$

For checking purposes, the fourth cable length is $L_4 = 50.04$ m.

   c.   A circular trajectory is specified with radius 20 m, centered in the middle of the workspace (with $x = y = 0$) and at a constant height of 2 m (see Figure 3). In this example, the IPK solution was calculated at each of 72 steps; given those cable lengths inputs (Figure 4), the FPK solution was calculated using the new three-spheres intersection algorithm. The FPK results are shown in Figure 5.

7. CONCLUSION

A novel simplified analytical three-spheres intersection algorithm was presented in this paper. It is useful in forward pose kinematics solutions of cable-suspended robots whose design includes at least three spheres that meet in one point. It can be applied to a variety of cable robots that were identified. The vertical center heights of all three spheres must be equal; otherwise the existing more-complicated algorithm three-spheres algorithm may be used. It was shown that the multiple solutions, algorithmic singularity, and imaginary solutions never interfere with practical implementations of this algorithm. The algorithmic singularity of the original three-spheres intersection algorithm regarding equal Z heights was eliminated. The new algorithm requires about 50% less computation compared with the original algorithm. Examples were presented to demonstrate the new three-spheres intersection algorithm.

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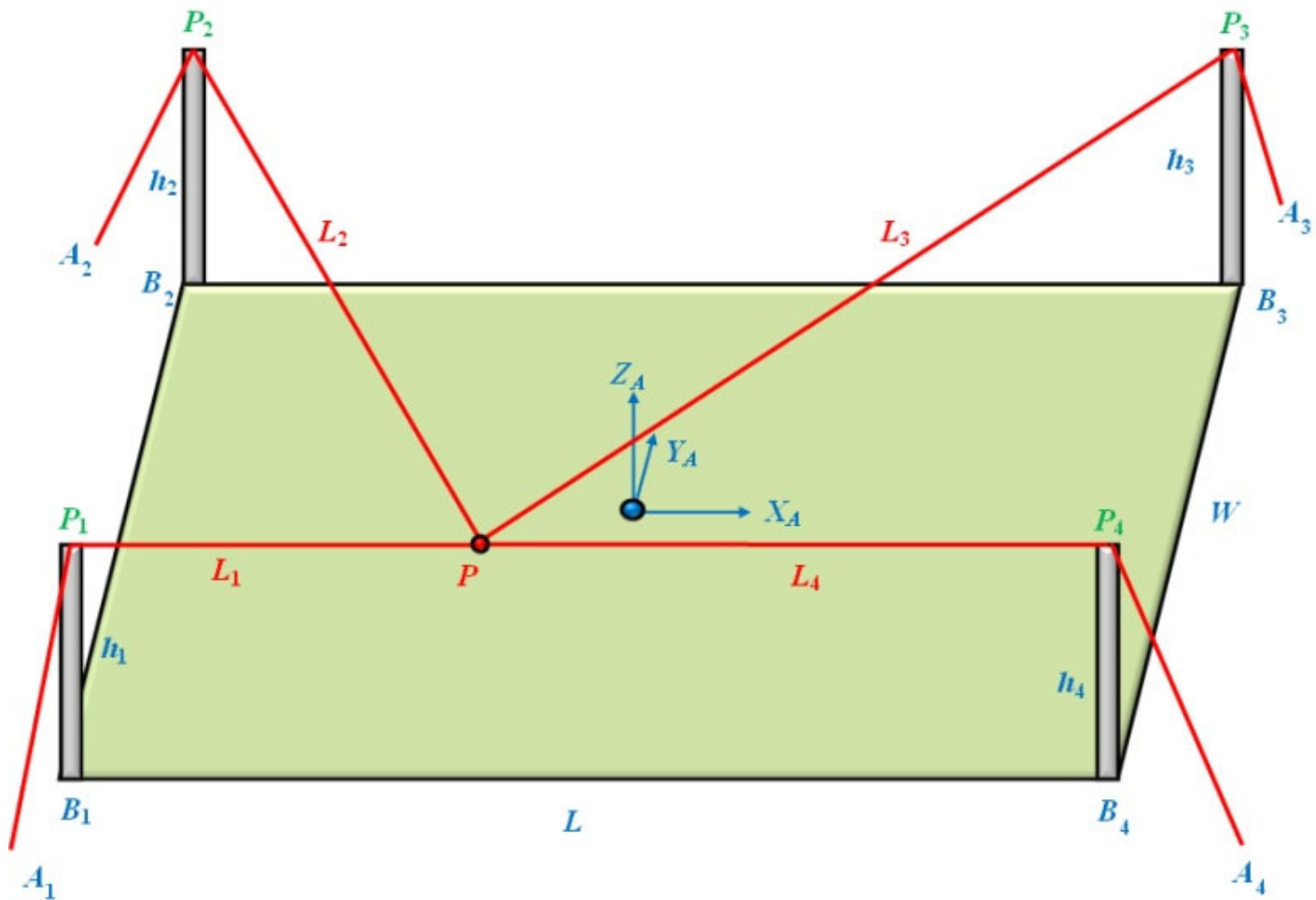R.L. Williams II, 1998, "Cable-Suspended Haptic Interface", International Journal of Virtual Reality, 3(3): 13 – 21.
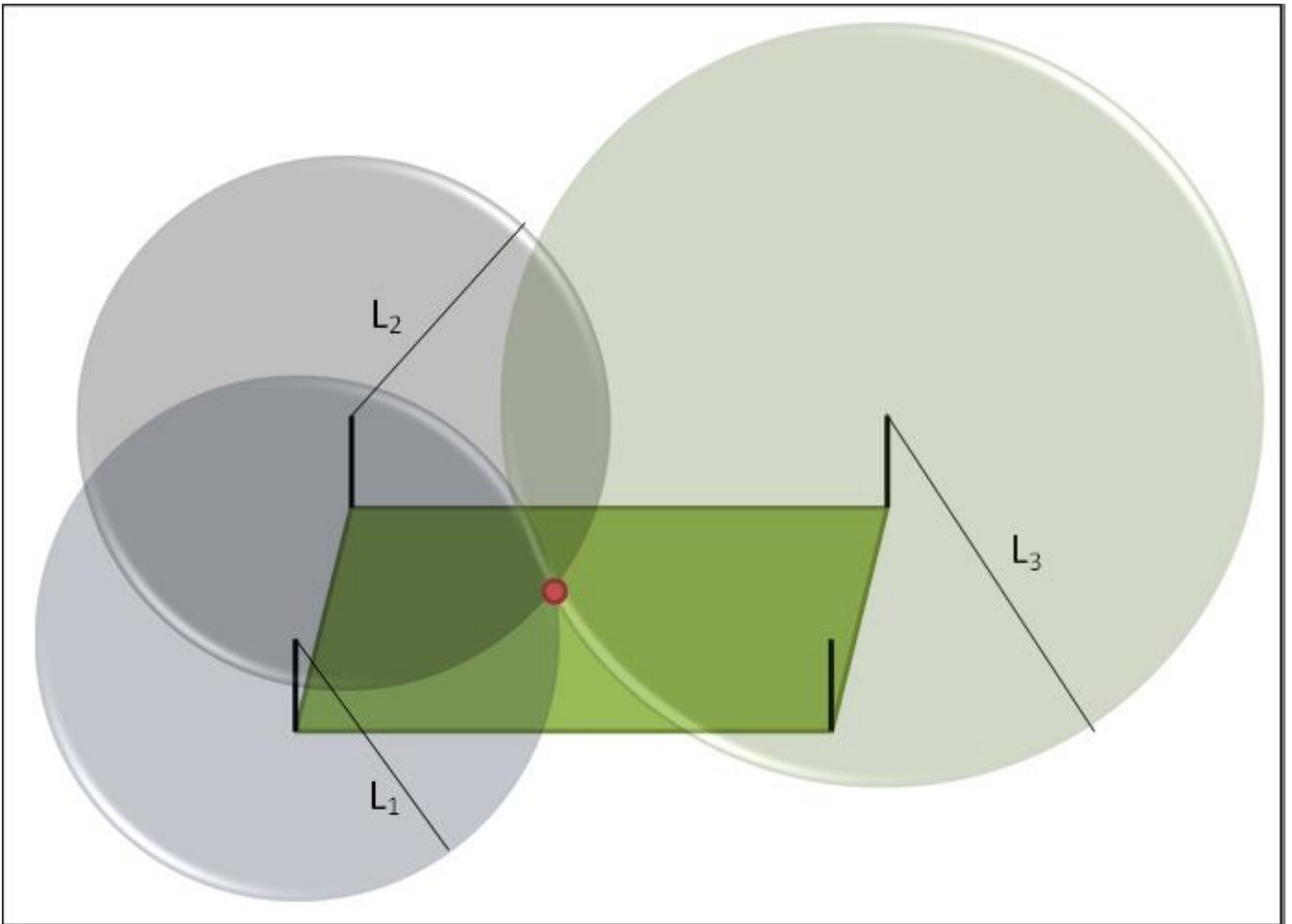
Figure 1. Four-Cable-Suspended Robot Diagram
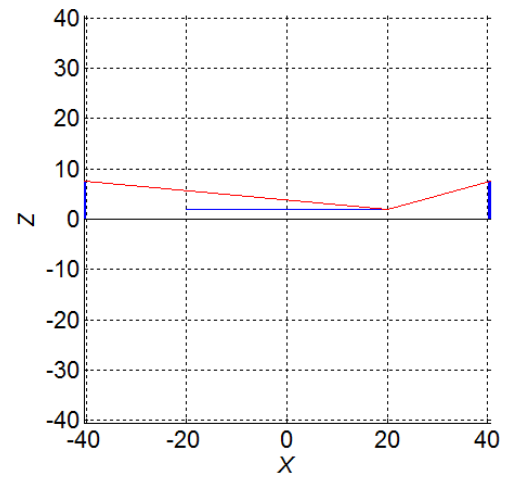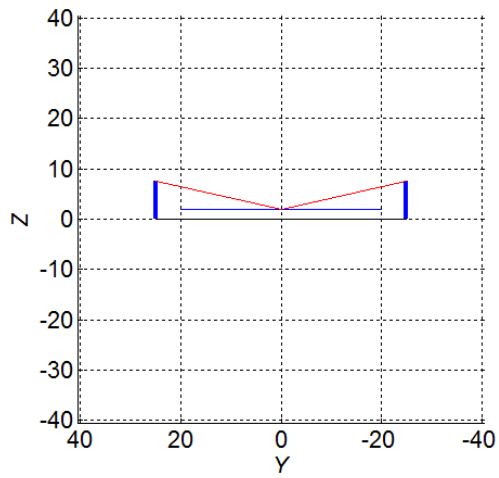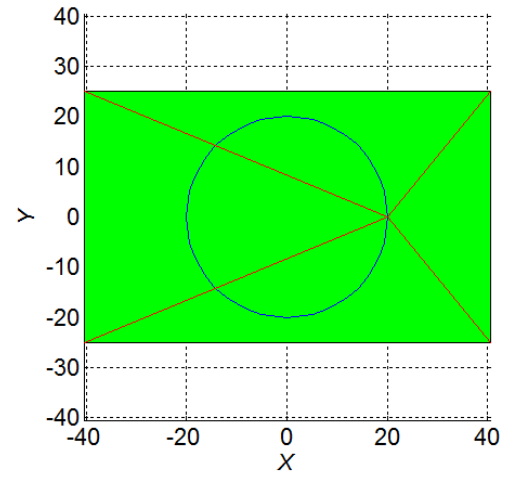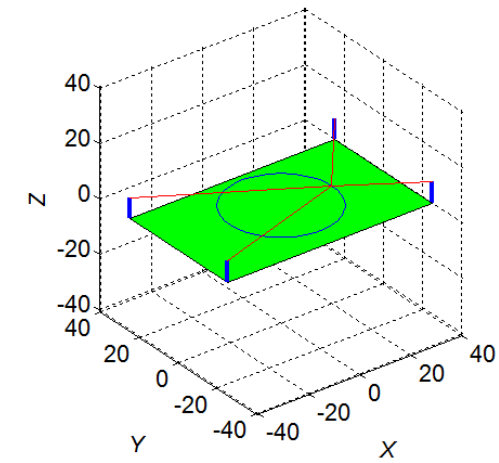
Figure 2. Three-Spheres Intersection Diagram
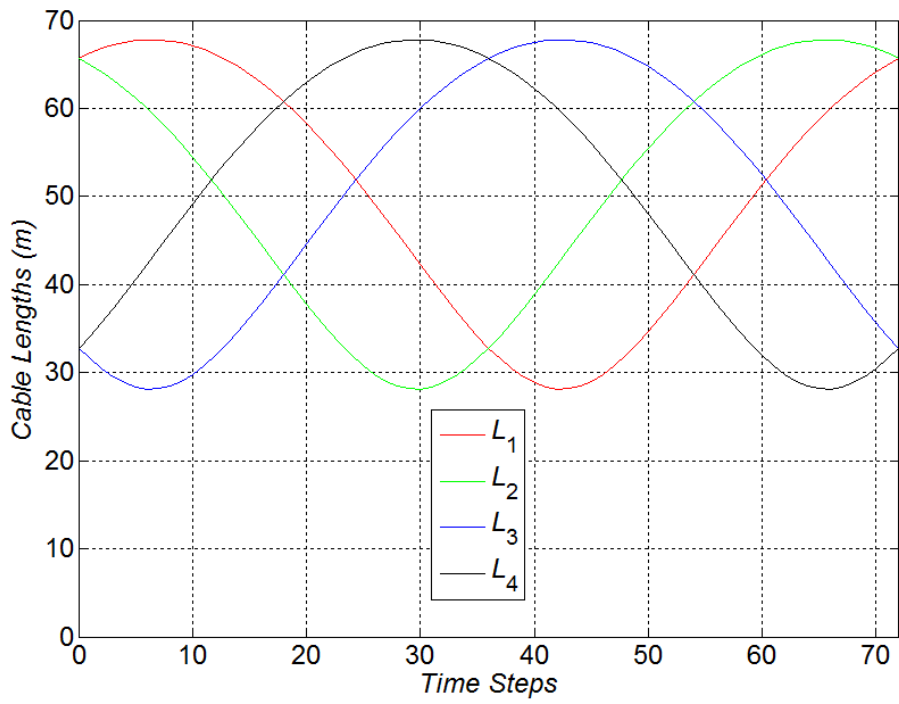
Figure 3. Final (and Initial) Circular Trajectory Configuration
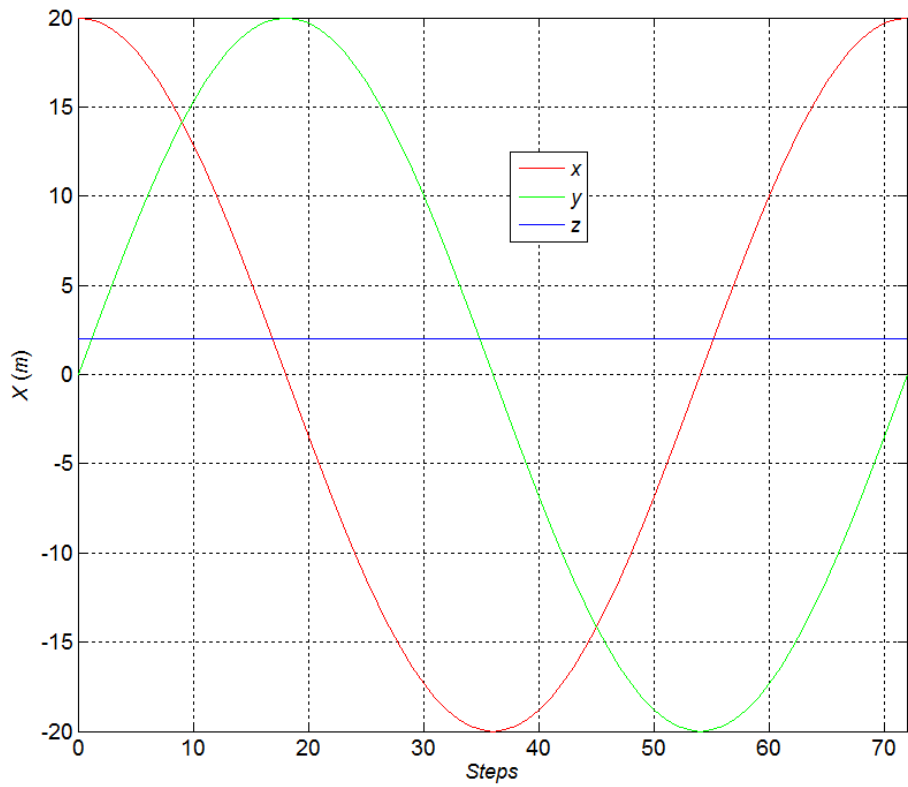
Figure 4. Cable Length Inputs, Circular Trajectory

Figure 5. FPK Cartesian Circular Trajectory Results