BOB WILLIAMS

Engineering Biomechanics of Human Motion

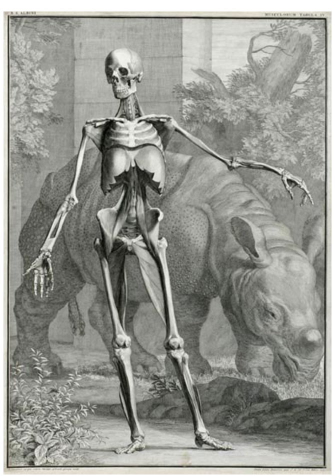
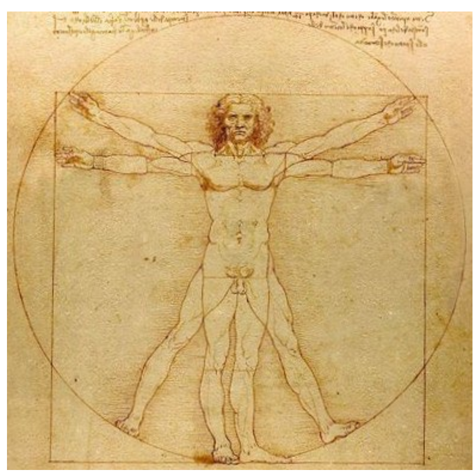
Engineering Biomechanics of Human Motion

Dr. Robert L. Williams II
Mechanical/Biomedical Engineering
Ohio University

NotesBook self-published for
ME 4670 / 5670 Engineering Biomechanics of Human Motion

williar4@ohio.edu

people.ohio.edu/williams


Resisting the unreasonable cost of textbooks since 2008

This ME 4670 / 5670 NotesBook is augmented by the on-line ME 4670 / 5670 Supplement.

Engineering Biomechanics of Human Motion

Author: **Robert L. Williams II, Ph.D.**
Mechanical Engineering
Ohio University
https://people.ohio.edu/williams

Holding a grudge is like taking poison and expecting your enemy to die.

Chief Editor: Dr. Bob
Sole Typist: Dr. Bob
Marketing Director: Dr. Bob
Production Manager: Dr. Bob
Project Supervisor: Dr. Bob
Technical Lead: Dr. Bob
Diversity Specialist: Dr. Bob
Layout Expert: Dr. Bob
Colour Consultant: Dr. Bob
Cover Designer: Dr. Bob
Water Boy: Dr. Bob
MATLAB Guru: Dr. Bob
Supplement Author: Dr. Bob

Cover photo: Human Musculoskeletal System from *Bodies: The Exhibition*.

The body text is set in 12-pt Times New Roman, and the headings, sub-headings, and sub-sub-headings are set in 16-pt, 14-pt, and 12-pt Arial, respectively.

This NotesBook is intended for ME 4670 / 5670 Engineering Biomechanics of Human Motion, a one-semester senior / graduate technical elective course in Mechanical and Biomedical Engineering at Ohio University. Covered are the anatomy and physiology of the human musculoskeletal system, followed by the kinematics, statics, and dynamics biomechanics of human motion, assuming rigid-body segments. MATLAB Software is used as a tool for human motion analyses and animations. Warning: my NotesBook concept serves both as textbook and notebook – some equations, figures, and examples are blank and must be completed in class. Readers external to Ohio University are welcome with that caveat in mind.

Keywords: biomechanics, kinesiology, human anatomy, human physiology, musculoskeletal systems, kinematics, statics, dynamics, biomedical engineering, mechanical engineering, MATLAB

ME 4670 / 5670 NotesBook Table of Contents

Dr. Bob

1. Introduction

1.1 Human Motion Images

Hamilton et al. (2008)

Citius, altius, fortius (Olympic motto); Latin for faster, higher, stronger.

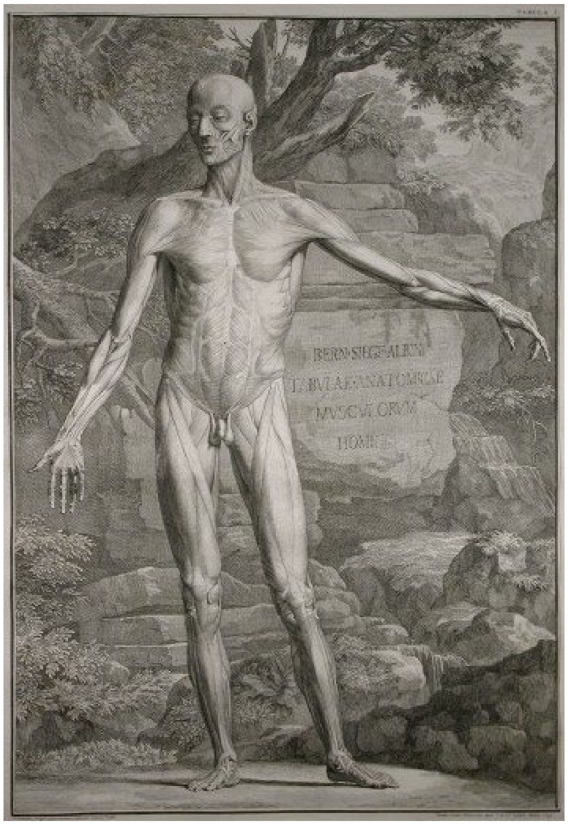
Bodies: The Exhibition – Images

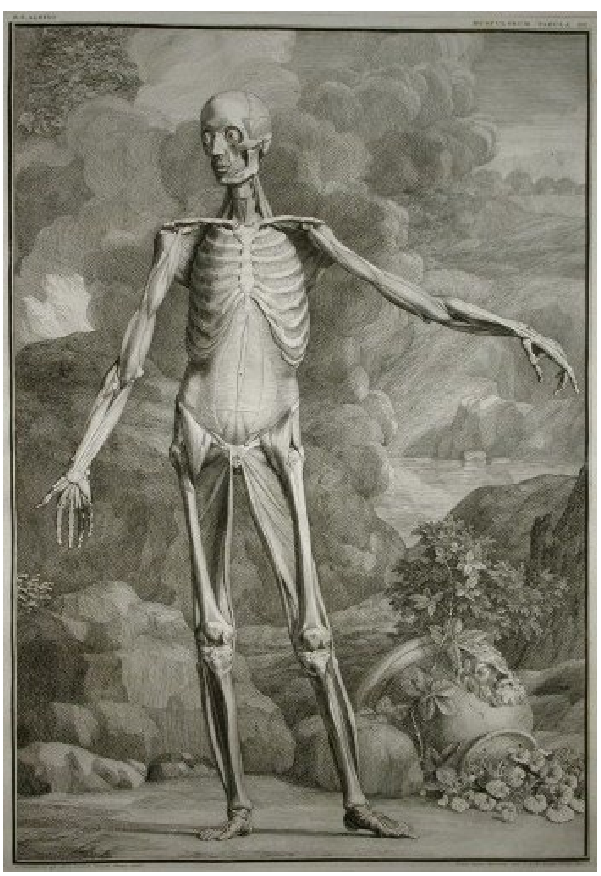
Bodies: The Exhibition

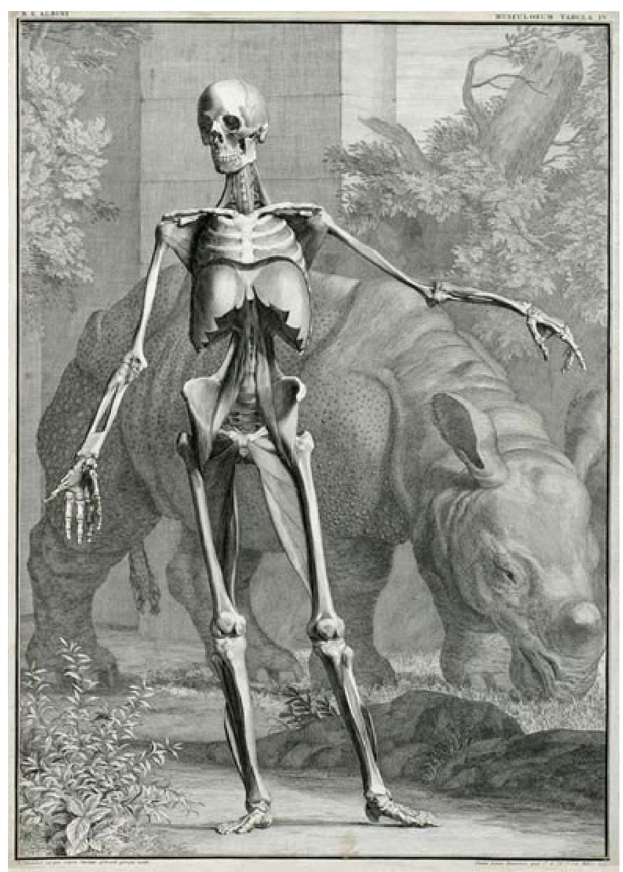
"What a piece of work is man! How noble in reason, how infinite in faculties! In form and moving how expressive and admirable! In action how like an angel, in apprehension how like a god! The beauty of the world, the paragon of animals!"

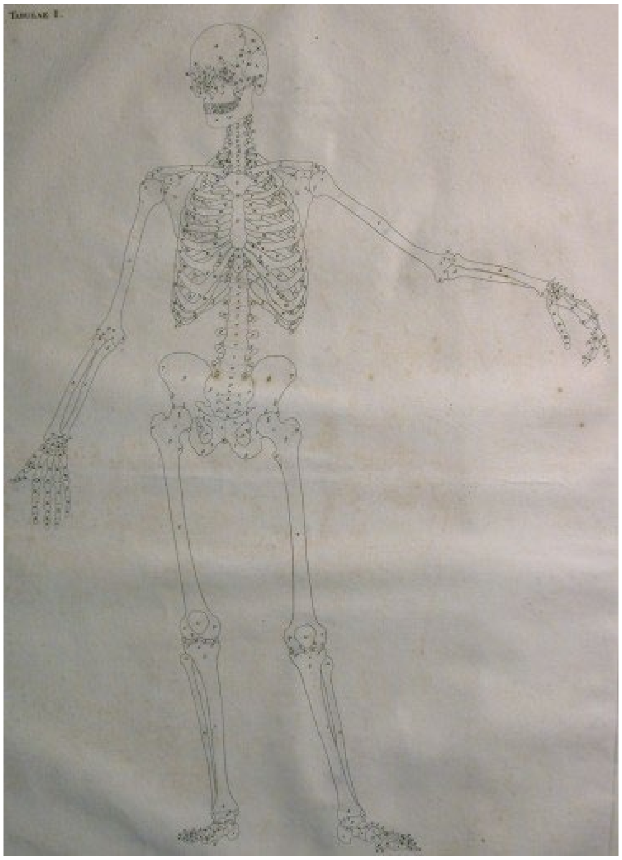
–Hamlet, William Shakespeare

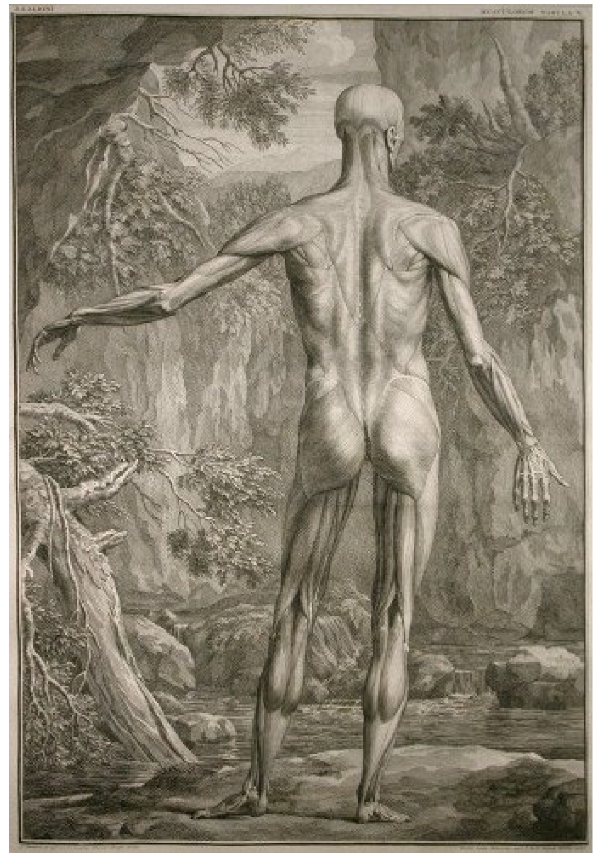
Anatomy Art by Albinus (Anatomist, 1697-1770) and Wandelaar (Engraver, 1690-1759)



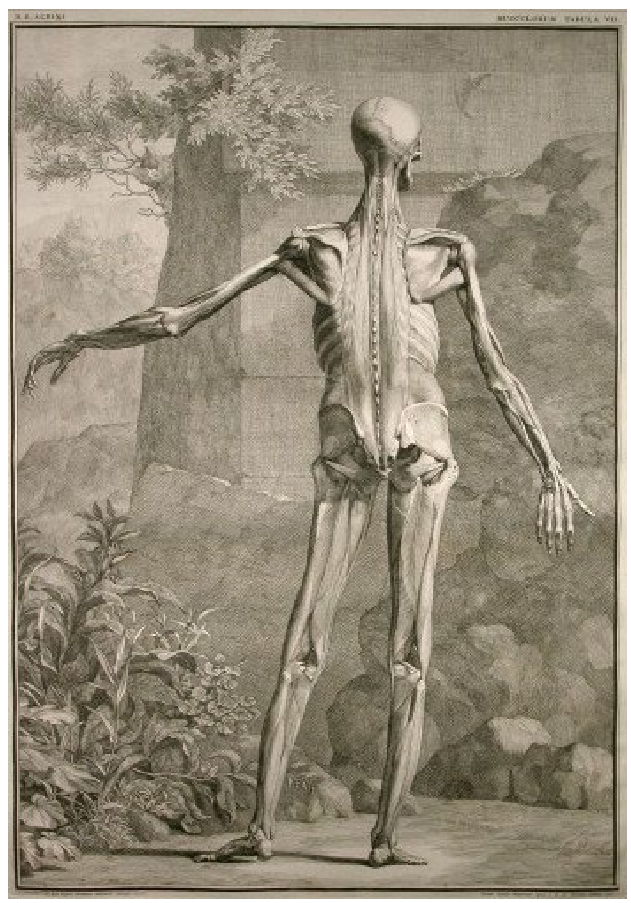


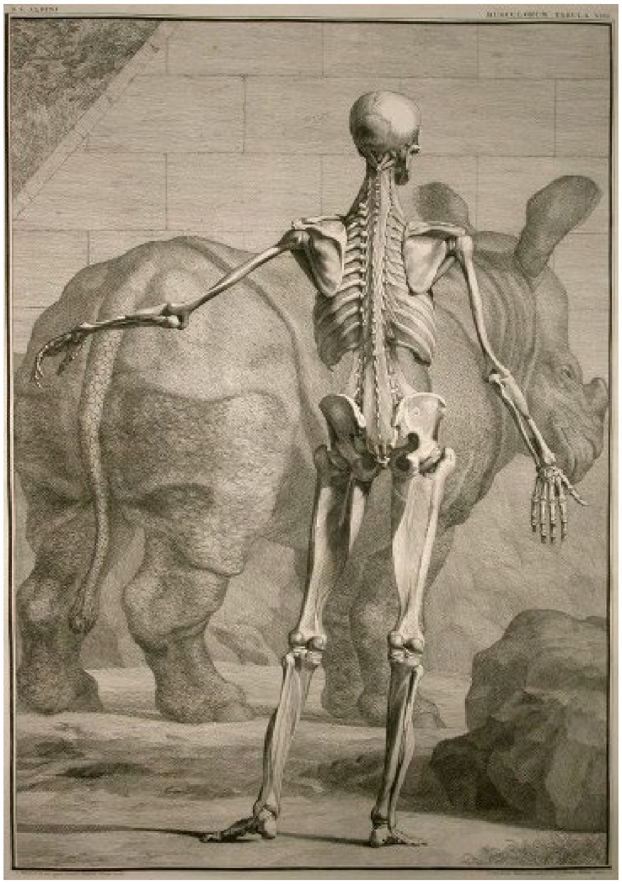


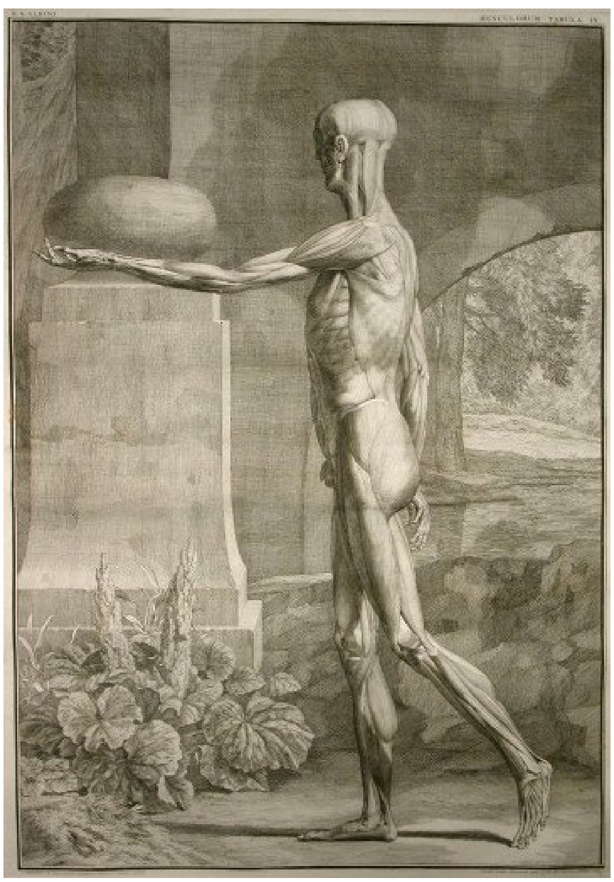

TABULAE I.

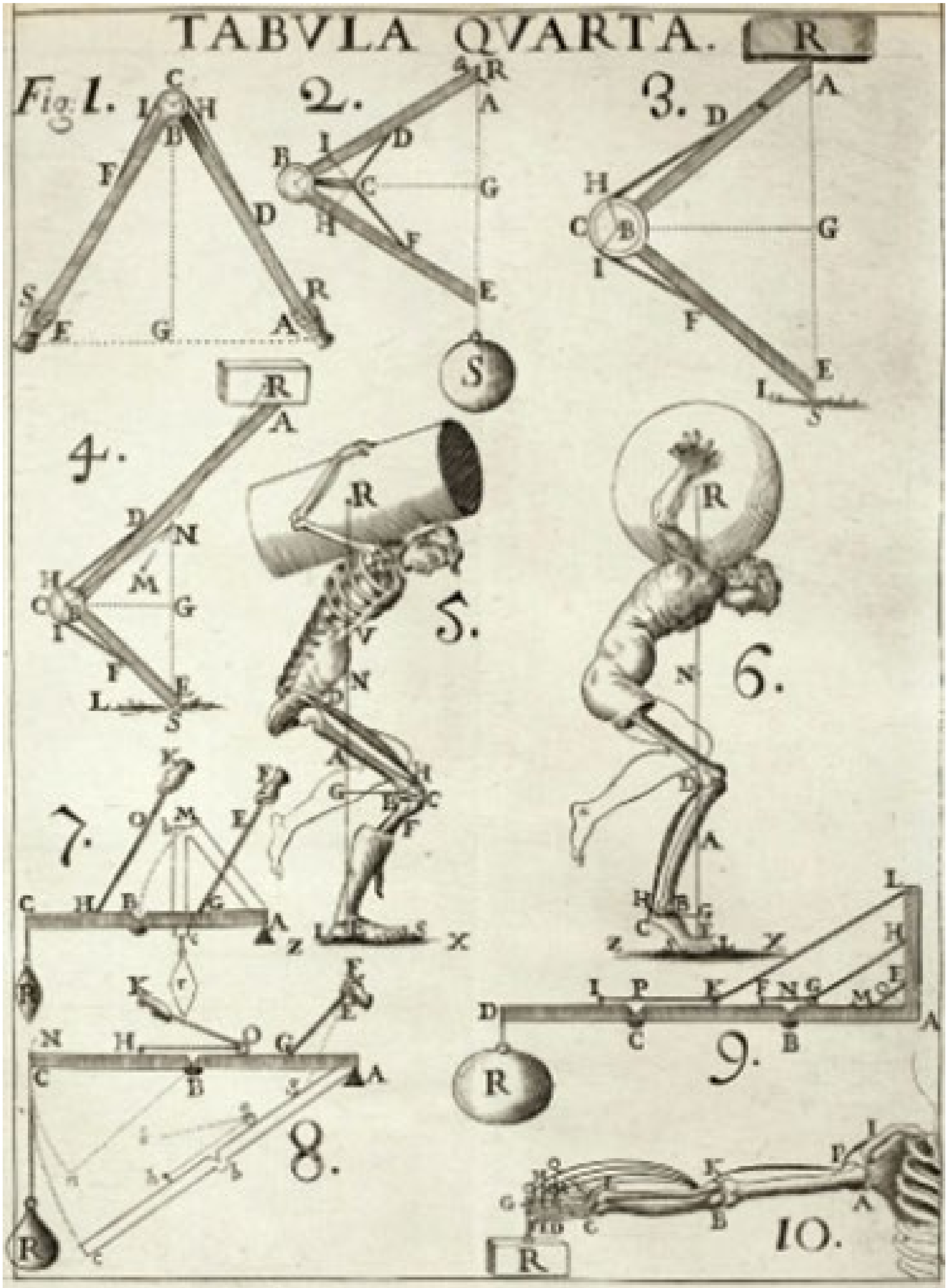



B.S. ALBINI
MUSCULORUM TABULA VI


B.S. ALBINI
MUSCULORUM TABULA VII





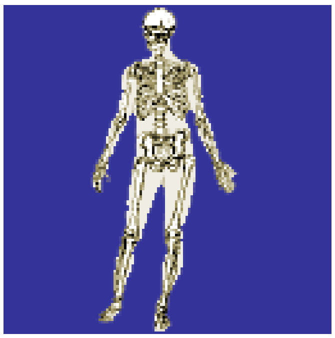
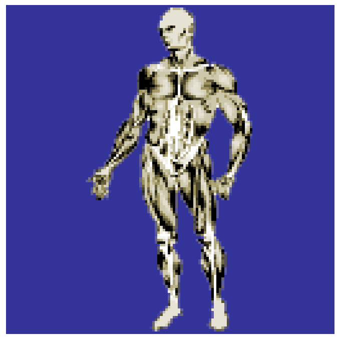


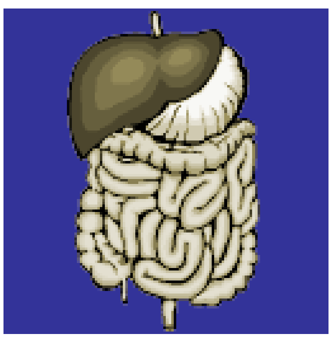
Static Analysis of Human Muscles and Joints by Giovanni Borelli (1608 – 1679)

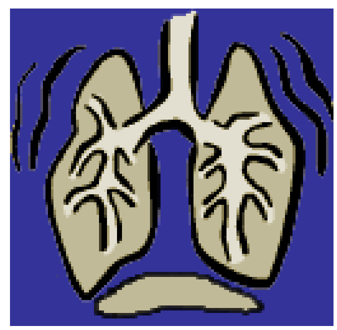
1.2 Concepts, Definitions, and History

Human Body Systems

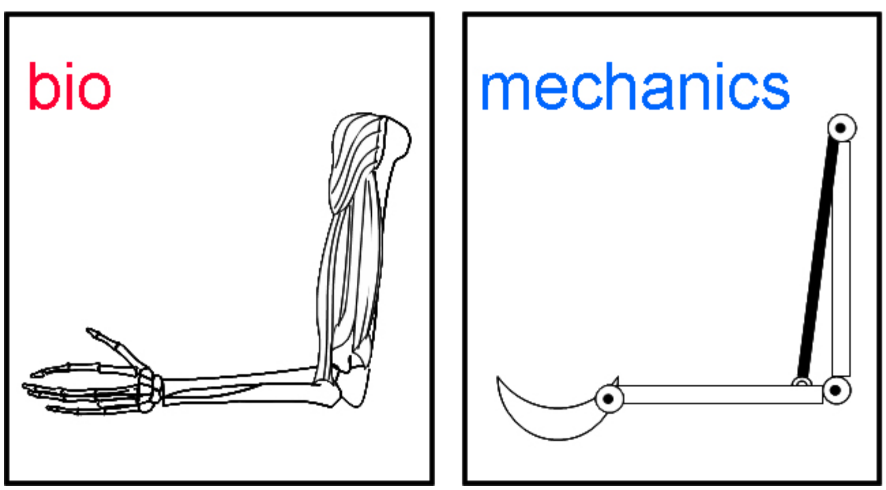
• Skeletal	• Fascial	• Reproductive / Urinary
• Muscular	• Integumentary (skin, hair, nails, and glands)	• Cardiovascular / Respiratory
• Ligaments/Tendons	• Nervous system with senses	• Lymphatic / Endocrine
• Surface	• Digestive	• Mind / Soul





Biomechanics, coined by researchers in the 1970s, is concerned with applying *engineering mechanics principles* to the study of the *human body*.



Hall (2007)

The subject of **biomechanics** is very broad and is applied to every system mentioned above, at all levels from the subatomic to atoms and molecules, cells, microscopic and macroscopic subsystems, and the entire body. The following disciplines can use biomechanics.

• Medicine (osteopathic, allopathic, orthopedic , chiropractors, and nursing). • Physical Therapy • Biology, Physiology, and Anatomy • Sports Training • Bioengineering including prostheses	• Psychology • Ergonomics and Occupational Therapy • Massage Therapy • Military • Automotive engineering including design, ergonomics, safety, pedestrians

The following also need biomechanics, even if they have never heard of the term.

• artists • actors • athletes	• dancers • musicians • fire, police, utility, service, construction, etc. personnel

This class will be entirely focused on **Kinesiology**, the study of human movement. We will call it the biomechanics of human motion and will generally focus on rigid body motions at the macro level.

Human motion is amazing, extremely varied, and it can be beautiful.

Additional interesting branches of biomechanics for mechanical engineers include modeling of human tissue, bones, and other flexible materials using the finite elements method, fluid dynamics and bioengineered materials for prostheses.

Back to **Kinesiology**, this class will be composed of anatomy, physiology, and engineering mechanics, specifically:

- musculoskeletal anatomy
- neuromusculoskeletal physiology
- rigid-body biomechanics

Many engineers are inspired by biological (*biomimetic*) and human (*anthropomorphic*) systems in design and control. This class will largely focus on human body motion for its own sake.

<u>Definitions</u>

kinesiology	The study of the mechanics of body movements. For this class, the study of human movement.
biomechanics	The study of the mechanical laws relating to the movement or structure of living organisms. For this class, applying engineering mechanics principles to the study of the human body.
anatomy	the biological study of the **<u>structures</u>** of an organism and the relationships of its parts
physiology	the biological study of the **<u>functions</u>** of an organism and the relationships of its parts
in vivo	in a living organism, indicating study of the organism as it exists
in vitro	study of organism tissue out of body context
in virtuo	modeling/study of an organism in virtual reality
mobility	the number of **<u>degrees-of-freedom</u>** (**dof**, number of parameters required to completely specify the location of a device) which a human or machine possesses.
statics	the study of the balance of forces/moments without motion
kinematics	the study of motion without regard to forces
dynamics	the study of motion with regard to forces
biomimetic	the application of biological methods and systems found in nature to the study and design of engineering systems and modern technology
anthropomorphic	the attribution of uniquely human characteristics and qualities to nonhuman beings, inanimate objects, or natural or supernatural phenomena
anthropometry	body measurement research, the study of human body measurements
goniometer	an instrument for measuring angles, useful for human joint angle range/limits determination
histology	the anatomical study of the microscopic structures of animal and plant tissues

See the following free website for human anatomy and physiology: https://www.biodigital.com/.

History of Biomechanics and Kinesiology

- **Kinesiology** is from the Greek 'kinein' (to move) and 'logos' (study of).

- **Aristotle** (384-322 B.C., Greek) is the Father of Kinesiology. His treatises, *Parts of Animals*, *Movement of Animals*, and *Progression of Animals*, described the actions and geometric analysis of muscles for the first time. He first analyzed and described walking; he noted the knee and lower leg form a right triangle when walking. Further he presented a precursor of Newton's three laws of motion.

- **Archimedes** (287-212 B.C., Greek) determined hydrostatic principles governing floating bodies. His inquiries included the laws of leverage and determining the center of gravity and the foundation of theoretical mechanics.

- **Galen** (131-201 A.D., Roman) is the first team physician in history (gladiators). His treatise *De Motu Musculorum* distinguished between motor and sensory nerves, agonist and antagonist muscles, described tonus, and introduced terms such as diarthrosis and synarthrosis. He thought muscular contraction resulted from the passage of 'animal spirits' from the brain through the nerves to the muscles. His treatise is the first textbook on kinesiology and he is the Father of Sports Medicine.

- **Leonardo da Vinci** (1452-1519, Italian), artist, engineer, and scientist, was interested in the structure of the human body as it relates to performance, center of gravity and the balance and center of resistance. He identified muscles and nerves in the human body that he retrieved from graveyards in the middle of the night. He described the mechanics of the body during standing, walking up and downhill, climbing stairs, rising from a sitting position, jumping, and human gait. Defined the term 'Kinematic Tree', now called 'Kinematic Chain'.

- **Galileo** (1564-1642, Italian) proved that the trajectory of a projectile through a non-resistant medium is a parabola. His work led to the study of mechanical events in mathematical terms.

- **William Harvey** (1578-1657, English) first demonstrated blood circulation through the body, although he attributed to the heart the function of recharging the blood with heat and 'vital spirit'.

- **Giovanni Alfonso Borelli** (1608 – 1679, Italian), physiologist, physicist, mathematician, was the first to realize the importance of levers in the human. He noted that muscles must produce much larger forces than those the body opposes. He also discovered that a muscle maintains constant volume while contacting.

- **Francesco Maria Grimaldi** (1618-1663, Italian Jesuit) was the first to hear sounds made by contracting muscles. *Physicomathesis de Lumine, Coloribus, et Iride, Aliisque Annexis*, was written 300 years before technology was available for studying these sounds.

- **Isaac Newton** (1642-1727, English) laid the foundation of modern dynamics in *Principia Mathematica Philosophiae Naturalis*, with his three laws of rest and movement.

- **Niels Stensen** (1648-1686, Danish) made the then-sensational declaration that the heart was merely a muscle, not the seat of 'natural warmth' nor of 'vital spirit'. He wrote *Elementorum*

Myologiae Specimum, an 'epoch-making' book on muscular function. He asserted that a muscle is a collection of motor fibers; that the center of a muscle differs from the ends (tendons) and is the only part that contracts. Contraction of a muscle is merely the shortening of its individual fibers and is not produced by an increase or loss of substance.

- **Nicolas Andry** (1658-1742, French) coined 'orthopedics' from the Greek 'orthos' (straight) and 'pais' (child). Andry believed skeletal deformities result from childhood muscular imbalances.

- **Bernhard Siegfried Albinus** (1697-1770, German) noted human anatomist, worked with artist/engraver **Jan Wandelaar** (1690-1759).

- **Étienne-Jules Marey** (1830-1904, French) scientist, physiologist, and chronophotographer, used chronophotography (antique photographic technique from the Victorian era, which captures movement in several frames of print) to study human and animal motion. He was a pioneer for the motion picture industry.

- **Archibald V. Hill** (1886-1977, English) physiologist, won the Nobel Prize for heat and mechanical work in muscles. He is the namesake for the Hill Muscle Model presented later in this NotesBook. He studied energy and efficiency for human motion. He tested the acceleration of human sprinters and discovered chemical and mechanical events in muscle contraction.

- **Ernst Otto Fischer** (1918-2007, German) chemist, won the Nobel Prize for pioneering organometallic chemistry. He studied the human walking gait and his work led to the development of prosthetics.

- **Yuan-Cheng (Bert) Fung** (1919-2019, Chinese/American) is the Father of Modern Biomechanics, focusing on soft-tissue biomechanics. He had an entire career in aircraft design in China before emigrating to the U.S. He coined the term *biomechanics* in the 1970s. Awarded the Russ Prize in engineering in 2007, he lectured in Baker Center on the Ohio University campus that year.

1.3 MATLAB Introduction

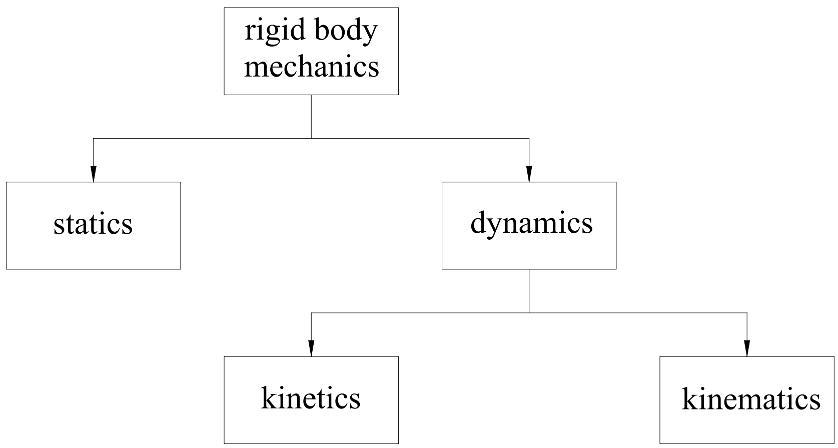
MATLAB is a general engineering analysis and simulation software. MATLAB stands for **MAT**rix **LAB**oratory. It was originally developed specifically for control systems simulation and design engineering, but it has grown over the years to cover many engineering and scientific fields. MATLAB is based on the C language, and its programming is vaguely C-like, but simpler. MATLAB is sold by Mathworks Inc. (www.mathworks.com) and Ohio University has a site license. For an extensive introduction to the MATLAB software, please see Dr. Bob's MATLAB Primer.

people.ohio.edu/williams/html/PDF/MATLABPrimer.pdf

1.4 Engineering Mechanics Overview

Again, biomechanics is concerned with applying *engineering mechanics principles* to the study of the *human body*. By this definition, virtually all disciplines in *mechanical engineering* could be involved, including statics, strength of materials, kinematics, dynamics, controls, vibrations, robotics, machine design, fluid dynamics, CAD, FEM, materials, thermodynamics, and heat transfer. The same could be said of much of *electrical engineering* and *chemical engineering*.

Traditionally, engineering mechanics is generally limited to the study of statics, strength of materials, kinematics, dynamics, vibrations, and continuum mechanics. For this class, we will further limit our scope (mostly) to rigid body statics, kinematics, and dynamics. This section presents a brief overview of some important terms, concepts, principles, and equations. Here is a diagram of rigid-body engineering mechanics components.



<u>Engineering Mechanics Terms</u>

Scalars

Name	Symbol	Definition	Formula	SI Units
mass	m			
volume	V			
density	ρ			
work	W			
energy	E			
power	P			
kinetic energy	T			
potential energy	V			
stiffness	k			
damping	c			

Vectors

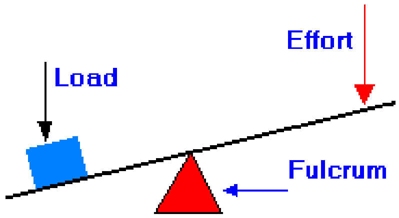
Name	Symbol	Definition	Formula	SI Units
force	f			
torque (moment)	τ			
center of mass	CG			
gravity	g			
weight	W			
pressure	P			
momentum	p			
angular momentum	L			
impulse	Imp			
angular impulse	Imp_A			
friction	f_f			
stiction	f_s			
position	P			
velocity	V			
acceleration	A			
jerk	J			
angular position*	θ			
angular velocity	ω			
angular acceleration	α			
angular jerk	β			

*Angular position is a vector for planar motion only.

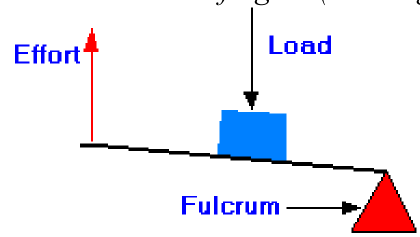
Other

Name	Symbol	Definition	Formula	SI Units
mass moment of inertia	I			
stress	σ			
strain	ε			

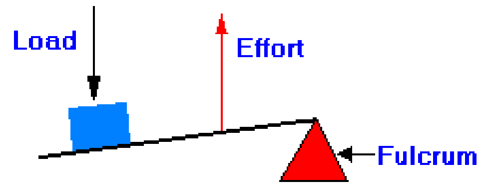
Simple Machines – pulley, lever, screw, inclined plane, wheel and axle.
What is the only simple machine found in the human body?
Exception: some anatomists identify a pulley system at the knees and fingers (see Trigger Finger)



Class 1 Lever

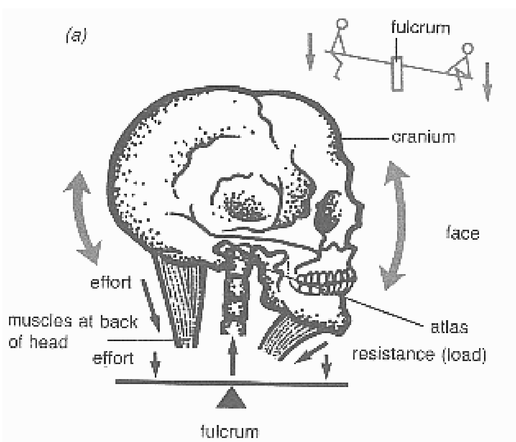


Class 2 Lever

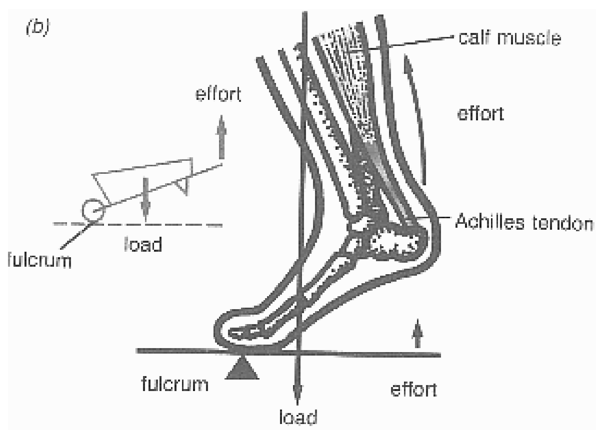


Class 3 Lever

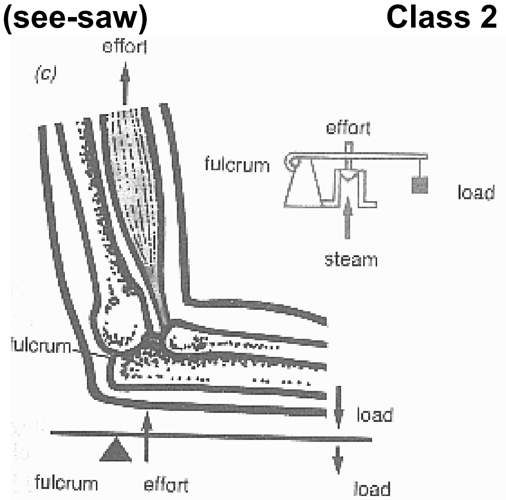
Human Musculoskeletal Examples for the Three Classes of Levers



Class 1 Lever: Head Nod (see-saw)



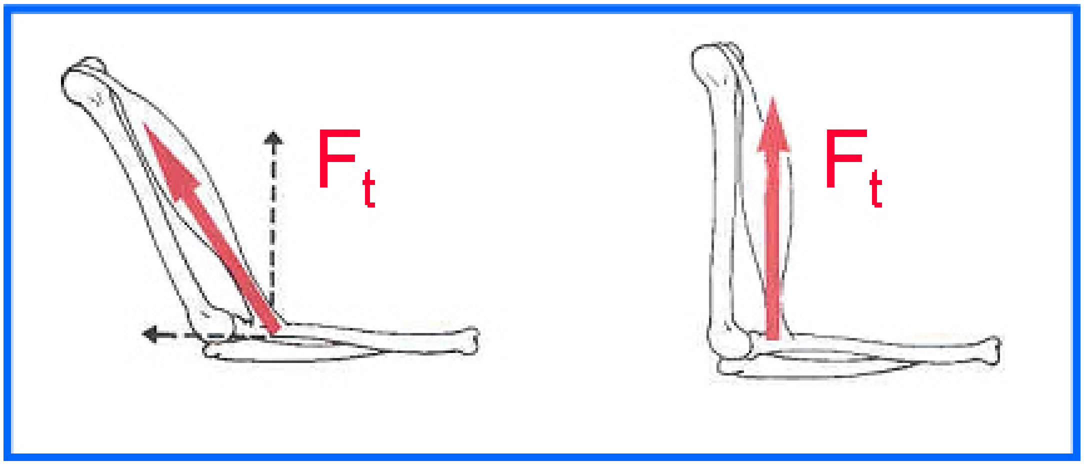
Class 2 Lever: Foot (wheelbarrow)



Class 3 Lever: Forearm-Bicep-Elbow (steam engine mechanism)

Mechanical Advantage

Mechanical advantage is the factor by which a mechanism multiplies the force put into it. The third class lever actually creates mechanical disadvantage! Why? In the arm poses below, the right image has more mechanical advantage than the left image. Why?



Third-Class Lever and Mechanical Advantage

<u>Engineering Mechanics Mathematics Review</u>

vector mechanics (addition, dot product, cross product), matrices and linear algebra, geometry, trigonometry, calculus, differential equations

<u>Engineering Mechanics Concepts</u>

translational and rotational motion; point mass, rigid body, flexible body; beam theory

<u>Engineering Mechanics Principles</u>

Newton's Laws

Newton's First Law

An object at rest tends to stay at rest and an object in motion tends to stay in motion with the same speed and in the same direction unless acted upon by an unbalanced external force.

Newton's Second Law

The acceleration a of an object as produced by a net force F is directly proportional to the magnitude of the net force, in the same direction as the net force, and inversely proportional to the mass m of the object; $F = ma$ (assuming constant mass).

Newton's Third Law

For every action, there is an equal and opposite reaction.

Free-Body Diagram (FBD)

Static / Pseudostatic Equilibrium

Newton-Euler Dynamics Equations of Motion

Newton's Second Law (translational) $\sum \underline{F} = m\underline{A}_G$

Euler's Law (rotational) $\sum \underline{M}_G = I_{GZ}\underline{\alpha}$

Conservation of Linear Momentum

This law states that the total vector linear momentum ($p = mv$) of a closed system of objects (with no external forces) is constant. One consequence is that the center of mass of any system of objects will continue with the same velocity unless acted on by an outside force.

This law is implied by Newton's first law of motion. Newton's third law of motion is also due to the conservation of momentum.

Conservation of Angular Momentum

This law states that the total vector angular momentum ($L = r$ x p) of a closed system of objects (with no external torques) is constant.

Work-Energy Method

Work applied to a body from time 1 to 2 equals the change in energy $^{1}W_2 = E_2 - E_1$

Work

translational $\underline{F} \bullet \Delta\underline{x}$ rotational $\underline{\tau} \bullet \Delta\underline{\theta}$

Potential energy (spring and gravity)

translational $\frac{1}{2}k\Delta x^2$ mgh rotational $\frac{1}{2}k_R\Delta\theta^2$ $mgL\sin\theta$

Kinetic energy

translational $\frac{1}{2}mv^2$ rotational $\frac{1}{2}I\omega^2$

Impulse-Momentum Method

This method is useful for collision dynamics and is derived from Newton's Second Law.

$$F\Delta t = m\left(v_2 - v_1\right)$$

The impulse $F\Delta t$ equals the change in momentum $m\left(v_2 - v_1\right)$.

1.5 Planes, Coordinates, Directions, and Motion Conventions

This section presents the standard anatomical terms for human body reference planes, coordinate axes, directional terms, and common motion terms. The same terms and conventions are also used for most animals in zoology and some plants in plant biology. The human figure below is shown in the **standard anatomical position**, standing with feet flat on the ground and palms supinated.

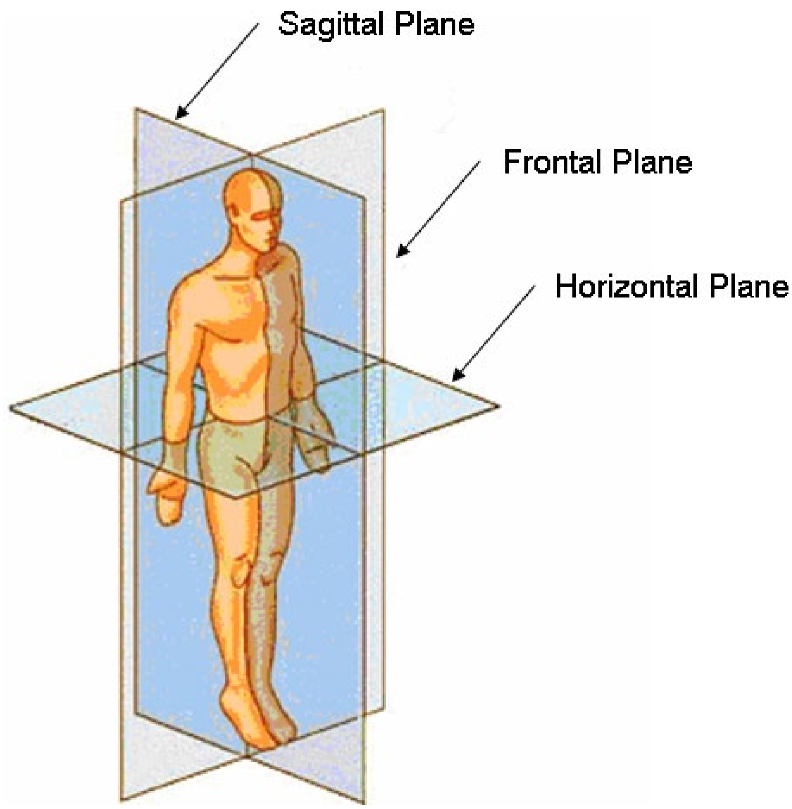
Human Body Reference Planes

Like most animals, humans have bilateral symmetry, meaning there is a plane that divides the organism into roughly mirror-image left and right halves. This is for external appearance only since not all internal organs are symmetric. The human below is posed in the **standard anatomical position**.

This plane of symmetry, the **sagittal** (or **median**) **plane**, divides the body in left and right parts.

The **frontal** (or **coronal**) **plane** divides the body into front and back portions.

The **horizontal** (or **transverse**) **plane** divides the body into top and bottom portions.



Body Reference Planes Conventions

How are the standard reference planes positioned in the body?

1. The **sagittal plane** is obvious – this is the one plane of symmetry in the human body, so it is placed in the middle. If the spine is straight, the sagittal plane bisects each vertebra. There are also other transverse sagittal planes of interest, offset to either direction and parallel to the mid-sagittal plane.
2. The **horizontal plane** may be placed at the umbilicus (navel). However, there are many possible horizontal planes of interest, e.g. mid cervical horizontal plane and pelvic horizontal plane.
3. The **frontal plane** may be referred to a gravity-neutral stance (i.e. stable standing). Then it would pass through the hips, roughly the same as passing through the center of the shoulders.

personal communication, Professor John N. Howell, 4/1/8

Human Body Coordinate Axes

From the origin point formed by the intersection of the three planes (*sagittal*, *frontal*, and *horizontal planes*), three Cartesian coordinate axes are defined:

The **anteroposterior axis** (**AP**) is horizontal from back to front. The *AP axis* is the intersection of the *sagittal* and *horizontal planes*.

The **bilateral** (or **mediolateral**) **axis** is horizontal from side to side (left to right). The *bilateral axis* is the intersection of the *frontal* and *horizontal planes*.

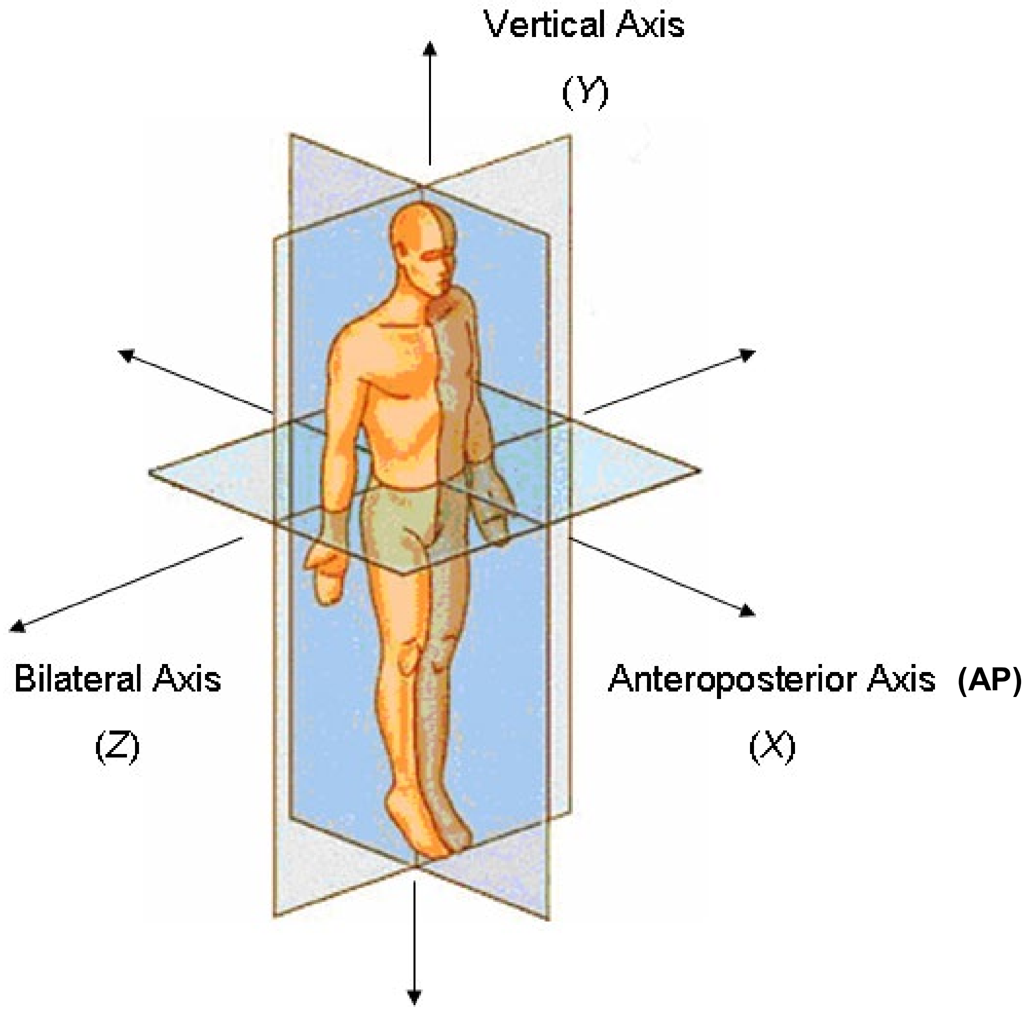
The **vertical** (or **longitudinal**) **axis** is vertical from bottom to top. The *vertical axis* is the intersection of the *sagittal* and *frontal planes*.

For convenience we will make the follow Cartesian axis definitions:

The $+X$ axis is the **AP axis**, toward the front (anterior).

The $+Y$ axis is the **vertical axis**, toward the top (superior).

By the right-hand rule, the $+Z$ axis is the **bilateral axis**, toward the right (starboard).



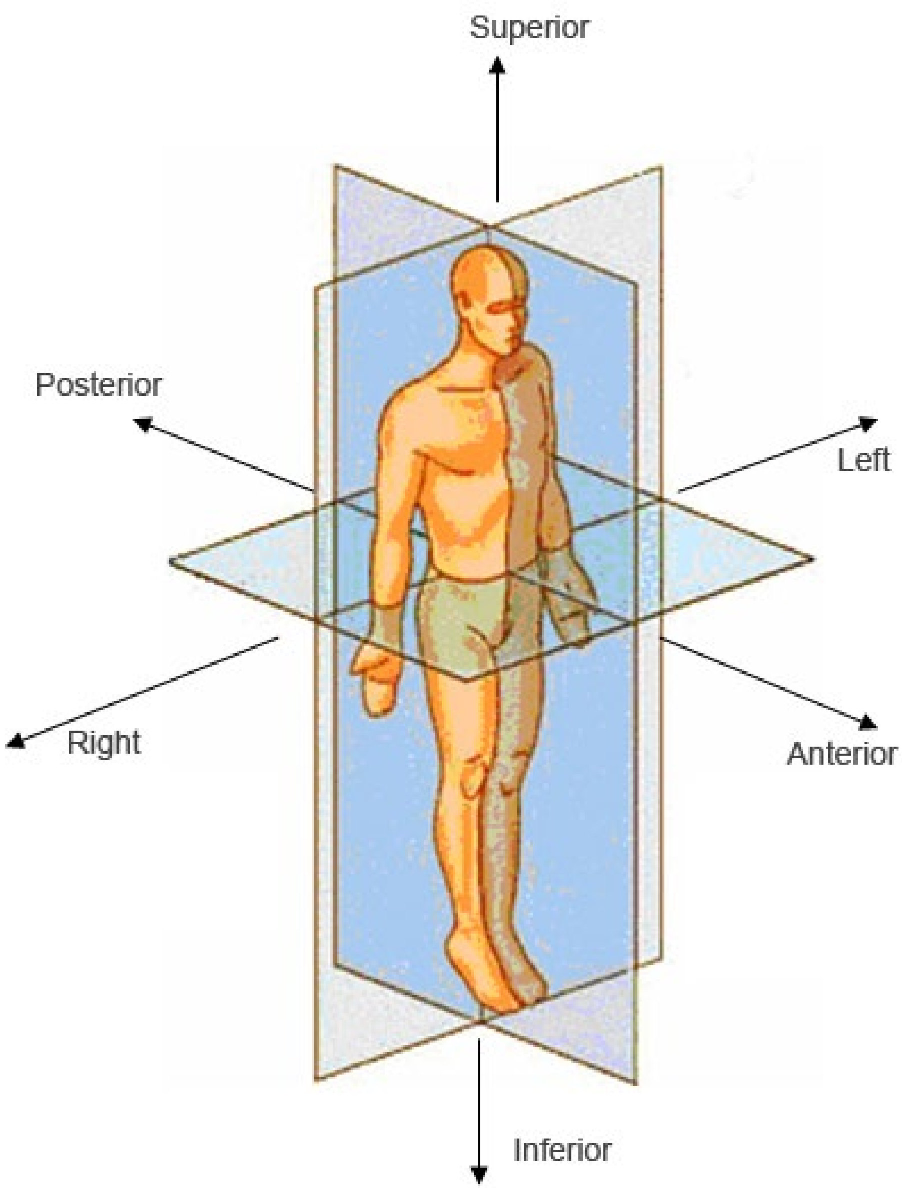
Cartesian Coordinate Axes Conventions

Most human motions are 3D (spatial) motions. What are some examples? However, many human motions can be approximated to one plane. What are some examples in the **sagittal plane**, **frontal plane**, and **horizontal plane**?

<u>Human Body Directional Terms</u>

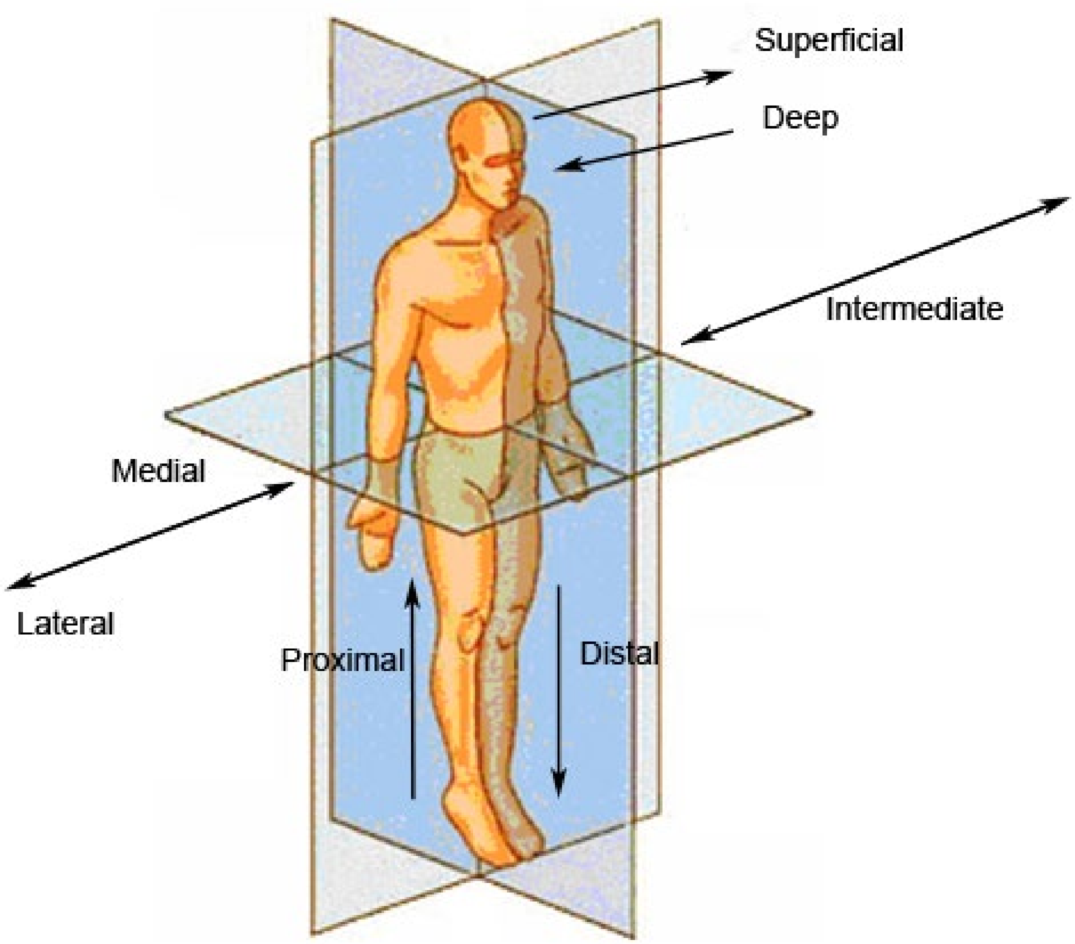
Anatomists use the following directional terms (with alternatives given) to describe directions in the body. These standard terms are generally used for a body at rest. See the figures on the next page. The standard terms for motion will be given next.

superior (**cranial**)	toward the head
inferior (**caudal**)	away from the head
anterior (**ventral**)	toward the front of the body
posterior (**dorsal**)	toward the back of the body
left (sinistral)	towards the port side of the body
right (dextral)	towards the starboard side of the body



Anatomical Directional Terms (1)

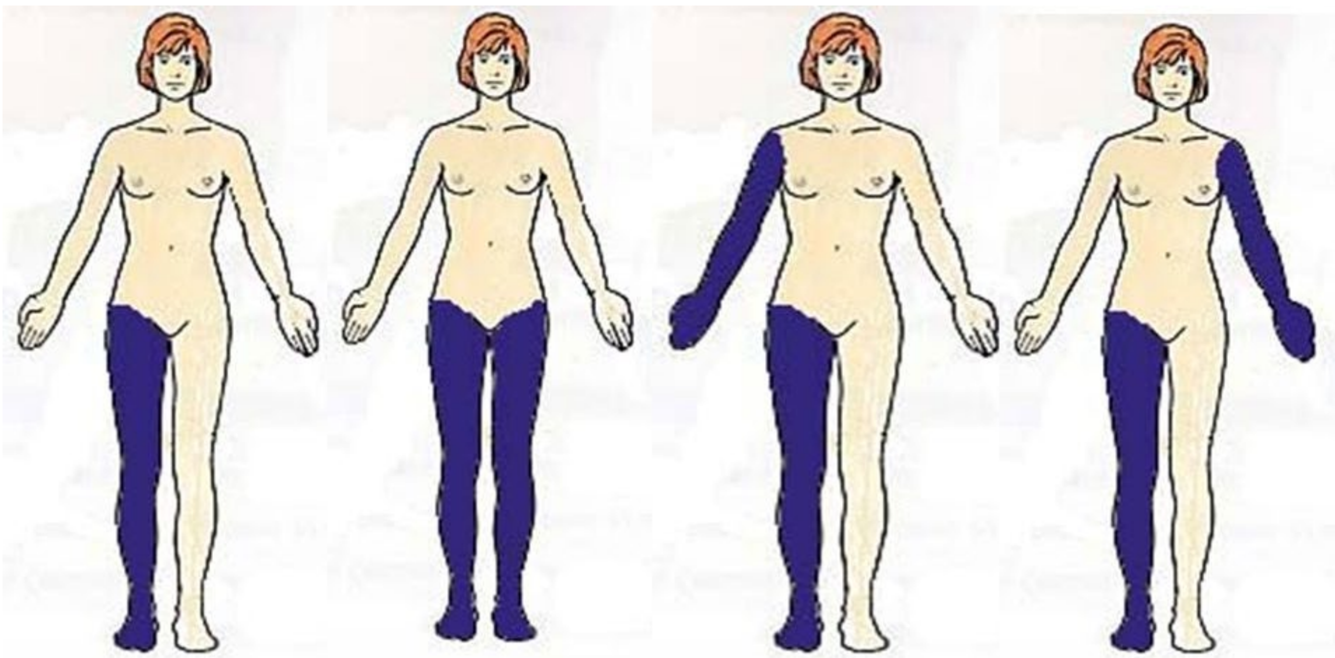
superficial	close to the surface of the body
deep (profundus)	away from the surface into the body
medial*	toward the midline of the body
lateral	away from the midline of the body
intermediate	between medial and lateral
proximal	closer to the trunk (torso)
distal	away from the trunk



Anatomical Directional Terms (2)

* Ophthalmology surgeons use **nasal** and **temporal** in place of **medial** and **lateral** (Dr. Erdey).

unilateral	on one side of the body (with respect to the sagittal plane)
bilateral	on both sides of the body (with respect to the sagittal plane)
ipsilateral	on the same side of the body (with respect to the sagittal plane)
contralateral	on the opposite side of the body (with respect to the sagittal plane)



www.cram.com

Anatomical Directional Terms (3)

Anatomists refer these directional terms to the **standard anatomical position**, which is shown in the standing human of the previous four figures. The terms may also be applied in other positions including seated, prone (lying horizontally, face down), and supine (lying horizontally, face up).

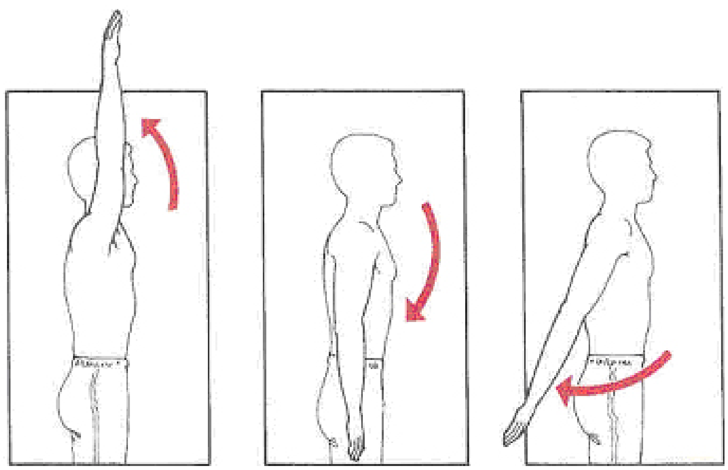
Directional terms examples

- The shoulder is *superior* (*cranial*) to the elbow and the elbow is *inferior* (*caudal*) to the shoulder.
- The nose is on the *anterior* (*ventral*) surface of the head and the spine is on the *posterior* (*dorsal*) side of the body.
- The nose is *medial* to the ears while the ears are *lateral* to the nose. The cheeks are *intermediate* between the nose and the ears.
- The knee is *proximal* to the foot and the foot is *distal* to the knee.
- The skull is *superficial* to the brain while the brain is *deep* to the skull.
- The left foot is *ipsilateral* to the left hand while the left foot is *contralateral* to the right hand.

Motion Terms

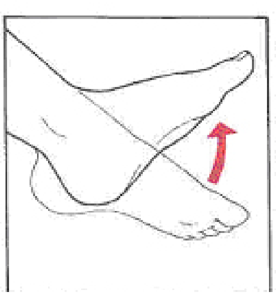
Anatomists and physiologists use the following standard terms to describe specific motions in the body, grouped by the three anatomical planes (*sagittal*, *frontal*, and *horizontal planes*). The sketch figures on the following two pages are from Hall (2007).

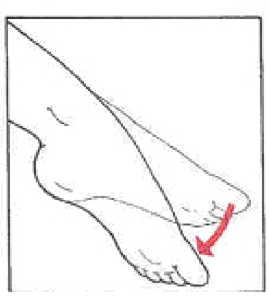
Sagittal Plane Motions



shoulder flexion extension hyperextension
(also wrist, neck, elbow, hip, knee)

Hyperextension is extension of a body limb beyond its normal range of motion. **Hyperflexion** is similarly defined.



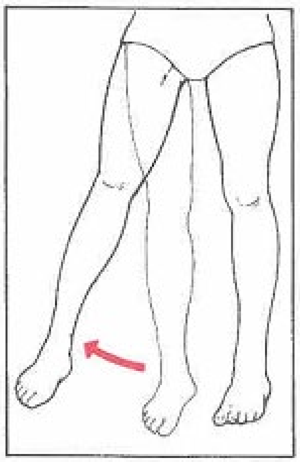


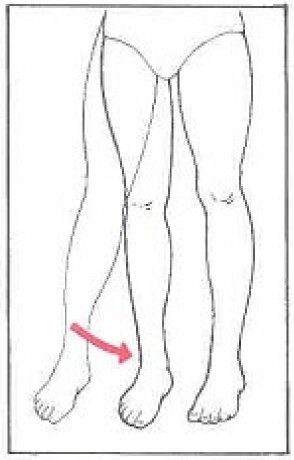
foot dorsiflexion plantar flexion

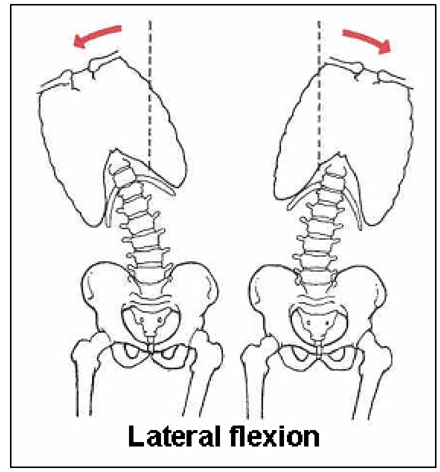
Mandible Protraction/Retraction octc.kctcs.edu

Protraction – forward motion of a body part such as the mandible.
Retraction – return motion of a body part such as the mandible.

Frontal Plane Motions







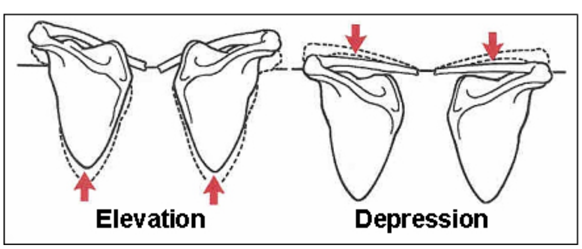
abduction **adduction** **spine**

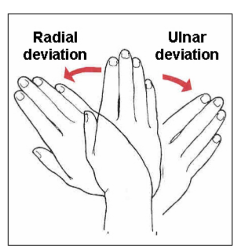
(also shoulder) **(right and left; also head)**

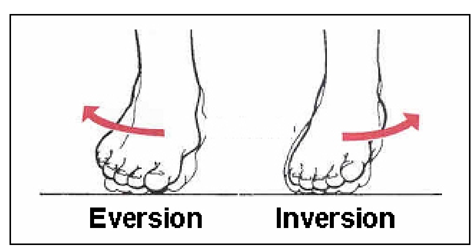
ab: away
ad: toward

fingers abduction **fingers adduction**

anatomystudybuddy.files.wordpress.com

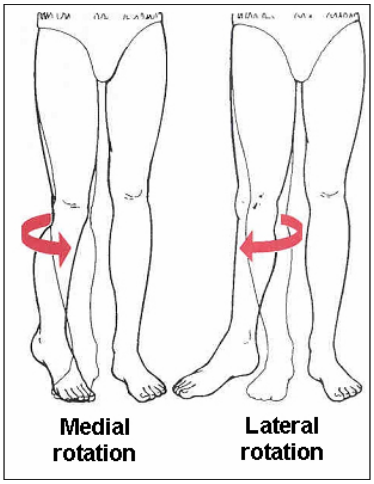


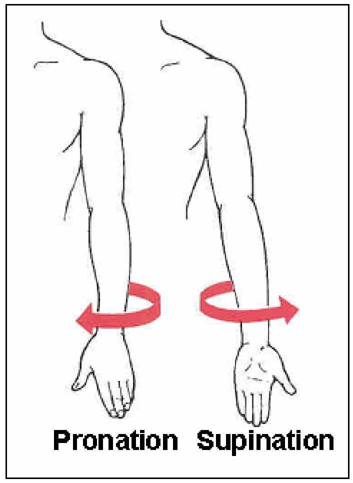


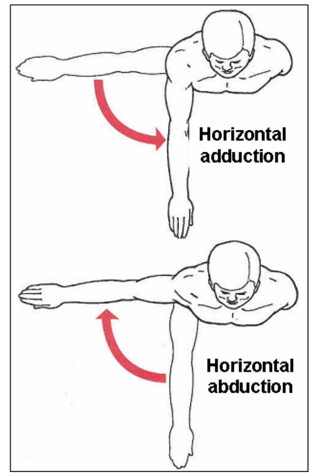


shoulder motions **wrist motions** **foot motions**

Horizontal Plane Motions







hip rotation (also shoulder)

elbow twisting

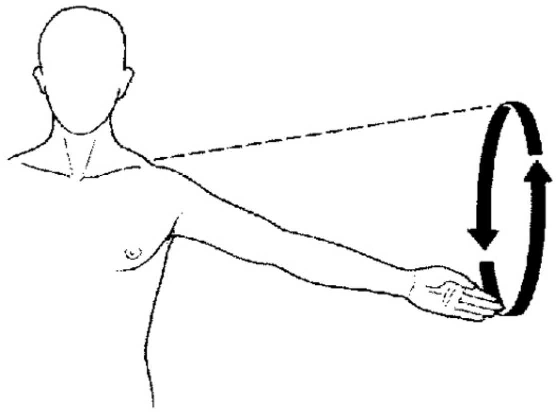
arm motions

Head Rotation Right

helpinghilda.com

Other Motions

Circumduction – tracing a cone, this is a combination of flexion, abduction, extension, and adduction. Circumduction is possible at the shoulder and hip, plus finger, wrist, head, and ankle.



circumduction

content.answers.com

opposition

eorthopod.com

"Well, there it goes again…and here we just sit without opposable thumbs."
The Far Side, Gary Larson

Opposition – opposable thumbs are NOT unique to humans as is often believed. All primates have opposable thumbs (and some toes), as do koalas, pandas, possums, some birds and some frogs; even some dinosaurs (ancestral birds) had opposable thumbs. The aye-aye lemur of Madagascar has an opposable thumb with a ball joint.

What is unique to humans? The human thumb metacarpal revolves to a large degree, allowing one-by-one digit opposition as shown above.

2. Human Skeletal Anatomy and Physiology

"When he (young Rossum) took a look at **human anatomy** he saw immediately that it was too complex and that a good engineer could simplify it. So he undertook to redesign anatomy, experimenting with what would lend itself to omission or simplification. Robots have a phenomenal memory. If you were to read them a twenty-volume encyclopedia they could repeat the contents in order, but they never think up anything original. They'd make fine university professors."

– Karel Capek, R.U.R. (Rossum's Universal Robots), 1920

"If you want to understand function, study structure."

– Francis Crick

2.1 Human Skeletal Anatomy

Again, anatomy is the study of the **structures** of an organism and the **relationships of its parts**. Etymology: from the Greek 'apart' and 'to cut', indicating the importance of dissection in anatomical studies. Public human cadaver dissections were held in medieval Europe. Leonardo da Vinci performed extensive anatomical human dissections to inform his artwork and for its own sake. Recently, machines (MRI, CAT Scans, Ultrasound) are allowing in vivo studies of anatomy.

This section presents a brief overview of human skeletal anatomy. A detailed study of joints and joint limits is presented later in the section on human skeletal physiology.

Human Skeleton Juxtaposed with its Muscular System *Bodies: The Exhibition*

Functions of the Skeleton

- **Support** – the skeleton is the body's framework, giving support to soft tissues and attachment points for the muscles.
- **Movement** – the skeleton joints (see the section on Skeletal Physiology) allow the bones to articulate and provide gross motion.
- **Protection** – the skeleton protects vital internal organs from injury. The brain is protected by the skull, the spinal cord by the vertebrae, the thoracic organs by the rib cage, and the bladder and internal reproductive organs by the bony pelvis.
- **Mineral Reservoir** – bones store calcium, phosphorus, sodium, potassium, and other minerals. These can be distributed by the vascular system as needed by the body. For example, during pregnancy, calcium from the mother's bones is used for fetal bone development (if the mother's diet does not include sufficient calcium).
- **Hemopoiesis** – the marrow of long bones produce the red blood cells, white blood cells, and platelets for the circulatory and immune systems.

Spence (1982)

The human skeleton is an endoskeleton, living, growing, adapting, and repairing within the soft tissues of the human body. The bones are held in place by ligaments. The average human baby is born with over 300 bones. The average adult human has 206 bones (some have more due to an extra rib or an extra lumbar vertebra). What happened to the 94+ bones in the baby?

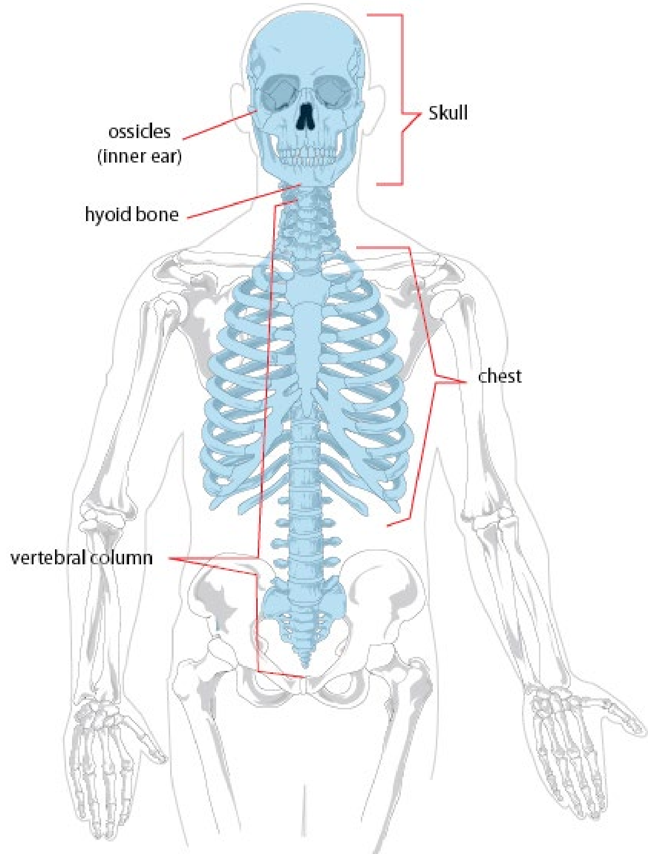
The adult human skeletal bones are grouped into the **axial skeleton** and the **appendicular skeleton** (see figure on the next page). The table below presents the numbers of bones in the subcategories of the adult human **axial** and **appendicular skeletons**. Clearly, the largest number of bones is in the appendicular skeleton, in the hands and feet of the upper and lower limbs, respectively.

Classification	**# Bones**	**Total**
Axial Skeleton		
Skull	29	
Vertebral Column	26	
Thorax (ribs and sternum)	25	
Subtotal		**80**
Appendicular Skeleton		
Shoulder girdle	4	
Upper limbs	60	
Pelvic girdle	2	
Lower limbs	60	
Subtotal		**126**
Total		**206**

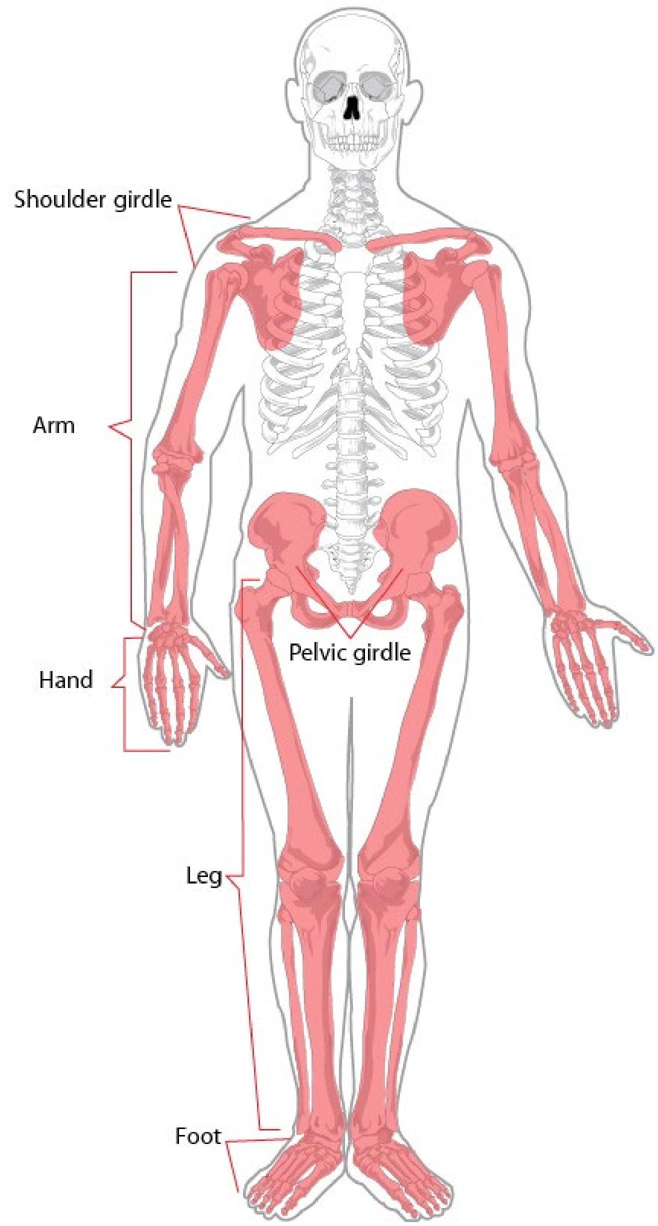
The figure following the axial/appendicular skeleton figures presents the names of the major bones in the adult human.

- The smallest bone in your body is the stirrup[1] in the ear. Less than one inch in length, it transmits sound from the eardrum to the inner ear.
- The largest bone in your body is the femur of the upper leg.
- The hyoid bone is between the voice box and mandible. It is V-shaped and is the only human bone not connected to another bone.

[1] Also called the stapes (Latin for stirrup) bone, the innermost (most medial) of the ossicles.



Axial Skeleton in Blue



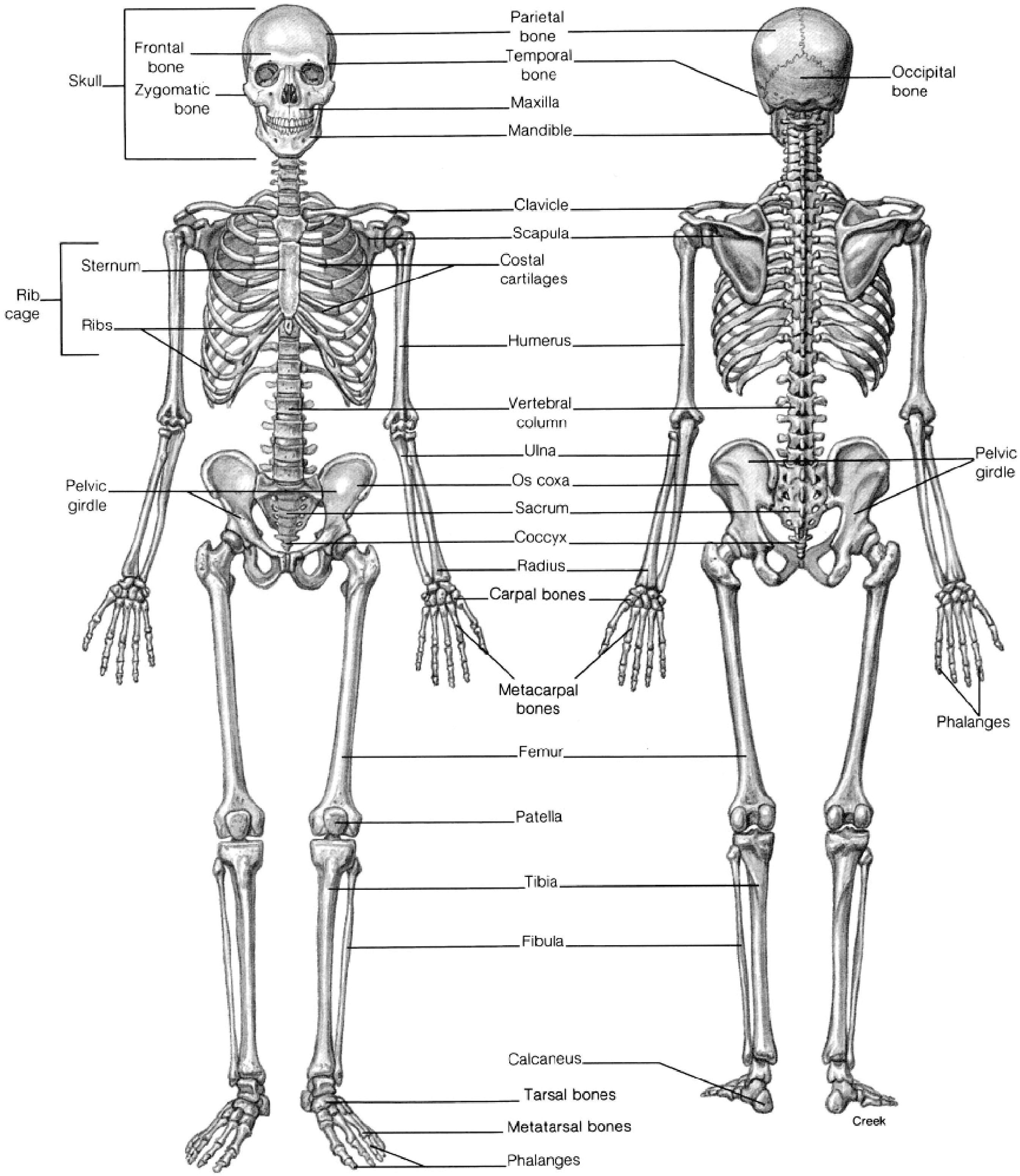
Appendicular Skeleton in Red

Wikipedia

Every bone cell is replaced every 7 years in a healthy adult human.

Human bone is harder than concrete and stronger and lighter than steel. The average adult human skeletal system has a mass of 10 kg; replaced with steel for equivalent strength, this would require 400 kg of steel! Plus, bone is more resilient than steel.

The human Tibia (shinbone, see next page) supports 100% of the body weight and the Fibula supports 0% (recent research suggests the Fibula may support up to 10% body weight). So what is the purpose of the Fibula? It provides muscle attachment points.



Major Bones of the Adult Human Skeleton

Hamilton et al. (2008)

<u>Bone Types</u>

Bones are classified according to their shape.

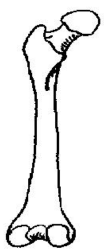
- **Long Bones** – most bones of the upper and lower limbs have a long axis: humerus, radius, ulna, femur, tibia, fibula, phalanges.

- **Short Bones** – bones that do not have a long axis: wrist (carpals) and ankle (tarsals).

- **Flat Bones** – thin bones: roof of cranial cavity, ribs, and sternum.

- **Irregular Bones** – various shapes: some skull bones, vertebrae, pectoral and pelvic bones.

Spence (1982)



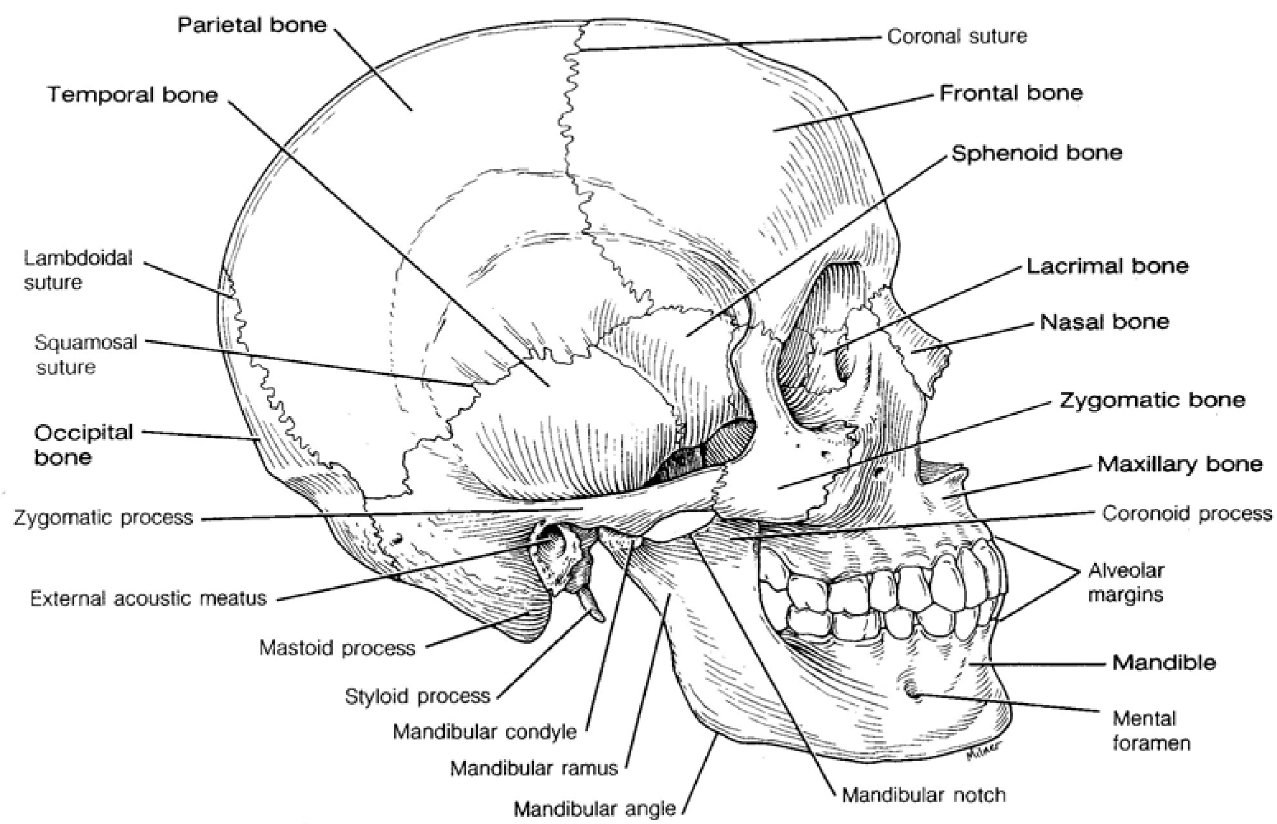
Long

Short

Flat

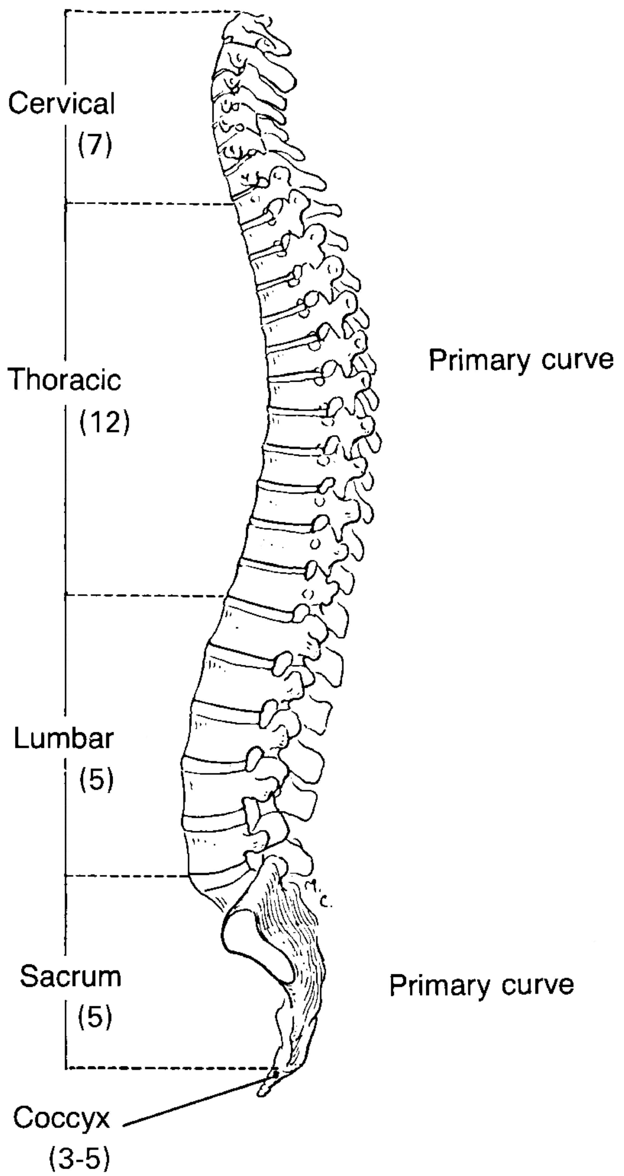
Irregular

<u>Human Skeletal Details</u>

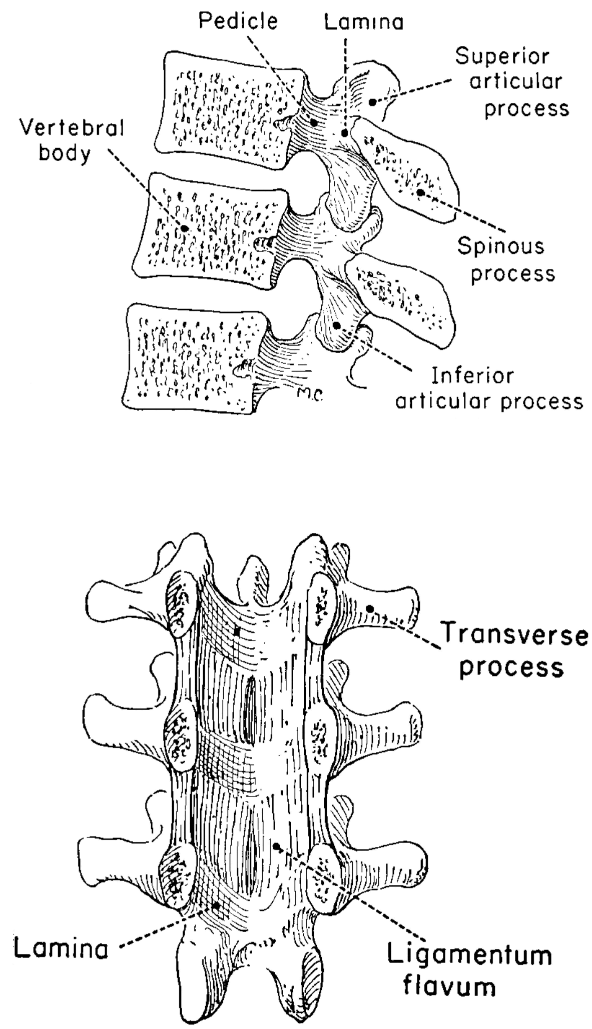


Skull Side View *Spence (1982)*

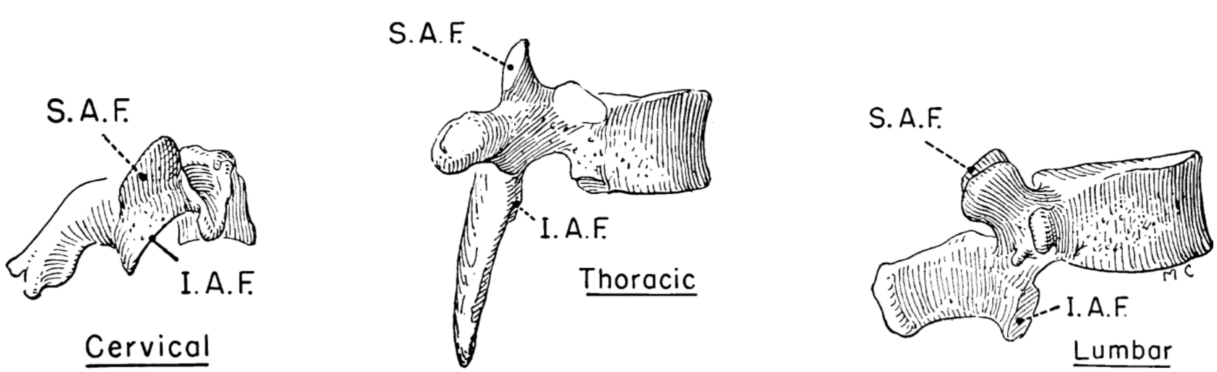
sinciput: the front of the skull from the forehead to the crown
occiput: the back of the skull



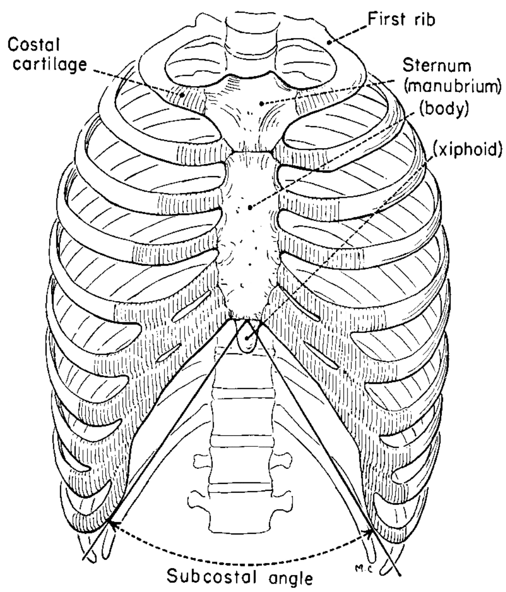
Human Spine showing normal Lordotic Curve



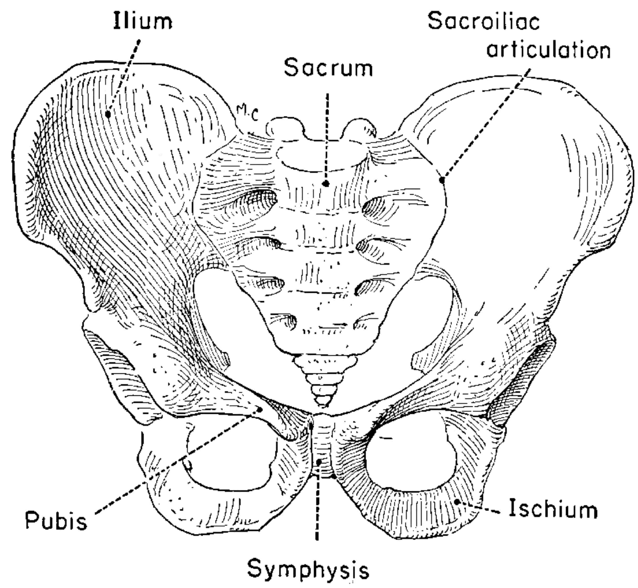
Vertebral Details



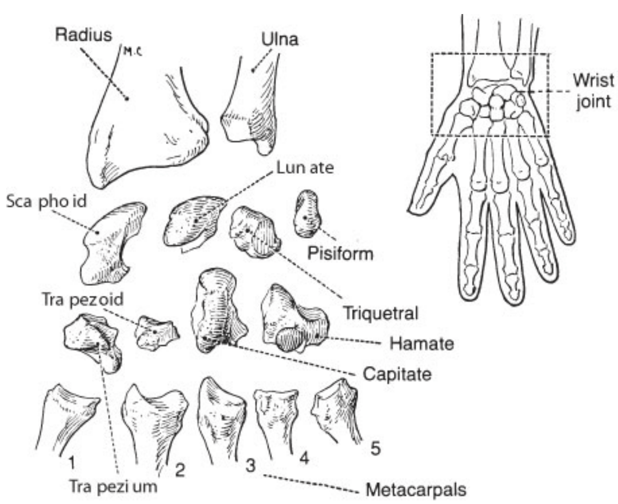
Hamilton et al. (2008)



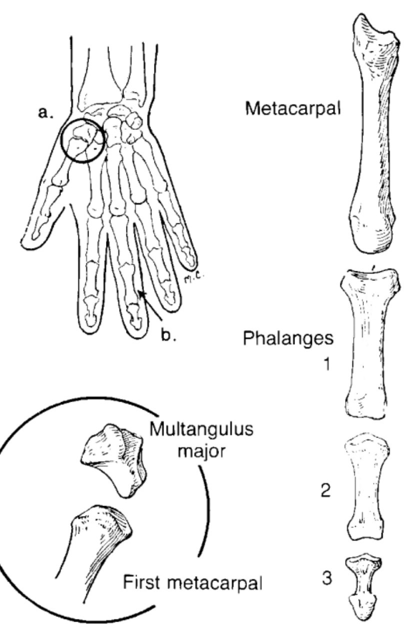
Rib Cage



Pelvic Bones



Wrist Details



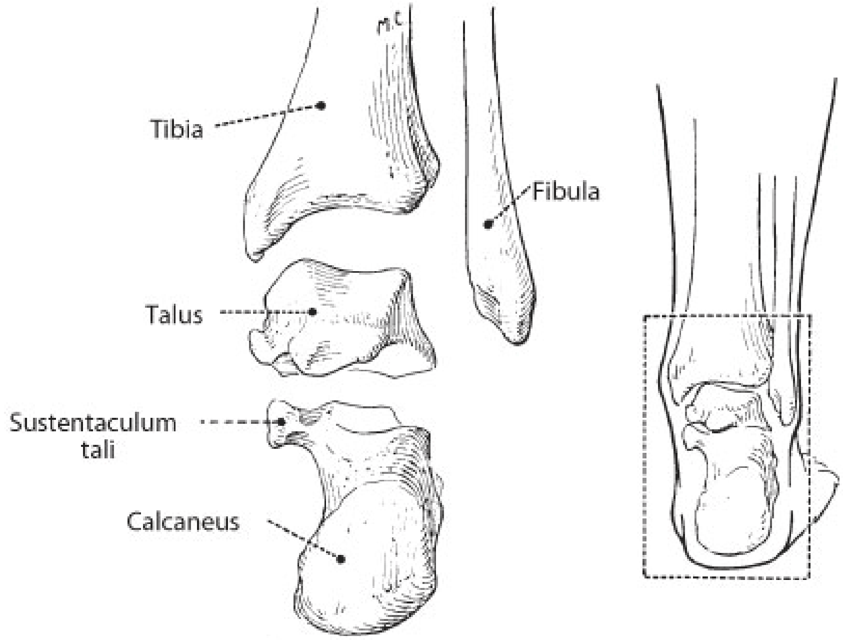
Finger Details

Hamilton et al. (2008)

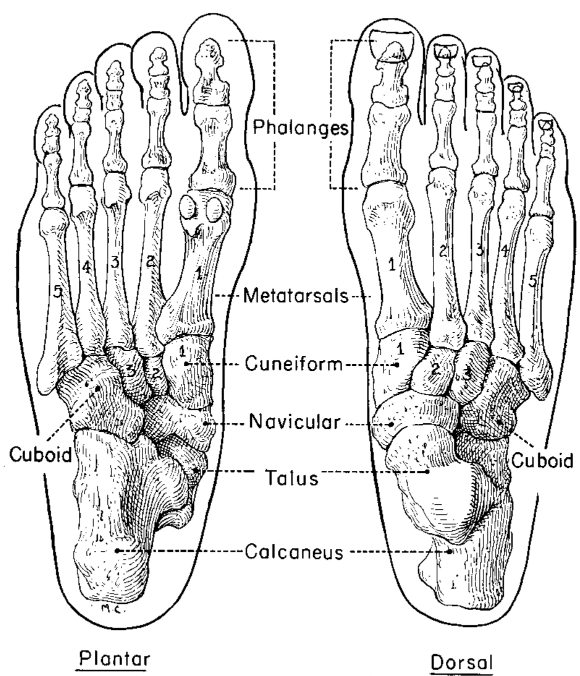
pollex: thumb, straight from Latin (plural pollices)

There are 28 phalanges in the human hand (1-proximal, 2-intermediate, 3-distal) counting both hands and including the thumbs (with no intermediate phalanges). There are also 28 phalanges in the human foot, distributed in the same exact manner. What is the singular form of phalanges? Hint: it is an ancient Greek military formation.

The basic human arm/hand morphology (single long bone from the shoulder, two forearm bones, cluster of wrist bones, five fingers) is repeated in various diverse animals including frogs, bats, dolphins, lions, and elephants, to name a few (National Geographic Magazine, May 2012).

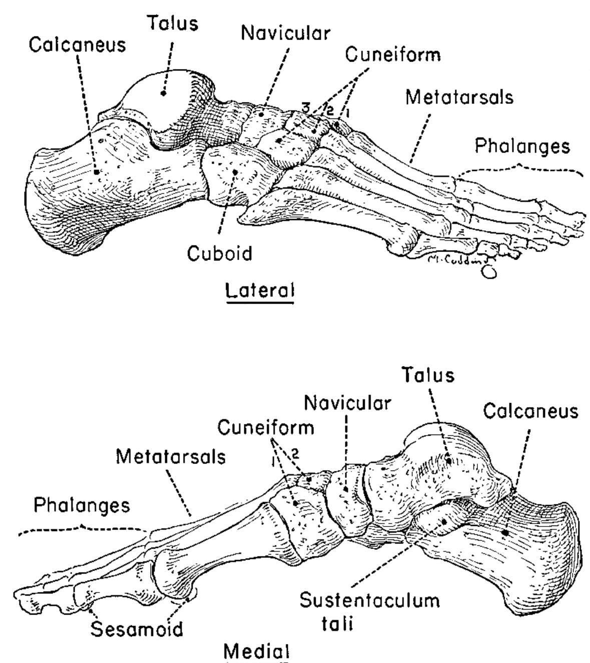


Ankle Details



Top Foot Details

hallux: big toe, straight from Latin (plural hallucis)



Side Foot Details

Hamilton et al. (2008)

Bone Composition

Bone is composed of an **organic framework** and **inorganic salts**. The *organic framework*, in which the *inorganic salts* are embedded, is formed of **collagenous fibers** like other connective tissues. The *inorganic salts* are mainly **calcium** and **phosphate**.

The *collagen fibers* give bone great tensile and torsional strength. The *salts* withstand compressive stresses. The combination of *fibers* and *salts* make the bone strong without being brittle. This is the same principle as reinforced concrete.

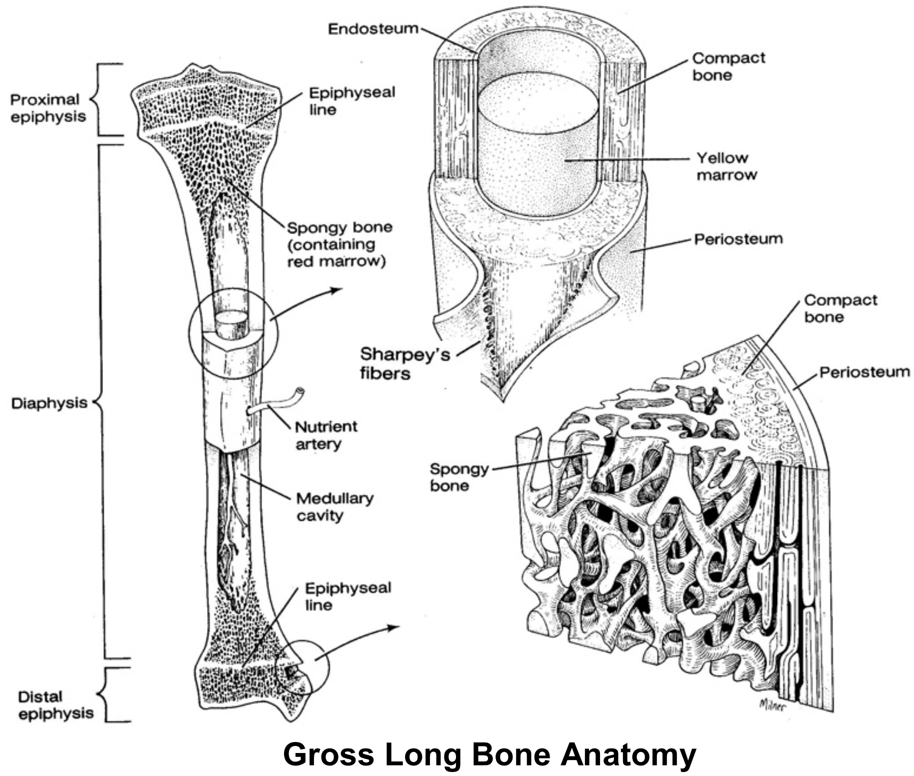
Human bone is a nonlinear, heterogeneous, non-isotropic material, with highly irregular geometry, unlike most engineering materials.

Long Bones Gross Anatomy

- **Shaft** (diaphysis) – a hollow cylinder of *compact bone* surrounding a *medullary cavity*, with two ends, the *proximal* and *distal epiphyses*.
- **Medullary Cavity** – yellow bone marrow cavity, stores fat. Lined by a thin connective-tissue layer called the *endosteum*.
- **Epiphyses** – outer surfaces are also compact bone. The inner structure is *spongy* (*cancellous*) *bone*, containing the red bone marrow.
- **Epiphyseal Cartilage** – bone growth plates for the young, transforms to bone in adults, called the *epiphyseal line*.

Flat bones have no *medullary cavity* – they are formed of two surface layers of compact bone encasing the **diploe** (*spongy bone*, also called *cancellous* or *trabecular* bone) containing red marrow.

Bones are covered with double-layered connective tissue – **periosteum** and **endosteum** (except at joints where the outer bone surface is covered with articular cartilage). The inner layer of the *periosteum* is connected to bone via collagenous bundles called **Sharpey's fibers**.



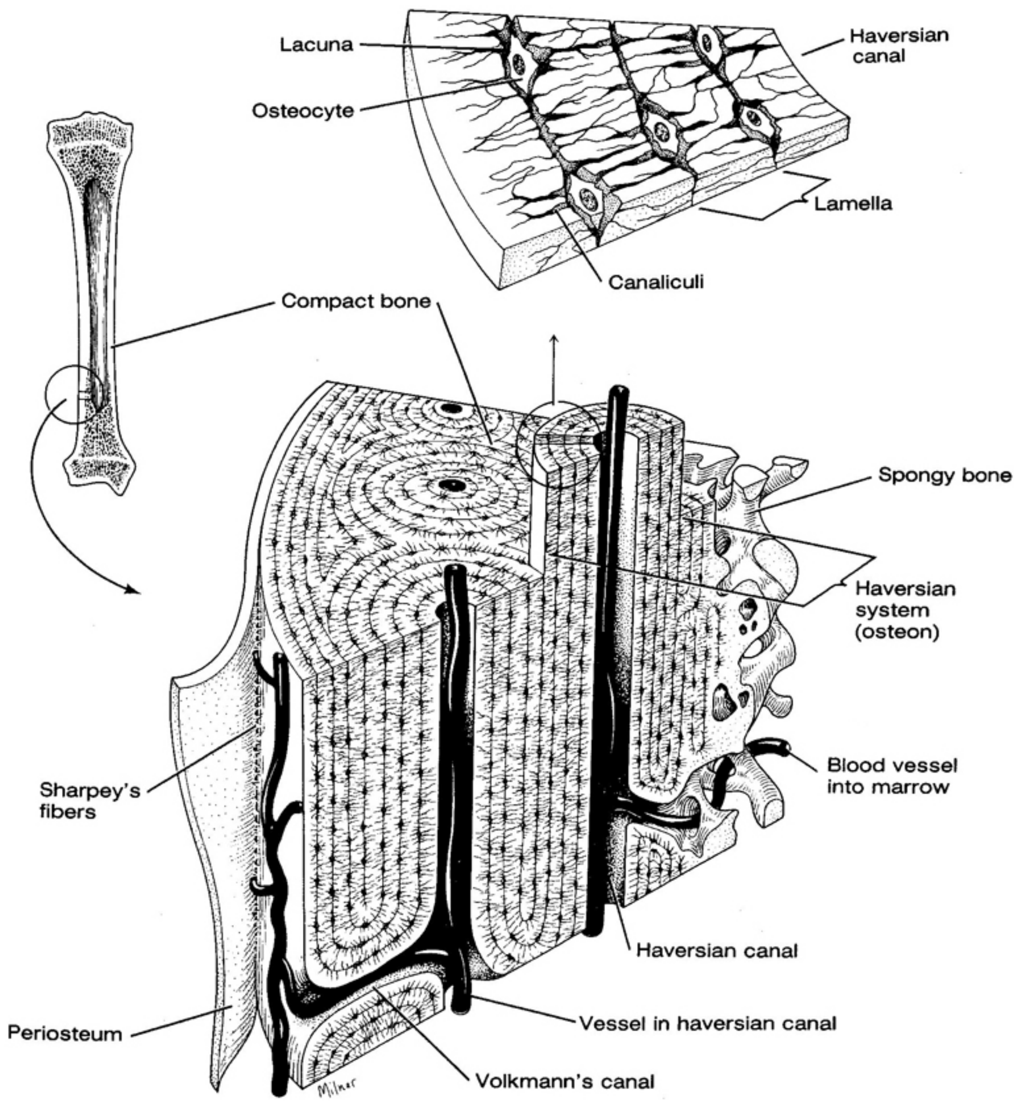
Gross Long Bone Anatomy *Spence (1982)*

Microscopic Bone Anatomy

Compact Bone

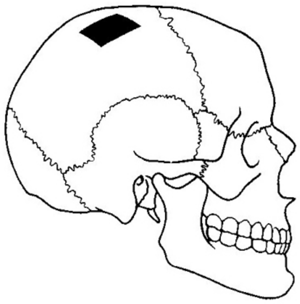
- Composed of many organized systems of interconnecting canals.
- The structural unit is called the **haversian system** (**osteon**).
- Each *haversian system* has a central **haversian canal** surrounded by concentric **lamellae** (layers) of bone, parallel to the long axis.
- Between adjacent *lamellae* are small cavities called **lacunae**.
- Each *lacuna* has a cell called an **osteocyte**.
- All *lacunae* in a *haversian system* are interconnected by tiny canals called **canaliculi**.
- Each *haversian canal* has at least one blood capillary for nutrients and waste removal.
- **Volkmann's canals** (perpendicular to *haversian canals*) allow blood to reach the *haversian canals* from the outer surface and from the marrow.

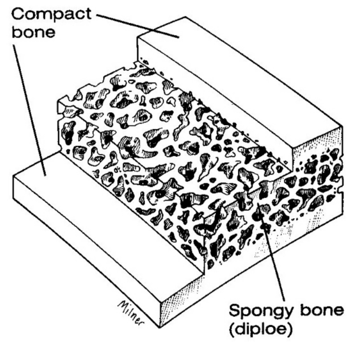
Every bone cell is replaced every seven years in a healthy adult human.



Microscopic Long Bone Anatomy *Spence (1982)*

Spongy bone (diploe) has the *osteocytes*, *lacunae*, and *canaliculi* of compact bone, but the *lamellae* are not arranged in concentric layers. They are arranged along lines of maximum pressure or tension.





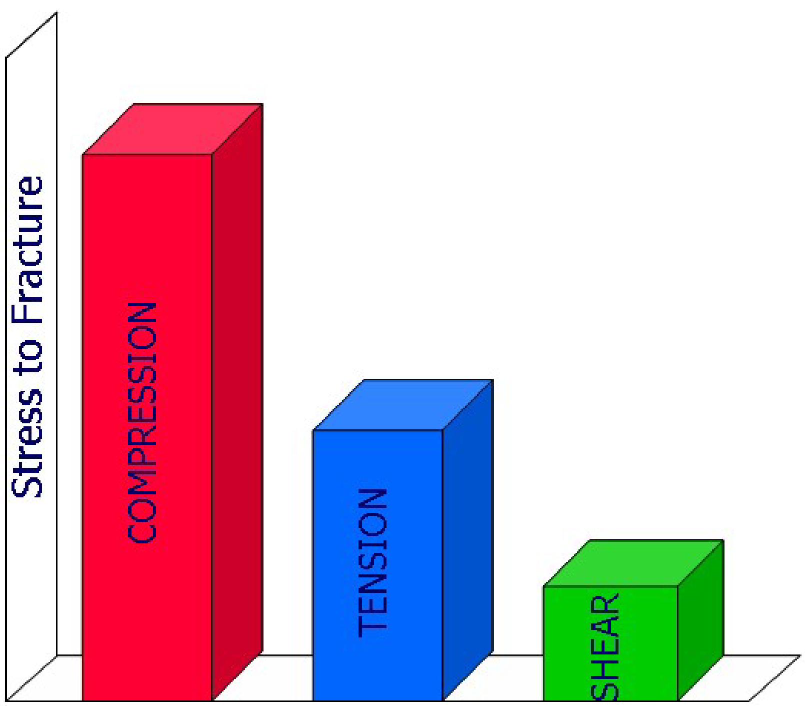
Gross Spongy Bone Anatomy *Spence (1982)*

The skeletal system is a living system with blood, minerals, nutrients, wastes, and marrow.

Bone Pathologies

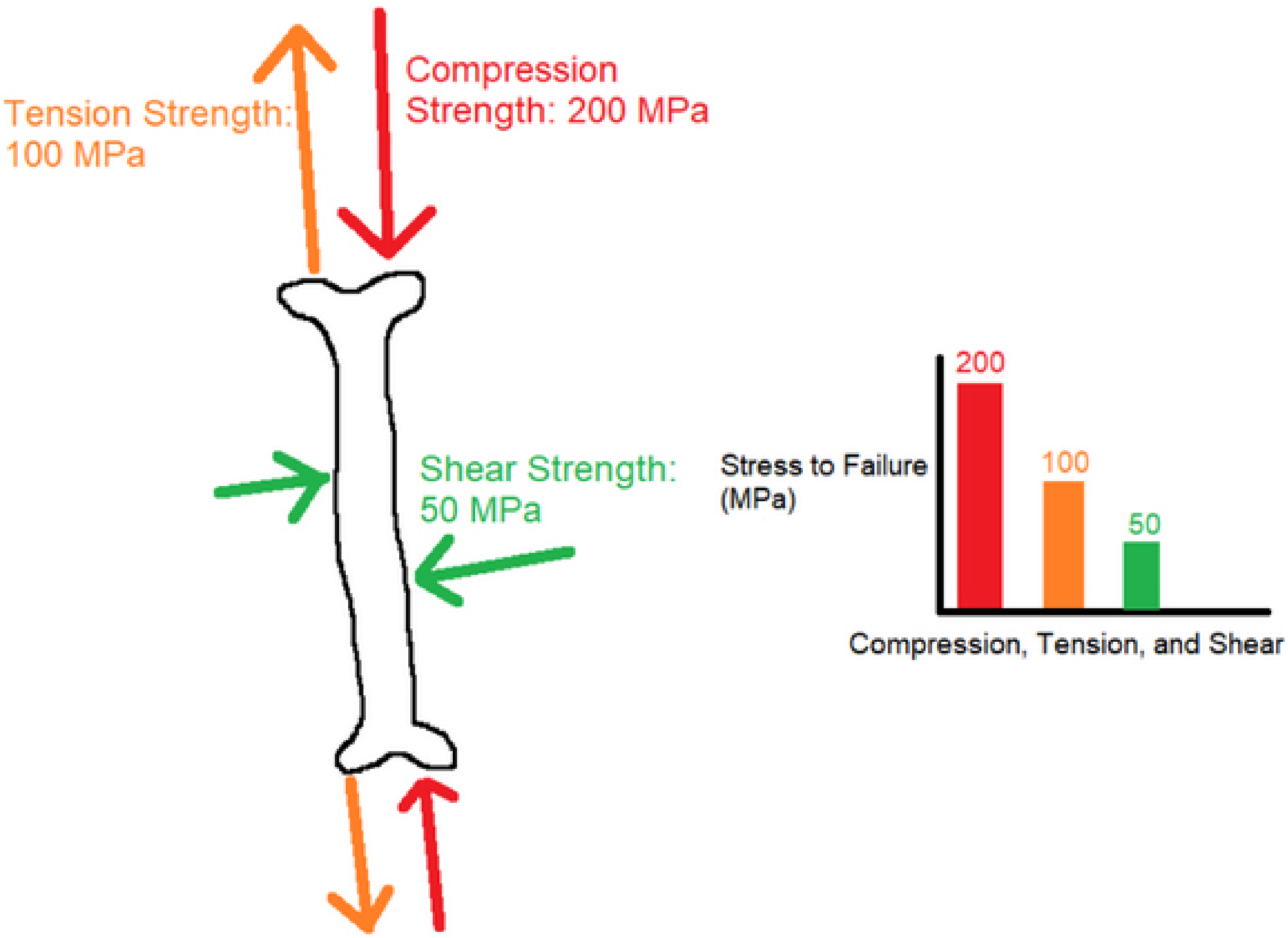
Fractures

Bone is strongest in resisting compression and weakest in resisting shear. The approximate ultimate strength (stress to fracture) for human long bones is 200 MPa in compression, 100 MPa in tension, and 50 MPa in shear.



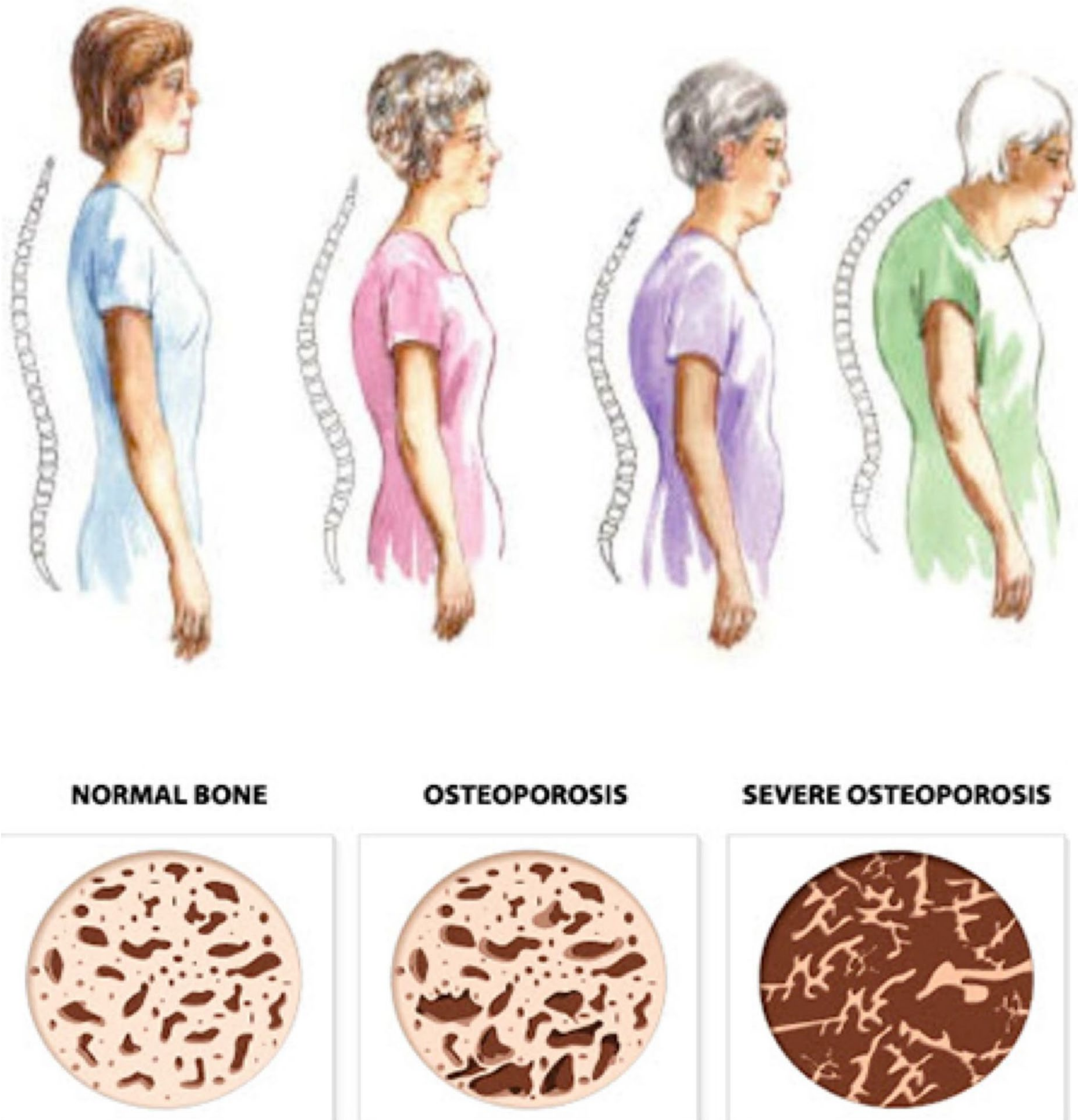
Relative Bone Strength for Different Loading Modes

Hall (2007)



Relative Bone Strength for Different Loading Modes

Osteoporosis is a gradual reduction of bone formation while bone absorption remains normal.



osteoporosis.jpeg (2025×2032) (drfitnessusa.com)

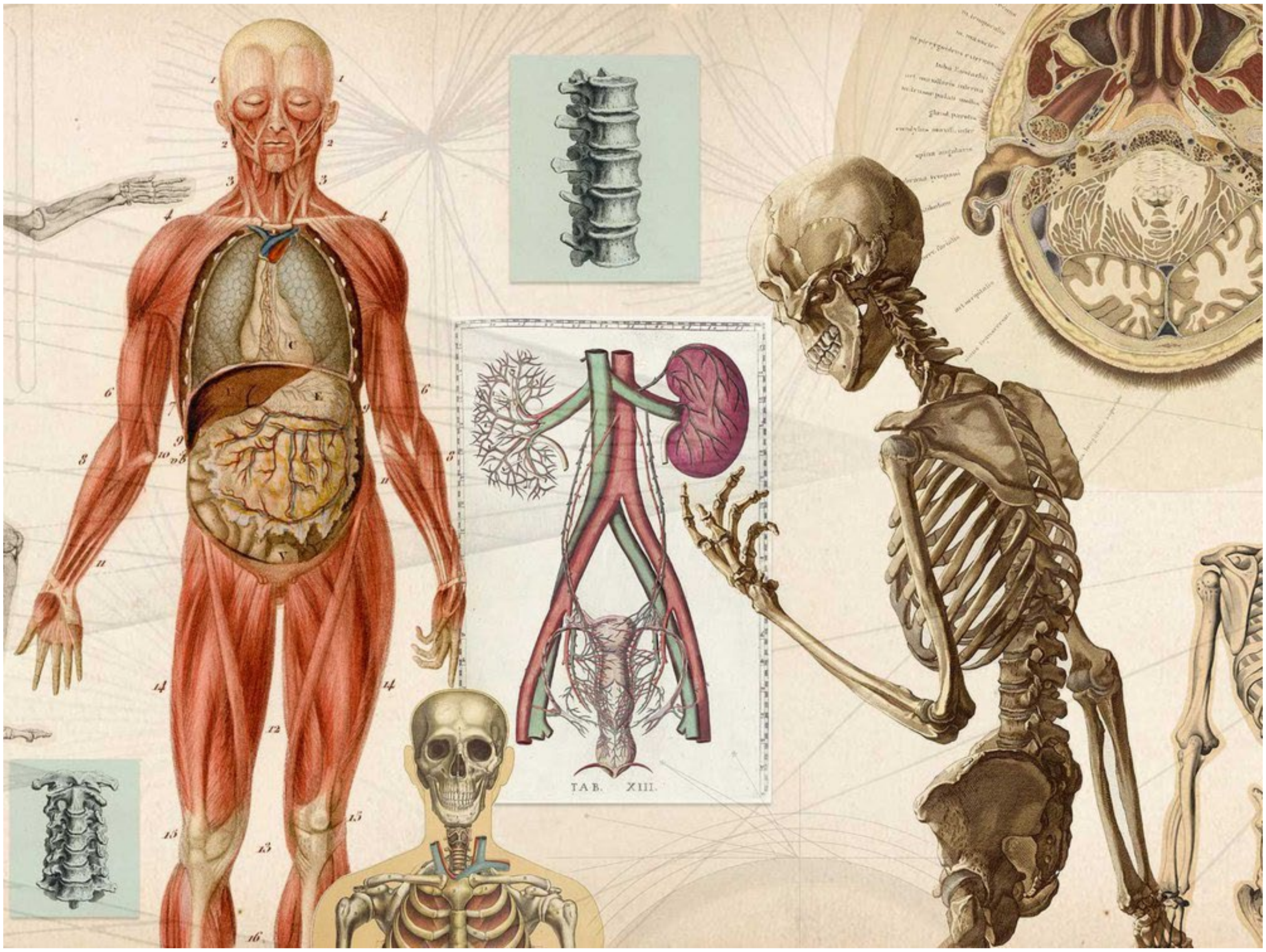
Bone with Osteoporosis *Hall (2007)*

For additional bone pathologies please see the online ME 4670 / 5670 Supplement.

Bones Communicate with other Parts of the Human Body

Recently biologists have discovered that bones of the human skeletal system have complicated chemical conversations with other parts of the human body, including fat and muscle, the kidneys, the brain, and gut microbes. This is a two-way communication. "It's as if you suddenly found out that the studs and rafters in your house were communicating with your toaster."

This new vision of a dynamic human skeletal system could lead to improved therapies for various maladies.



Human Bones Communicate with Other Body Systems

[How Bones Communicate With the Rest of the Body | Science| Smithsonian Magazine]

March 2022

2.2 Human Skeletal Physiology

Again, physiology is the study of the **functions** of an organism and its parts. This section presents the physiology of the human skeleton, focusing on the joints, whose degrees-of-freedom, nature, and functions are very important in the field of kinesiology.

Overview

The human skeleton is a framework of bones, cartilage, ligaments, and joints. The skeletal system provides the levers (bones) and joint axes of rotation about which human movements are generated via muscles. There are about 244 degrees-of-freedom (dof) in the adult human body.

There are three different types of human skeletal joints:

- Immovable joints (synarthroses)
- Slightly-movable joints (amphiarthroses)
- Freely-movable joints (diarthroses)

Degrees-of-Freedom (dof)

In engineering mechanics, the mobility, or number of degrees-of-freedom (dof) is defined as the number of parameters required to completely specify the location of a machine. Structures have a mobility of 0 (or even negative), mechanisms have 1 dof, while robots have greater than 1 dof. One way to determine the mobility is to simply count the number of motors required to move a machine.

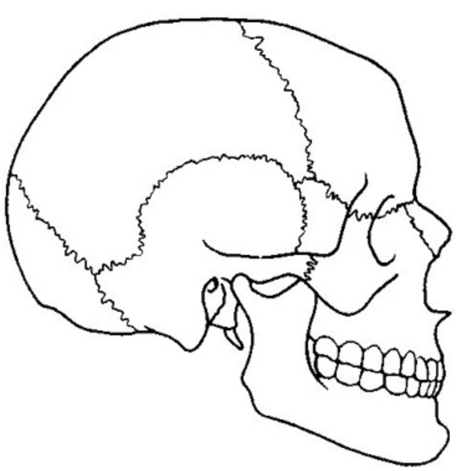
This approach will not work in biomechanics since there is actuation redundancy (agonists and antagonists and more) in the human musculature. We will consider the degrees-of-freedom for each type of joint.

Immovable Joints

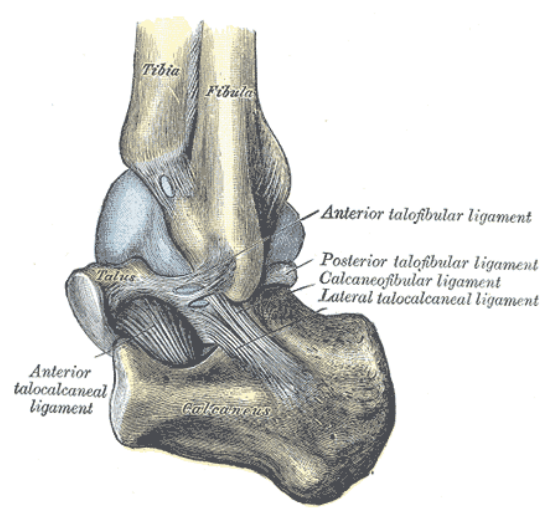
Also called **fibrous joints** or **synarthroses**, immovable joints are fixed joints that can absorb shock but they allow little or no motion. There are two types of immovable joints:

1. **Sutures** – only found between the flat bones of the skull, these fuse by early adulthood.
2. **Syndesmoses** – hold bones together by fibrous ligaments. They permit some give, but not gross motion. e.g. distal ends of the tibia and fibula are held together by a syndesmosis joint.

Immovable joints have 0 dof, like welds, bolts and nuts, rivets, or other fixed connections in engineering mechanics.



Skull Sutures *Spence (1982)*



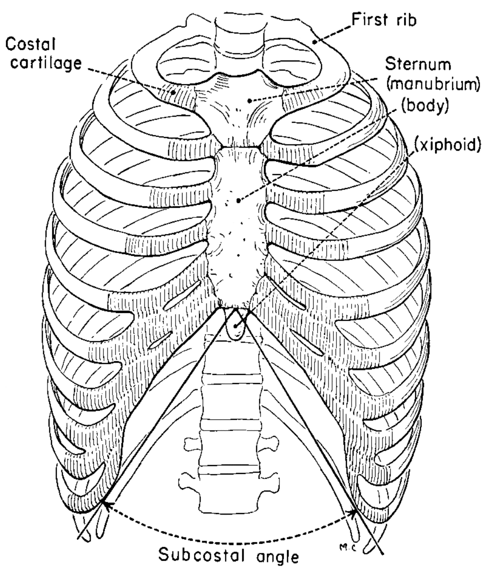
Tibiofibular Syndemosis wikipedia.org

Slightly-Movable Joints

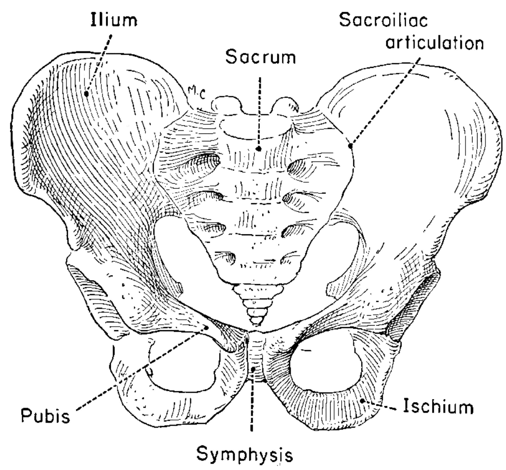
Also called **cartilaginous joints** or **amphiarthroses**, slightly-movable joints are formed by cartilage. There are two types of slightly-movable joints:

1. **Synchondroses** – many of these are temporary and eventually replaced by bone, e.g. between the epiphyses and diaphysis of long bones and between certain skull bones. The joints between the first ten ribs and the sternum are permanent synchondroses.

2. **Symphyses** – fibrocartilaginous pad, such as between the two pubic bones and between all vertebra of the spinal column.



Rib Synchondroses



Pubic Bone Symphysis

Hamilton et al. (2008)

Slightly-movable joints also have 0 dof, for engineering mechanics rigid-body analysis purposes. The exception is the spine – the Ohio University Virtual Haptic Back team modeled each vertebra as having 6 dof; we also have a 3-dof per vertebra rotational-only spine model (see Appendix B for the spine vertebral angular joint limits).

Freely-Movable Joints

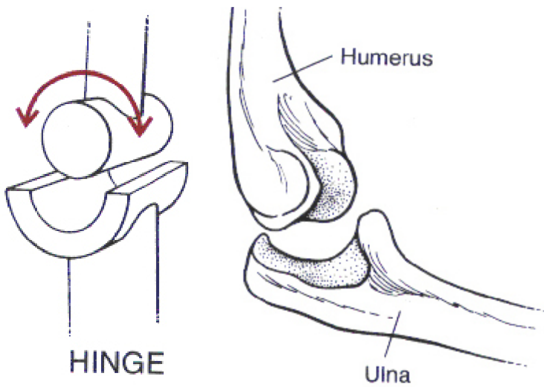
Most of the joints in the adult human body are **freely movable joints**, also called **synovial joints** or **diarthroses**. Of course the range of motion for all joints is not unlimited – this is discussed at the end of this section. Freely-movable joints each have a cavity filled with synovial fluid, with very low friction, for lubricating and nourishing each joint.

There are of six types of synovial joints, based on the type of movement provided and the number of degrees of freedom allowed. These are subdivided by anatomists and physiologists into **Uniaxial**, **Biaxial**, **Nonaxial**, and **Multiaxial** joints. Engineers have a different categorization.

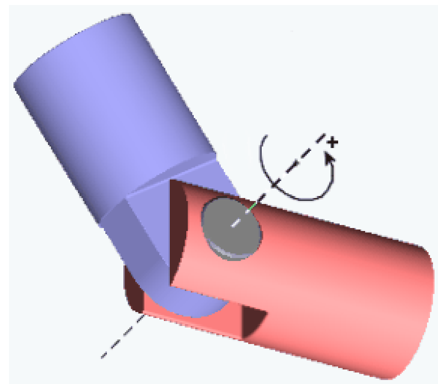
Uniaxial Joints

1. **Hinge** (**ginglymus**) **joints** – only flexion or extension about a single rotation axis is allowed. The elbow joint (see the left figure below, also called ulnohumeral articulation), the knee, and the interphalangeal joints of the fingers and toes are examples.

In engineering mechanics these are called **revolute** (**R**) **joints**, allowing one-dof of rotational motion about a single axis (see the right figure below).



Elbow Hinge Joint
Hamill and Knutzen (1995)

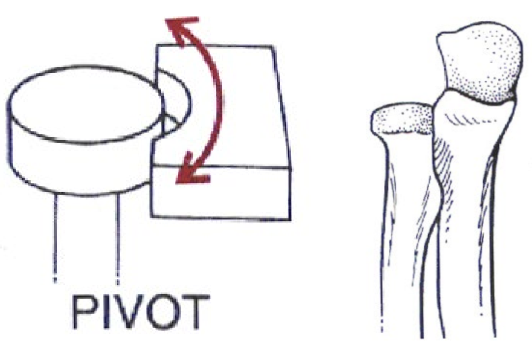


Engineering Revolute Joint
www.mathworks.com

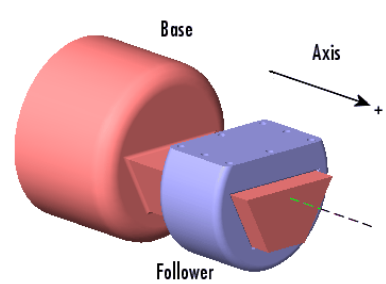
2. **Pivot** (**trochoid**) **joints** – these are also uniaxial with one rotational dof about a single axis. The only motion allowed is about the longitudinal axis of the bone. For example a) rotation of the first cervical vertebra (atlas) around the odontoid process of the second cervical vertebra (axis) to rotate the head in the horizontal plane; and b) proximal articulations between the radius and ulna, allowing forearm pronation and supination (radio-ulnar joint, see the left figure below).

In engineering mechanics **pivot joints** are also called **revolute** (**R**) **joints**, allowing one-dof of rotational motion about a single axis.

Engineers call 1-dof sliding joints **prismatic** (**P**) **joints** – there are no 1-dof P joints in the human body.



Radius / Ulna Pivot Joint
Hamill and Knutzen (1995)

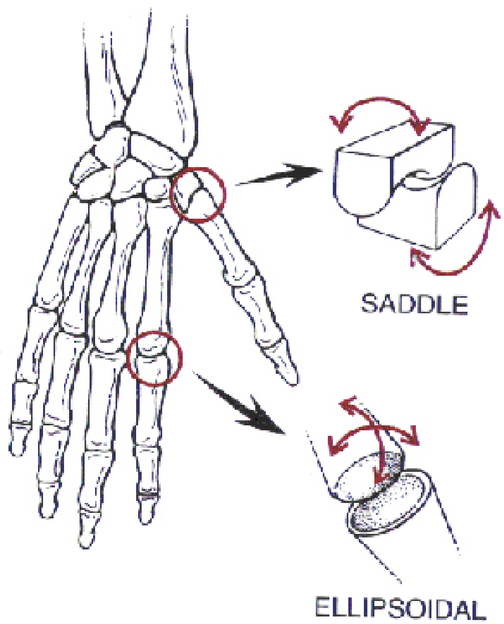


Engineering Prismatic Joint
www.mathworks.com
No human body examples

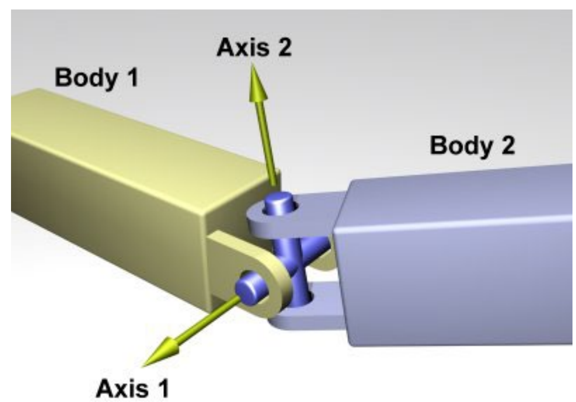
Biaxial Joints

3. **Ellipsoidal** (**condyloid**) **joints** – these provide rotational motion about one axis and a second rotational motion about a second, perpendicular axis, allowing two-dof. Flexion and extension are enabled in the first plane and abduction and adduction are enabled in the second plane. As shown in the left figure below, the articular surface of one bone is concave and the articular surface of the mating bone is convex. The radiocarpal joint at the wrist, the skull occipital condyles on the first cervical vertebra, and the metacarpophalangeal (shown in the left figure below) and metatarsophalangeal joints are all ellipsoidal joints.

In engineering mechanics these are called **universal** (**U**) **joints**, allowing two-dof of rotational motions about perpendicular axes (see the right figure below).



Ellipsoidal and Saddle Joints of the Hand
Hamill and Knutzen (1995)



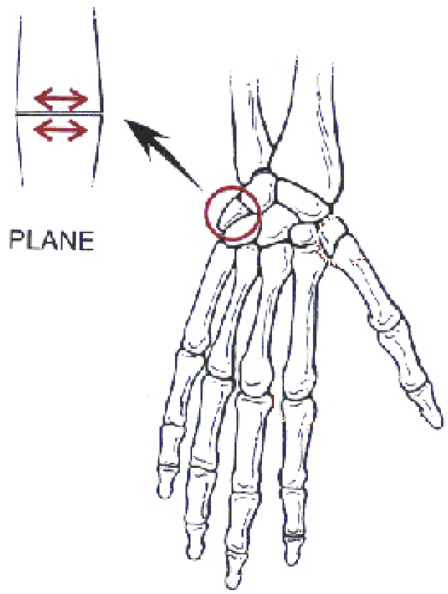
Engineering Universal Joint
[ode.org/pix](ode.org/pix)

4. **Saddle** (**sellar**) **joints** – these joints are functionally similar to the ellipsoidal joints; they are also two-dof U-joints. The articular surface of each bone is concave in one direction and concave in the other so they fit together like two saddles. The trapeziocarpometacarpal joint of the thumb is the only saddle joint in the body (shown in the left figure above).

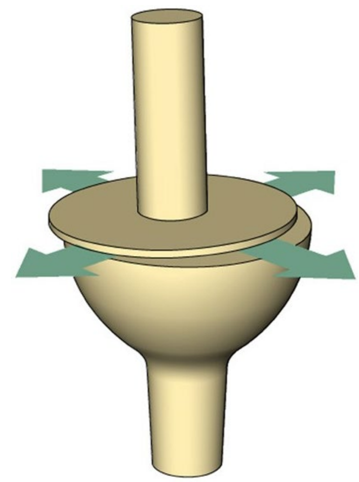
Nonaxial Joints

5. **Gliding** (**arthrodial**, **plane**) **joints** – formed by two nearly flat planes, sliding is allowed in any planar direction. Gliding joints are between the articular processes of vertebrae and between carpal bones of the wrists (shown in the left figure below) and tarsal bones of the ankles. The scapulae (shoulder blades) also move with plane joints, in the frontal plane.

In engineering mechanics these are also called **plane joints**, allowing two-dof of sliding motion. The name **Nonaxial** is not correct since we can attach *XY* motion axes to describe the **Biaxial** motion.



Intercarpal Plane Joint in the Hand
Hamill and Knutzen (1995)

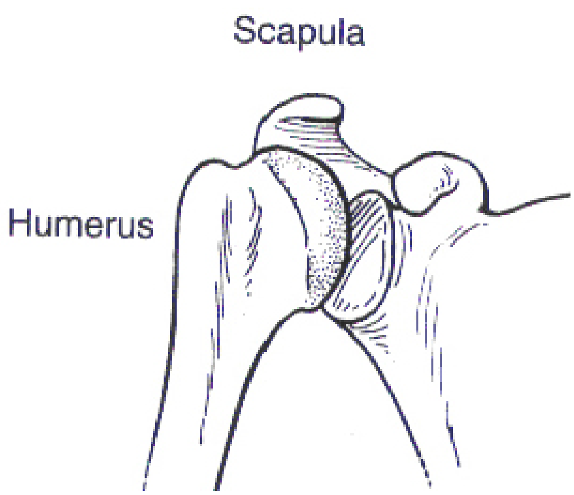


Engineering Plane Joint
dorlingkindersley-uk.co.uk

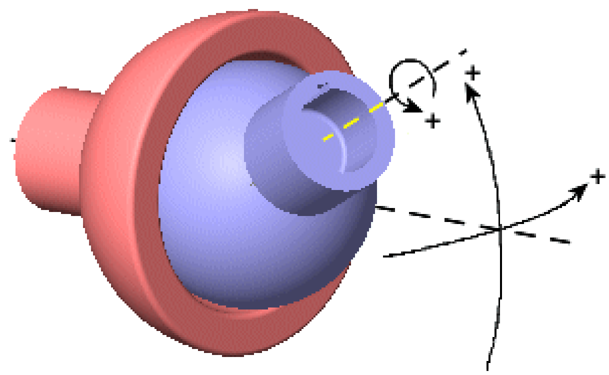
Multiaxial Joints

6. **Ball and socket** (**spheroidal**) **joints** – these are the most mobile joints in the human body with three-dof. The movement is in three planes. Flexion and extension, abduction and adduction, and medial and lateral rotation are the motions allowed. The shoulder (glenohumeral joint, shown in the left figure below) and hip (coxal) joints are the only ball and socket joints in the body.

In engineering mechanics these are also called **ball and socket** or **spherical** (**S**) **joints**, allowing three-dof of rotational motions about perpendicular axes (see the right figure below). Anatomists say this is multiaxial with rotations about infinite axes, but engineers say there are only three independent axes, so this category should be called **Triaxial** instead of **Multiaxial**.



Ball and Socket Joint of the Shoulder
Hamill and Knutzen (1995)



Engineering Spherical Joint
www.mathworks.com

The neck, wrist and ankle joints also demonstrate spherical joint motion. Why are these not considered to be ball and socket joints?

"The shoulder joint is largely a myth. It's less a joint, more of a bony fit held together with musculotendon tension and a modest bit of cartilage. It is extremely mobile. And highly vulnerable."

– Gregg Hurwitz, Orphan X

Main Joints of the Human Body

This section has discussed all of the human body joint types in detail, under the heading of skeletal physiology. Examples were given for each joint type. For a more complete listing of the major joints in the adult human body, please see Appendix A.

Human Joint Motion Ranges

The movement of synovial joints is limited by ligaments, muscles, tendons, and/or adjoining bones and tissue. This limitation in motion affects the kinematic ranges and dynamics characteristics achievable in human motion. Human synovial joint ranges are dependent on age, gender, body type, disease, level of activity, and past injury, among other factors.

Appendix B presents a table of human synovial joint motion ranges limits, based on averaged data from healthy adults. You can check your own mobility vs. this data.

For additional bone physiology topics, please see the online ME 4670 / 5670 Supplement.

3. Human Muscular Anatomy and Physiology

OpenSIM Musculoskeletal Model for Recumbent Cyclist

Uchida and Delp (2021)

Muscles contract to exert tension; like cables they cannot push. Therefore, muscles must occur in antagonistic pairs to move a joint both ways, e.g. biceps (flexion) and triceps (extension) to move the forearm about the elbow joint. As one muscle contracts and shortens (the agonist), developing tension, the partner muscle must relax and lengthen (the antagonist), while still maintaining tension.

In this class we will model muscles as perfect massless, 1D cables (the significant muscle mass can be lumped to neighboring skeletal structure for more fidelity). In the real world there are three assumptions for this to hold good.

1. The muscle path is independent of muscle force;

2. The muscle path is defined by a series of line segments; and

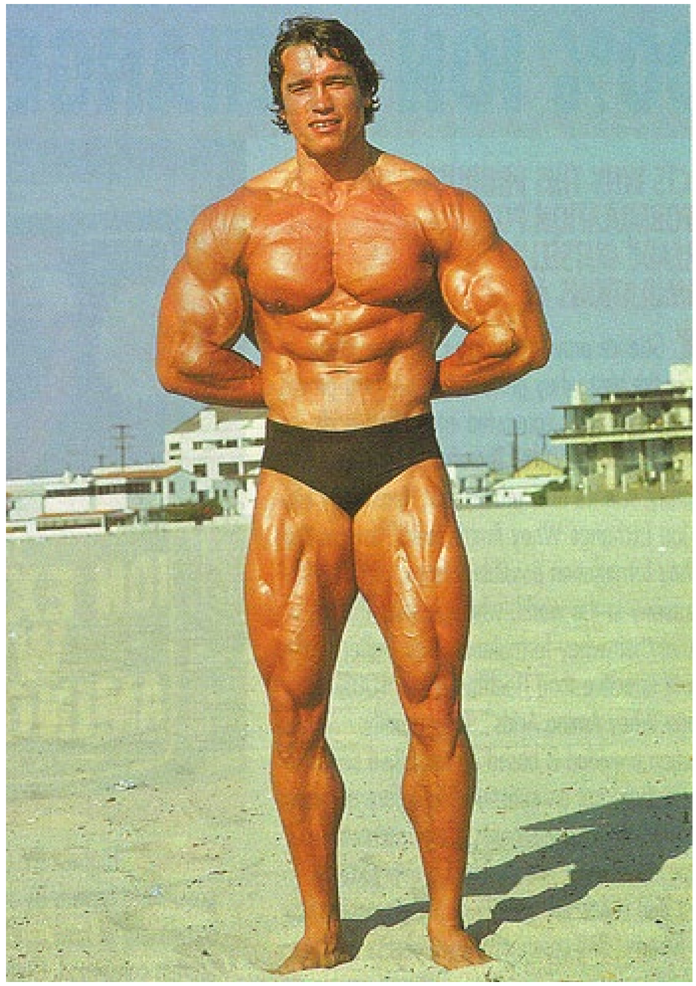
3. The muscle / tendon pair slides without friction over other muscle / tendon pairs.

In reality all of these assumptions may not hold well, 1. for muscles that bulge as they contract; 2. For large muscles with complicated geometry and broad attachments; and 3. For muscles attached to neighboring muscles via connective fascia.

However, the resulting simplified musculoskeletal kinematics, statics, and dynamics models will capture the essence of human motion adequately. Experimental validation should be done for all models.

3.1 Human Muscular Anatomy

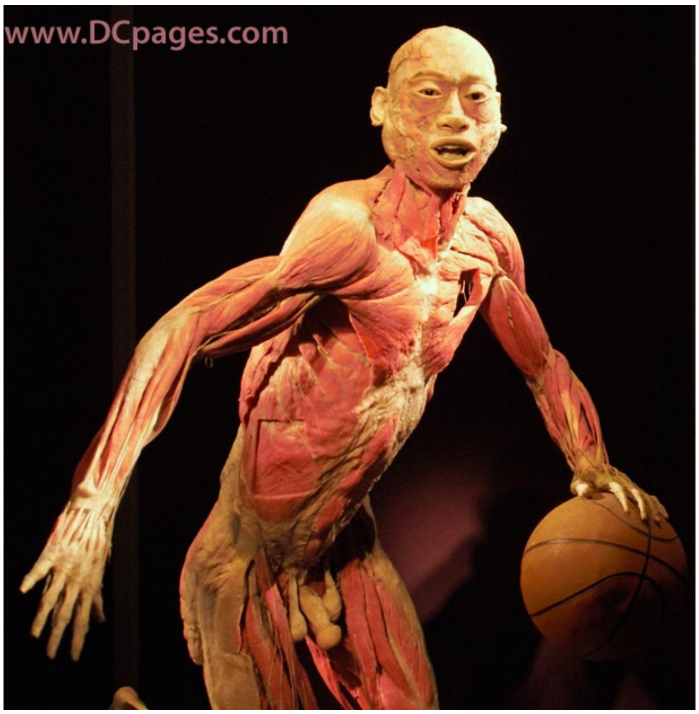
Michelangelo's David (ca. 1504)
sculpturegallery.com



Arnold Schwarzenegger (ca. 1980)
flickr.com

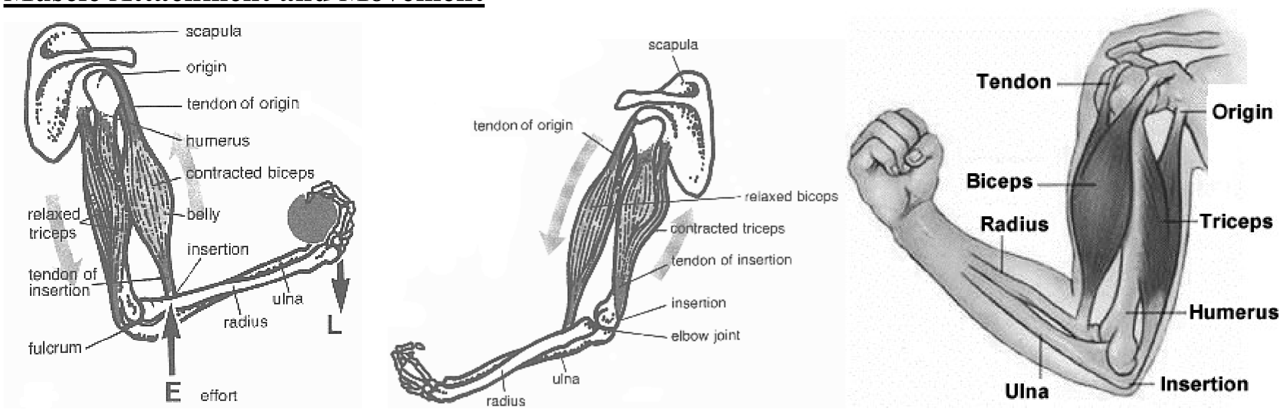
Anatomy is the study of the **structures** of an organism and the **relationships of its parts**. The human muscular system accounts for about half of the body's weight. The muscles give the body most of its visible surface appearance. This section presents a brief overview of human muscular anatomy. A study of neural muscle control is presented later in the section on human muscular physiology.

Human Muscular System Juxtaposed with its Skeleton



Bodies: The Exhibition

Muscle Attachment and Movement



Biceps / Triceps Flexion (left) Extension (center), and Details (right)

Muscle attachment

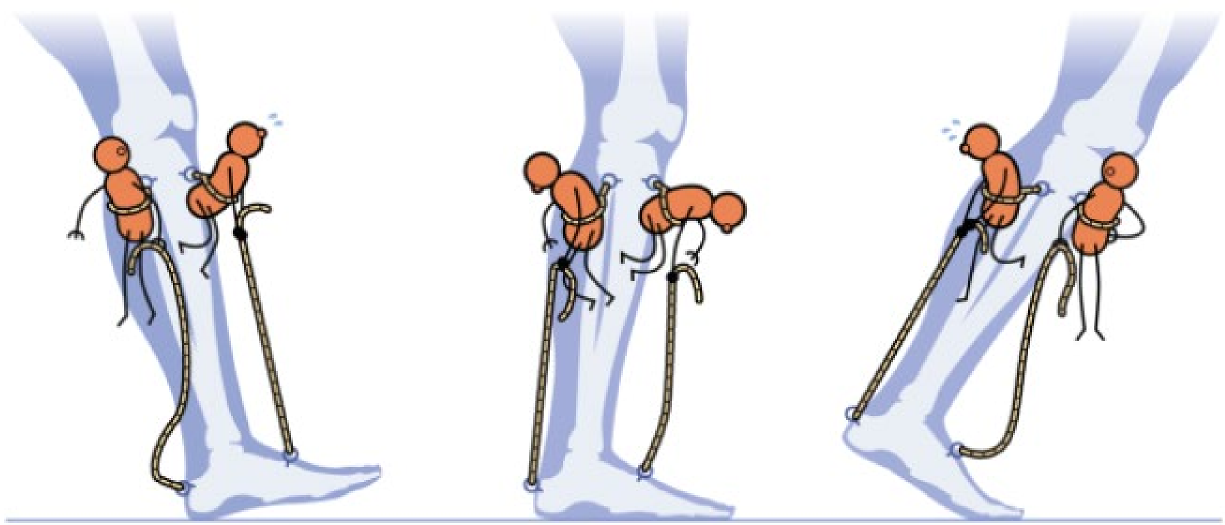
- **origin** point of muscle attachment on the stationary bone
- **insertion** point of muscle attachment on the moving bone

Muscle movement

Most skeletal muscles work in pairs or groups.

- **agonists** muscles primarily responsible for an action due to their contraction.
- **antagonists** muscles that relax while providing tension to smooth the action of the agonists.

iadeaf.k12.ia.us



Agonist and Antagonist in Human Walking

Uchida and Delp (2021)

For an interactive website featuring many muscles including origin, insertion, action, and nerve details, see meddean.luc.edu/lumen.

Five Functions of the Human Muscular System

Muscle cells are the only cells in the body that can contract and develop tension to perform the following functions.

- **Movement** of body parts
- **Change diameter of tubes** in the body
- **Propulsion of materials** through the body
- **Excretion of substances** through the body
- Produce significant **heat** for maintaining body temperature

Spence (1982)

Five Major Properties of the Human Muscular System

Skeletal muscles are:

- **Excitable** capable of receiving and responding to stimulation from nerves
- **Contractible** after receiving stimulation, capable of contracting (shortening)
- **Extensible** capable of stretching by the application of force without damage
- **Elastic** able to return to resting shape and length after contraction or extension
- **Adaptable** able to be changed in response to how it is used

Classification of Muscles

Muscles are classified by location, microscopic appearance, and type of control.

- **Skeletal muscles** attach to bones
- **Visceral muscles** are associated with internal body structures
- **Cardiac muscles** form the wall of the heart

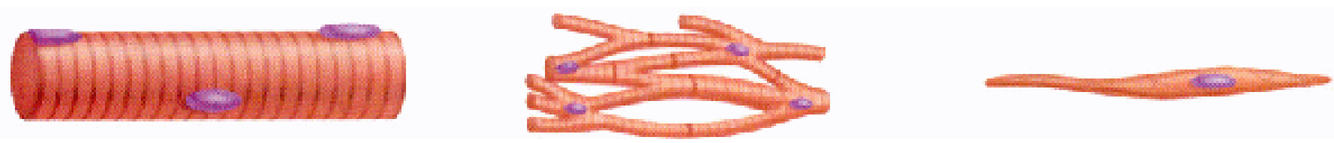
Skeletal (multinucleated) and *cardiac* (uninucleated) muscles are **striated** (striped) when viewed under the microscope.

Visceral muscles (uninucleated) are **smooth** (non-striated).

Voluntary muscles are contracted by the will of the human. **Involuntary muscles** work autonomously.

Muscle Classification Summary

1. Striated multinucleated skeletal voluntary muscles
2. Striated uninucleated cardiac involuntary muscles
3. Smooth uninucleated visceral involuntary muscles



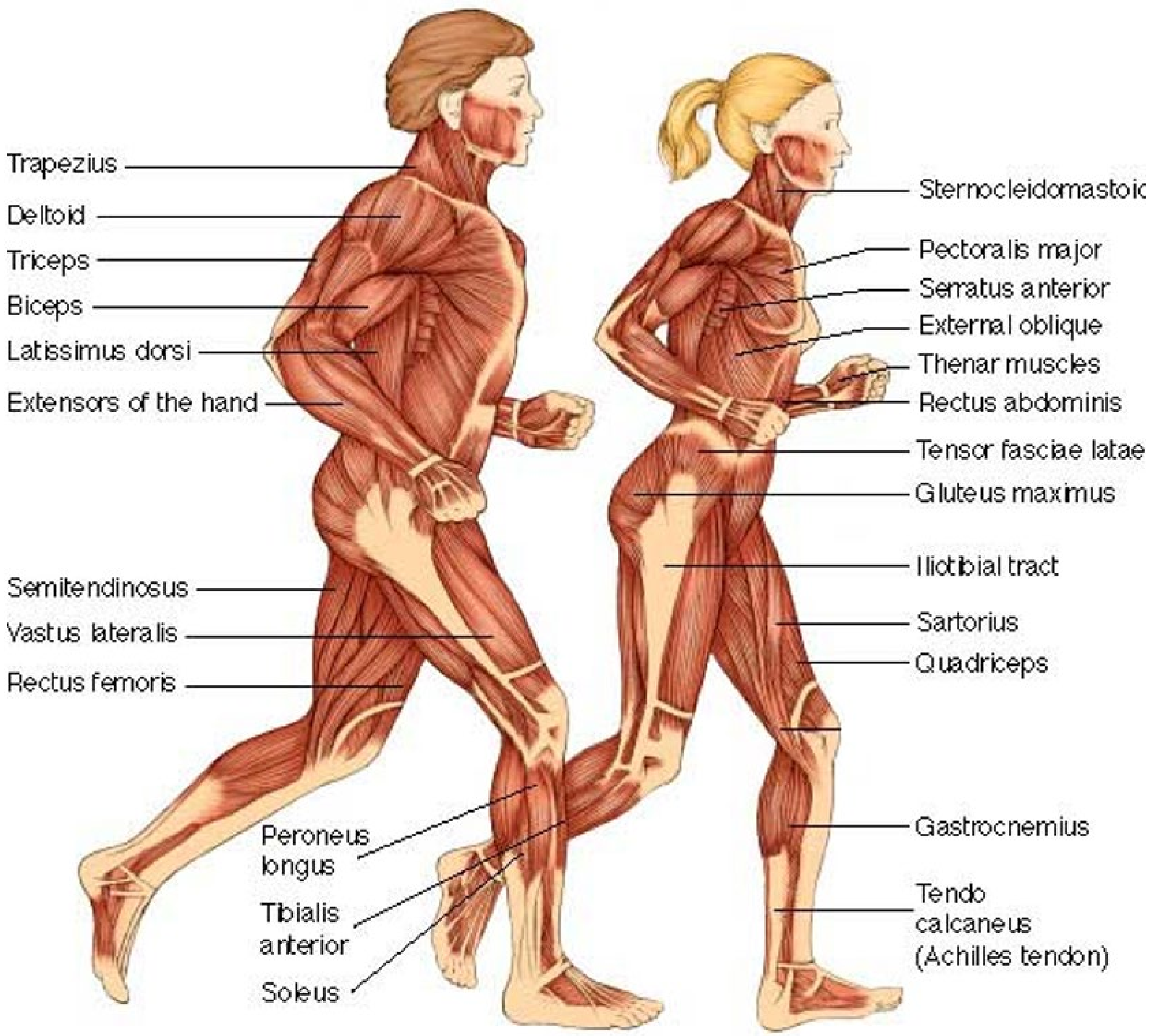
Skeletal Cardiac Smooth

There are more than 600 skeletal muscles in the adult human body (650-850, depending on opinions as to what constitutes an individual muscle), almost all occurring in bilaterally symmetric pairs. So there are a minimum of 325 muscles to memorize!

The involuntary smooth visceral muscles are located in arteries and veins, the bladder, uterus, male and female reproductive tracts, the gastrointestinal and respiratory tracts, the ciliary muscle (middle layer of the eye), the kidneys, and the irises. They are difficult to enumerate since, due to their smooth nature, one muscle blends into the next.

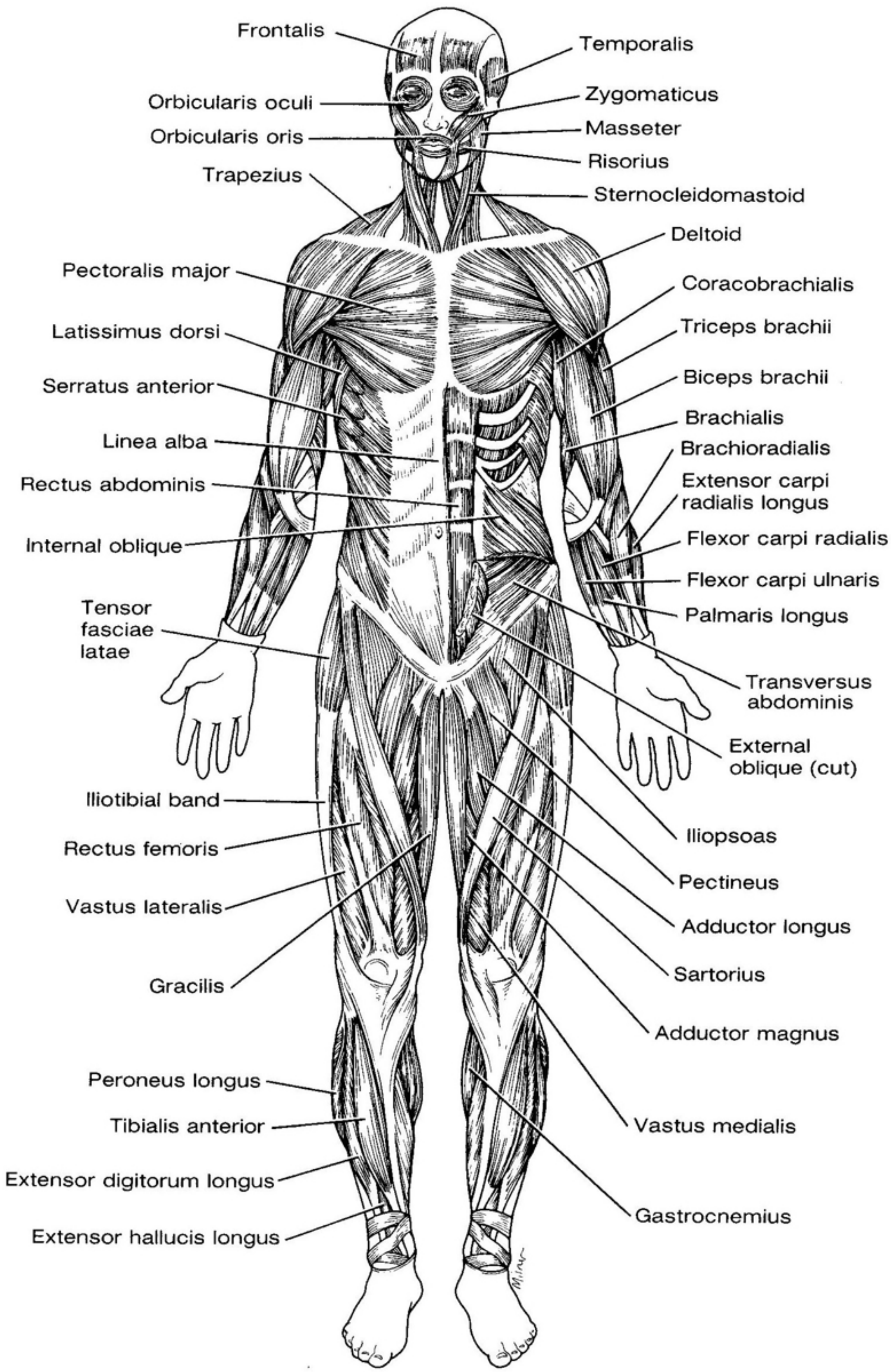
Babies are born with all the muscle fibers they will have as an adult – muscle fibers can grow larger or smaller with working out or disuse, respectively. We only use about one-third of our total muscle power, even in strenuous situations. Survival emergencies yield much higher muscle use.

Human and animal skeletal muscles are **multinucleated** (having multiple nuclei for one cell), **postmitotic** (they do not do cell division nor mitosis after fetal development is complete), and **syncytial** (describing cells which have been formed by the fusion of progenitor cells into a single cell). Other **syncytial** cells end up **uninucleated** after cell combination: a zygote (in which sperm and egg nuclei fuse into one); and cardiac muscle cells, which are **uninucleated**, created by the fusion of multiple cells as **multinucleated** skeletal muscle is.



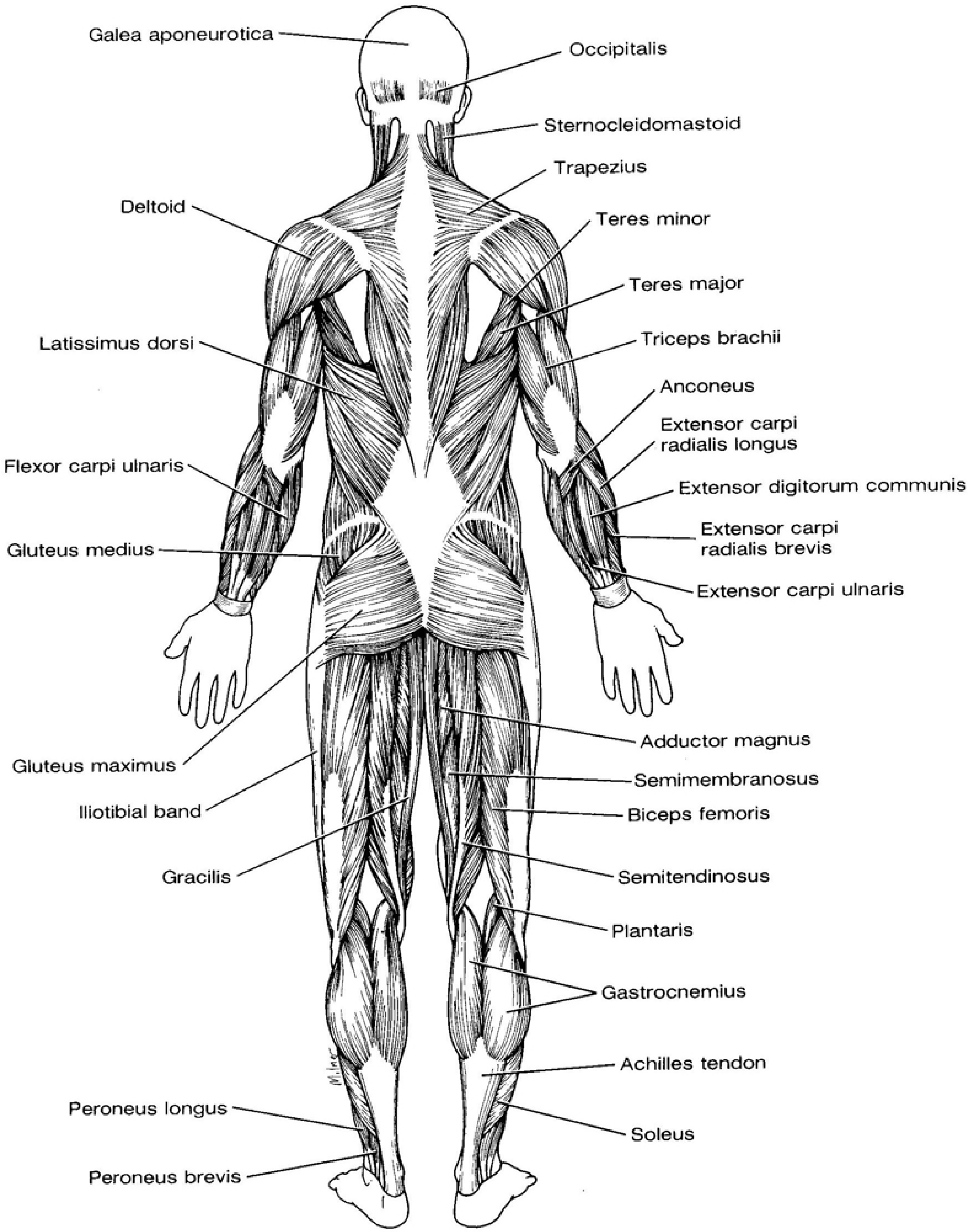
Human Musculature, Dextral Side View

professionalhealthcaretrainer.com



Major Skeletal Muscles (anterior)

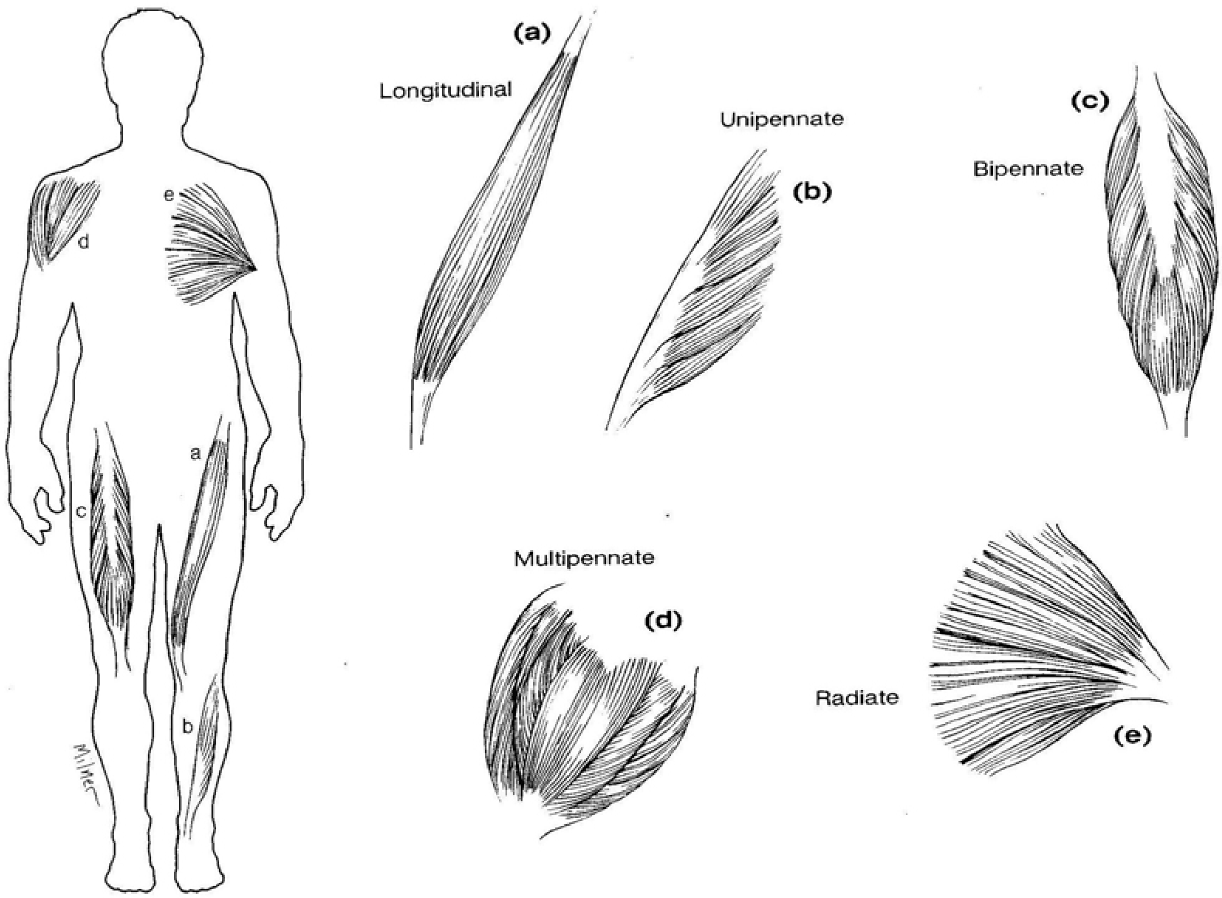
Spence (1982)



Major Skeletal Muscles (posterior)

Spence (1982)

Muscle Shapes



Spence (1982)

Examples

a.	Longitudinal	sartorius
b.	Unipennate	tibialis anterior
c.	Bipennate	rectus femoris
d.	Multipennate	deltoid
e.	Radiate	pectoralis major

The **angle of pennation** is the angle from the axis of the muscle to the direction of the muscle fibers.

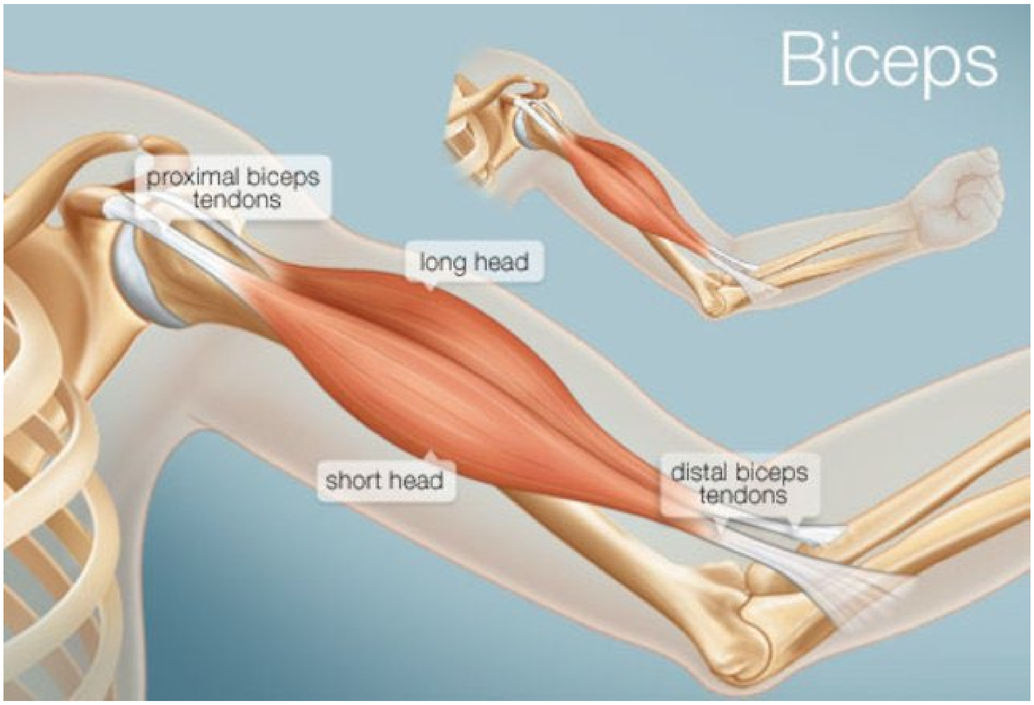
Extreme Human Muscles

- The busiest muscle is the levator palpebrae superioris, causing blinking
- The largest muscle is the gluteus maximus (buttock)
- The smallest muscle is the stapedius deep in the inner ear
- The strongest muscle relative to size and weight is the masseter, for mastication.
- The longest muscle is the sartorius of the thigh (the ‘tailor muscle’)

Skeletal Muscles are named according to the following categories

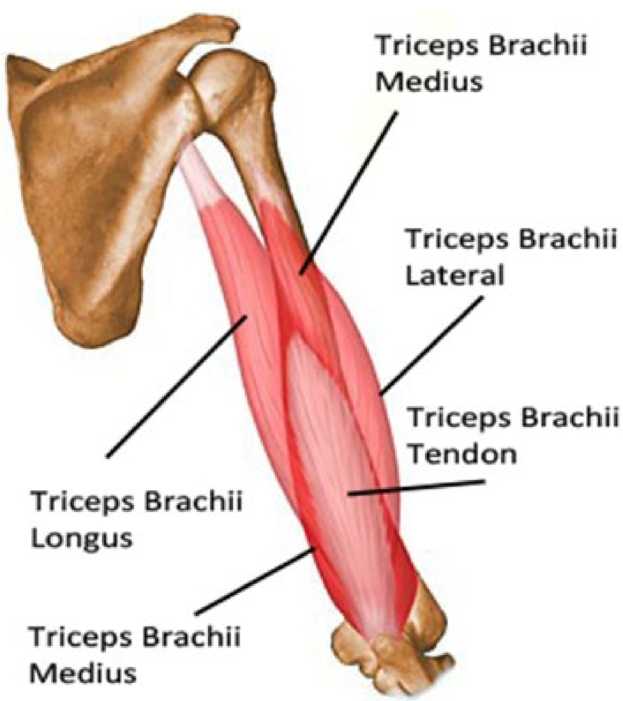
- direction of fibers (e.g. oblique)
- location or position (e.g. superficial)
- number of divisions (e.g. triceps)
- shape (e.g. deltoid)
- origin and/or insertion (e.g. iliocostalis)
- action (e.g. levator scapulae)
- size (e.g. major)
- combinations

Why are the Biceps and Triceps muscles so named (with regard to prefix)?



img.webmd.com/dtmcms/live/webmd/consumer_assets/site_images/articles/image_article_collections/anatomy_pages/493x335_bicep

The Biceps Brachii Muscle is composed of two divisions, the Long Head and Short Head.



howtorelief.com/wp-content/uploads/2017/05/triceps-muscle-275x300

The Triceps Brachii Muscle is composed of three divisions, the Longus (long head), the Lateral (short head), and the Medius (medial head).

Selected Muscles / Motions (see Appendix C for a more complete list)

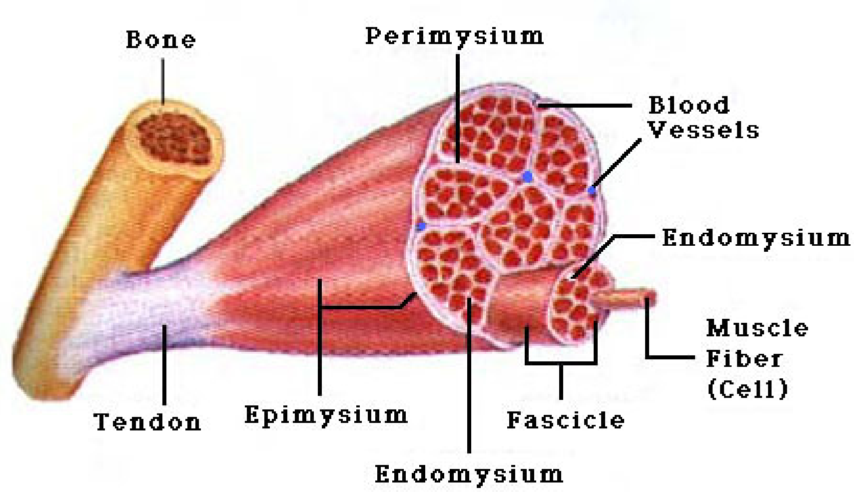
Arms	Legs	Trunk
Deltoid abducts the arm **Biceps brachii** flexes the forearm and supinates the forearm from neutral **Triceps brachii** extends the forearm	**Sartorius** flexes hip and knee **Rectus femoris** extends knee **Gluteus maximus** extends the hip and rotates the thigh **Biceps femoris** flexes the knee **Gastrocnemius** flexes the ankle and stabilizes the ankle and knee for standing	**Pectoralis major** adducts the humerus **Rectus abdominus** produces trunk motions **Trapezius** elevates and rotates the scapula **Latissimus dorsi** rotates the humerus

Skeletal Muscle Anatomy *Spence (1982)*

- **Skeletal muscle** moves the skeleton and is responsible for voluntary movements. It also provides involuntary movements such as standing, holding up the head, and breathing. Skeletal muscles are the body's motors, brakes, and shock absorbers. Muscles are heaters when the body shivers. Muscles also store protein reserves.

- As seen in the next two figures, skeletal muscle anatomy consist of five levels, going down in detail to smaller scales. **a) muscle → b) fiber bundles (fascicles) → c) fibers (muscle cells) → d) myofibrils → e) filaments**. Muscle cells are the longest cells in the body, generally running the entire muscle length.

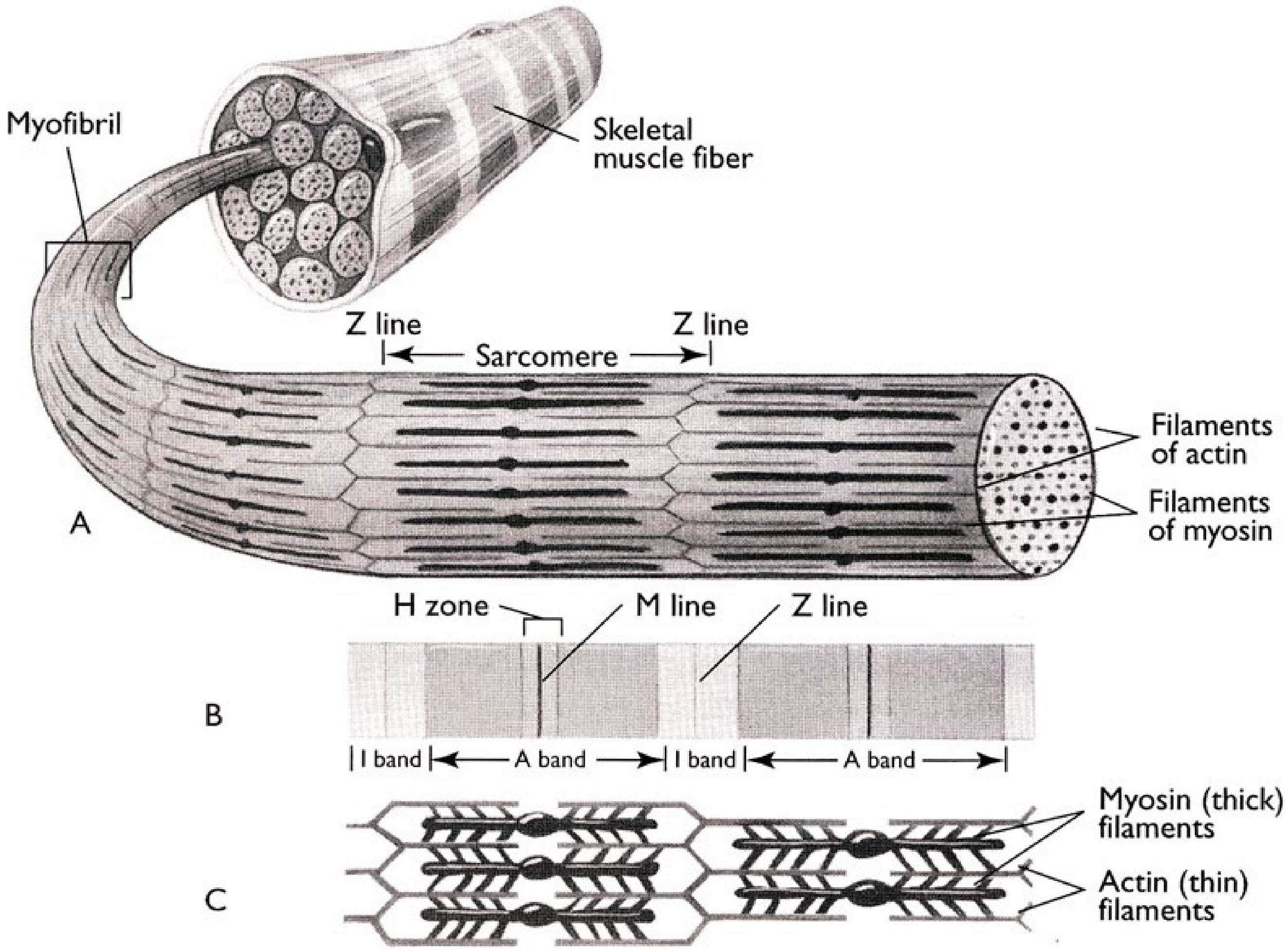
- **Skeletal muscle** is held together by thin collagenous connective tissue called **fascia**. The **epimysium** is the *fascia* that envelops an entire muscle. Bundles of muscle fibers are called **fascicles** (Latin for bundle), surrounded by **perimysium** *fascia*. Muscle is composed of many individual muscle cells called **muscle fibers**. The **endomysium**, very thin *fascia*, envelop the cell membrane of each individual muscle fiber. Blood vessels and nerves enter the muscle with the *fascial* sheaths.



Skeletal Muscle, Fascicle, Fiber *Spence (1982)*

- Muscles are made up of large numbers of large multi-nucleated cells (**muscle fibers**). There are about 100,000 muscle fibers in the biceps brachii. The fibers are as thick as a hair (50 µm) and 10-100 mm long, many as long as the overall muscle. Muscle fascicles are generally angled from the axis of the muscle by the angle of pennation. Individual muscle fiber cells in the human sartorius muscles (thigh) are up to 0.6 m in length.

- **Muscle cells** (**fibers**) have a regular honeycombed subcellular structure, composed of threadlike proteins called **myofibrils**, running the whole length of the muscle fiber. *Myofibrils* are cross-striated with light (**isotropic, I bands**) and dark (**anisotropic, A bands**) bands. Crossing the center of each *I band* is a dense **Z line**, which divides the *myofibrils* into repeating units called **sarcomeres**, of 1–2 µm diameter. In the center of a *sarcomere* (and thus in the center of the *A band*) is a less-dense region called the **M line**. **Sarcomeres** are the lowest level of contraction.



Skeletal Muscle Fiber, Myofibril *Hall (2007)*

- A *sarcomere* consists of **thick filaments** (composed of the protein **myosin**) in the *A band* and **thin filaments** (composed of the proteins **actin**, **tropomyosin**, **troponin**) in the *I band*. *Thin filaments* also exist in the *A band* up to the *M line*. The **H zone** is a lighter region on either side of the *M line* that contains thick filaments.

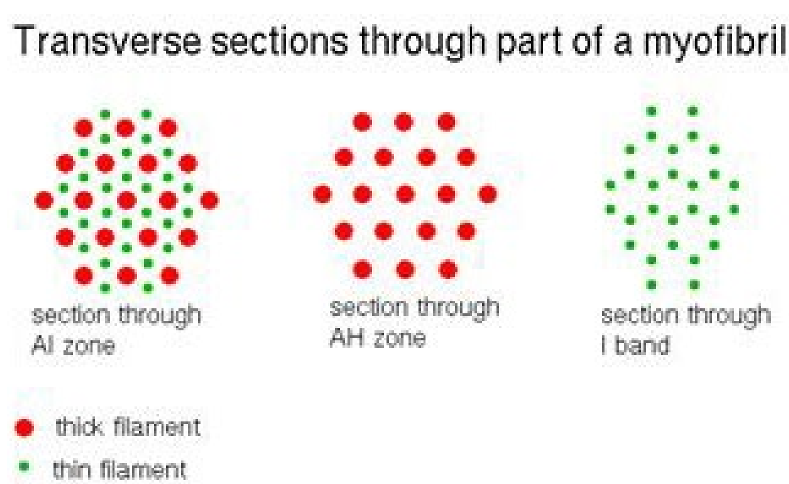
- The contraction takes place by an interaction of the *actin* with projections on the *myosin* molecules (**crossbridges**), similar to Velcro. Each *crossbridge* can develop 5×10^{-12} N (5 picoN) of force and can pull the thin filament past the thick filament by 1×10^{-9} m (one billionth of a meter, i.e. a nm). The net effect of many of these small movements and small forces is to shorten the *myofibrils*, and thus the whole muscle shortens and generates force. A skeleton part is moved, by means of the attachment of the muscle at each end to bone, directly or via tendons.

- Human voluntary, striated, multinucleated skeletal muscles have sarcomeres that are approximately 2 μm in length. This length does not vary very much. It is also amazingly unvarying (2 μm) for all of the vertebrate animals (from mice through cows, elephants, and blue whales). However, the arthropods see up to 7x variation in sarcomere length for different species.

Sarcomere Name Origins *Hall (2007)*

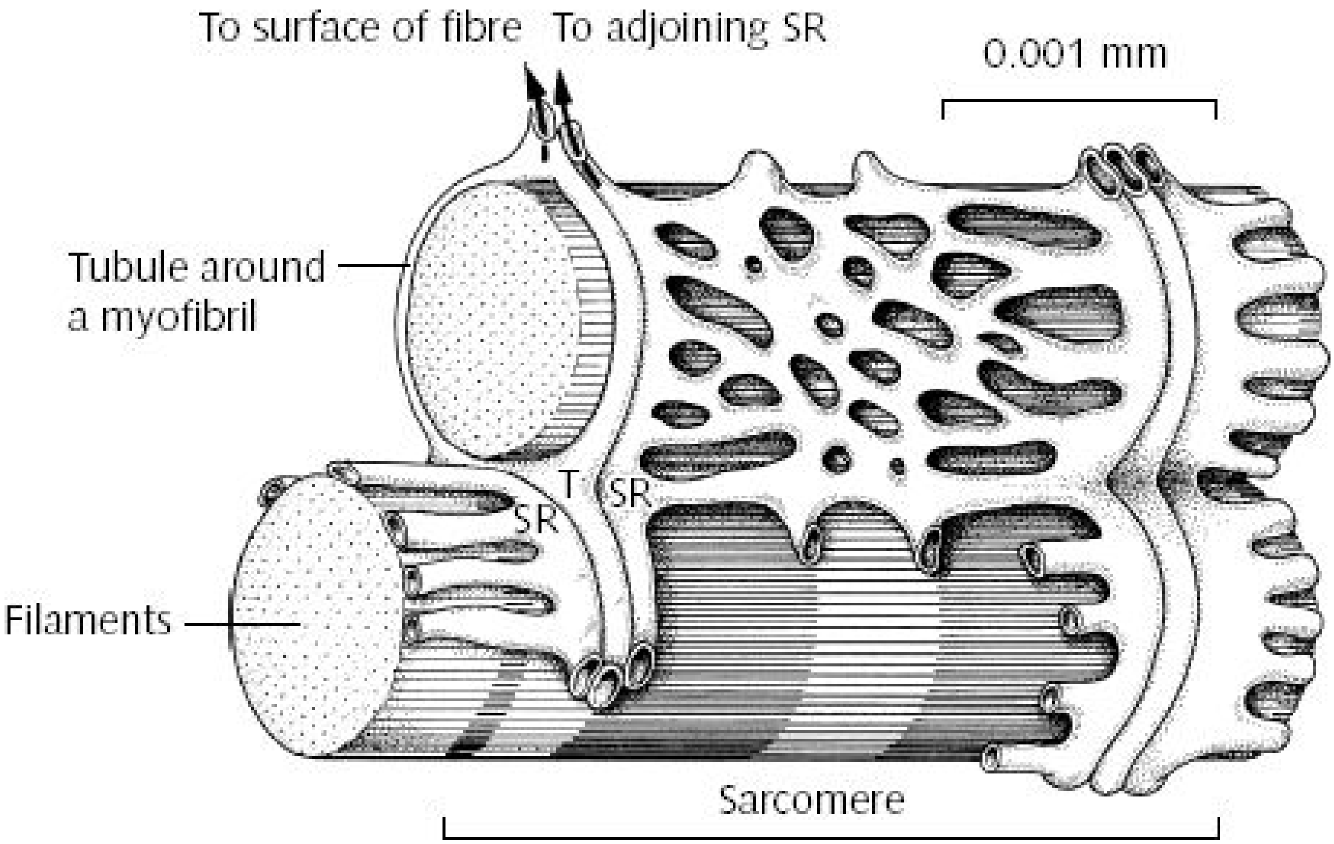
Component	Origin
A band	Polarized light is **Anisotropic** as it passes through this region
I band	Polarized light is **Isotropic** as it passes through this region
M line	**Mittelscheibe** (German) middle or intermediate
H zone	Discovered by **Hensen**
Z line	**Zwischenscheibe** (German) the disk in between the **I** bands

Many images and videos illustrating the action of sarcomeres appear flat. In fact, sarcomeres (with thin filaments of actin and thick filaments of myosin) are arranged in a 3D hexagonal structure as shown in the adjoining figure.



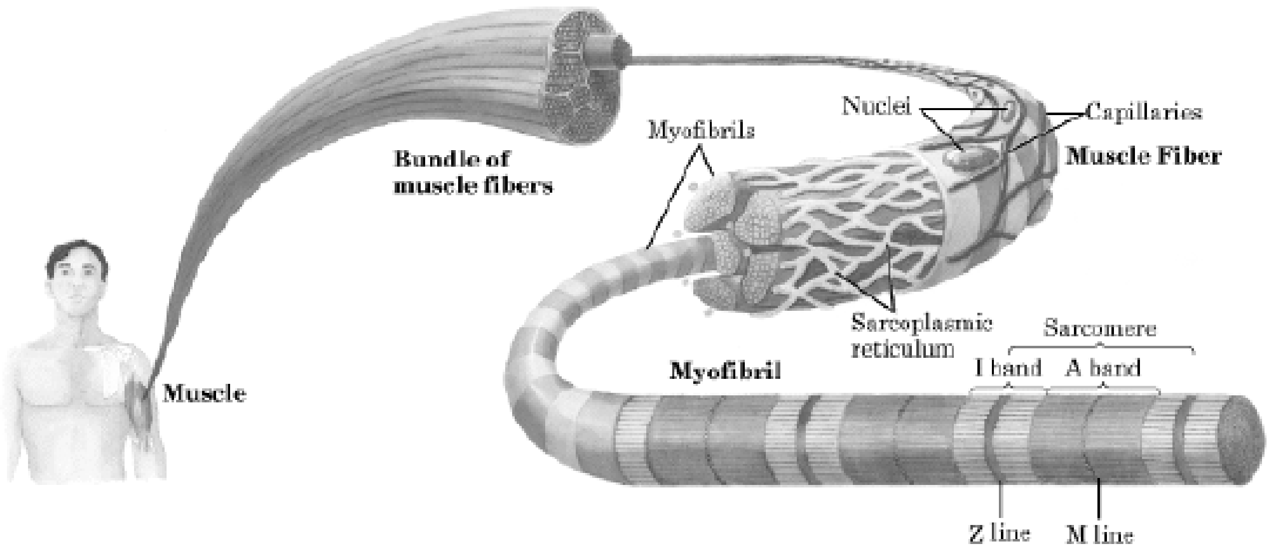
biocadmin.otago.ac.nz

- The **myofibrils** (the muscle's contractile apparatus) have a system for controlling contraction through changes in calcium concentration. This system, a membranous network called the **sarcoplasmic reticulum** (**SR**), is a closed set of tubes containing a high concentration of calcium. Tubules called **transverse tubules** (**T**) run deep into the muscle cell from the muscle cell membrane to deliver and collect calcium.



Transverse Tubules (T) and Sarcoplasmic Reticulum (SR)

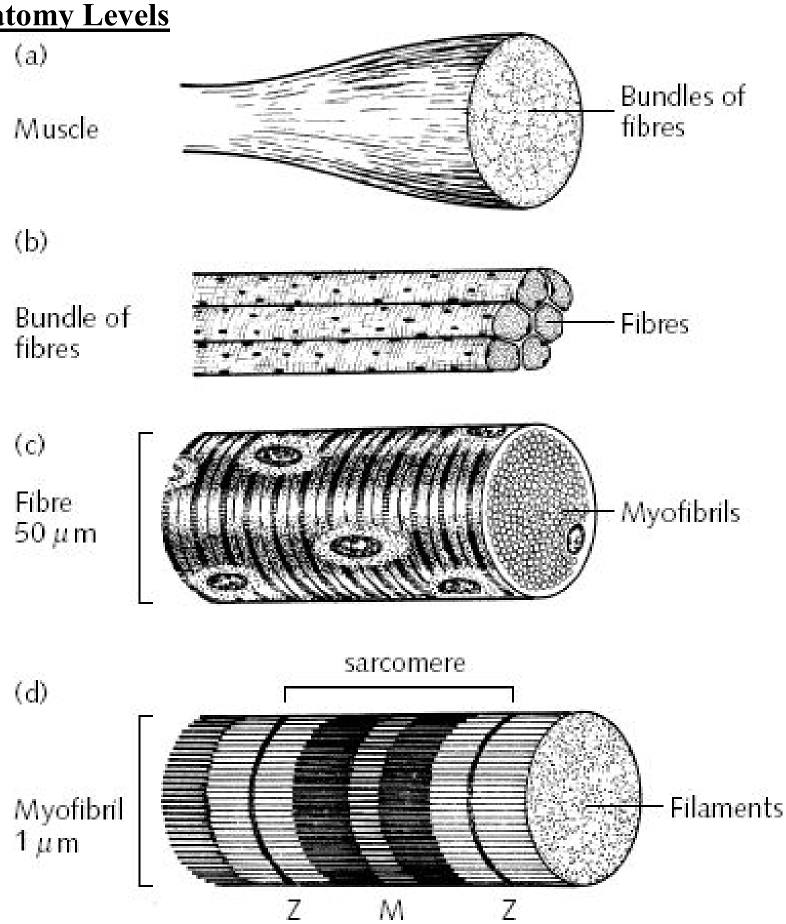
Spence (1982)



Striated Muscle Structure from Macro- to Micro-Levels

Hamilton et al. (2008)

<u>Skeletal Muscle Anatomy Levels</u>



Skeletal Muscle from the Gross to the Microscopic Level *Spence (1982)*

This figure shows skeletal muscle anatomy at five levels, going down in detail to smaller scales

a) muscle → b) fiber bundles (fascicles) → c) fibers (cells) → d) myofibrils → e) filaments

Muscle Pathologies

- **muscle atrophy** – shrinkage and death of muscle cells caused by reduction in blood flow, interference with the nerves, or disuse. Atrophy can also occur in the absence of cell death, but rather with just an overall reduction in protein content within a given cell. This also occurs for a variety of other reasons such as aging and numerous disease states (e.g. AIDS, cancer cachexia).

- **cramps** – painful involuntary muscle contractions that are slow to relax. May occur at rest or during exercise and may be caused by low oxygen or low sodium and chlorine ions, respectively.

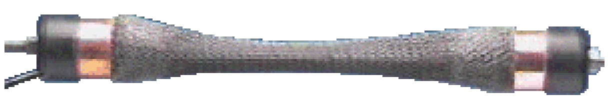
- **muscle dystrophy** – group of hereditary diseases with progressive muscle weakness, caused by degeneration of muscle cells, an increase in connective tissues in muscle, or the replacement of muscle cells by fat.

- **myasthenia gravis** – a rare chronic condition with extreme muscle weakness, caused by an immunological blockade of acetylcholine receptors at the neuromuscular junction.

Spence (1982)

Artificial Muscles

Artificial McKibben muscles were developed in the 1950s and enjoyed a resurgence in the 1990s due to improvements in nonlinear control theory and improvements in computing. These engineered muscles consist of a bicycle inner tube surrounded by a weave mesh. As the tube is pressurized with air, the diameter expands, the length shortens, and the mesh develops a contractile force, not unlike a human muscle. Pneumatic artificial muscles do not approach actual human muscles in terms of functionality, efficiency, number of muscles intertwining and interacting, and strength.



McKibben Muscle Relaxed

McKibben Muscle Inflated

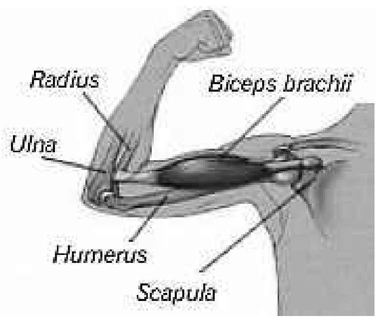
University of Washington Arm with Artificial Muscles

3.2 Human Muscular Physiology

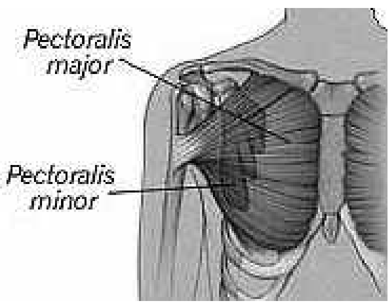
Again, physiology is the study of the **functions** of an organism and its parts. This section focuses on the physiology of voluntary, striated, multinucleated skeletal muscles, presenting some specific muscle motions, followed by the voluntary neural muscle control mechanisms and several important details regarding muscles and human motion.

Muscle Motions

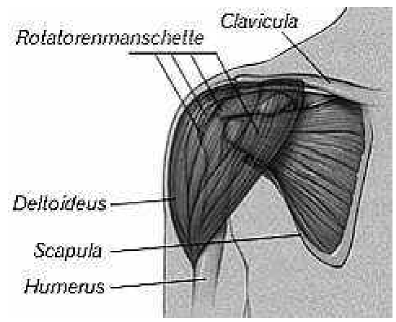
The following 12 images from Bartel (2006) show skeletal muscle motions, listed by which body part is moved and the view.



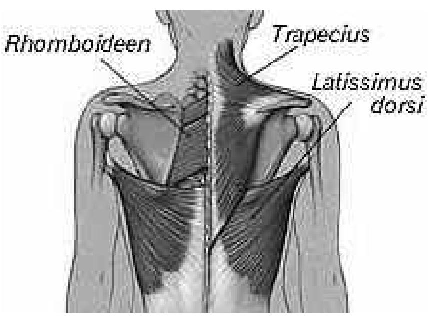
Forearm, Anterior



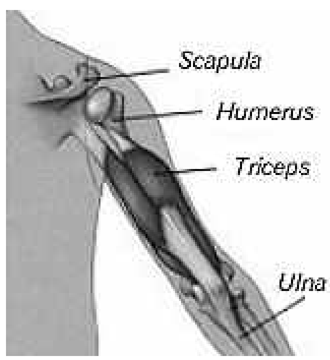
Upper Arm, Anterior



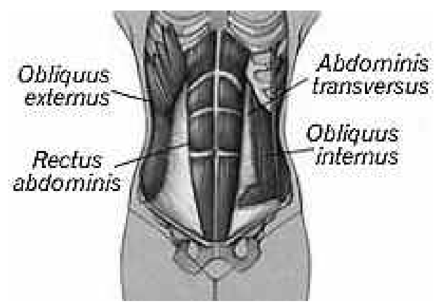
Upper Arm, Posterior



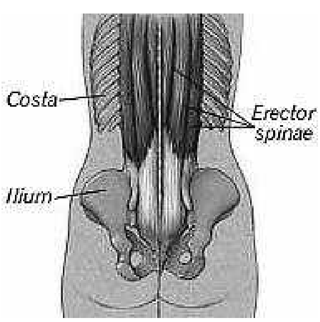
Neck, Upper Arm, Posterior



Upper Arm, Posterior



Trunk, Anterior



Spine, Posterior

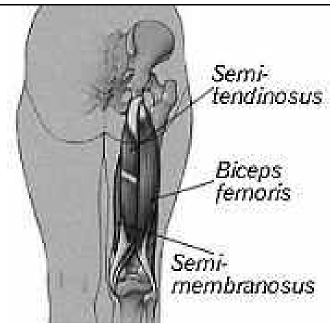
Ankle, Foot, Posterior

The hamstrings consist of four muscles:

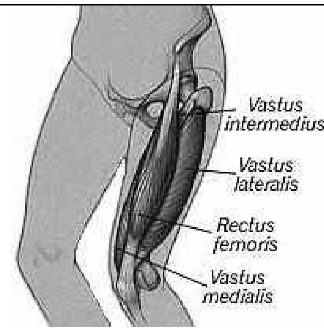
- biceps femoris, long head
- biceps femoris, short head
- semitendinosus
- semimembranosus

The quadriceps femoris consist of four muscles:

- rectus femoris
- vastus lateralis
- vastus medialis
- vastus intermedius

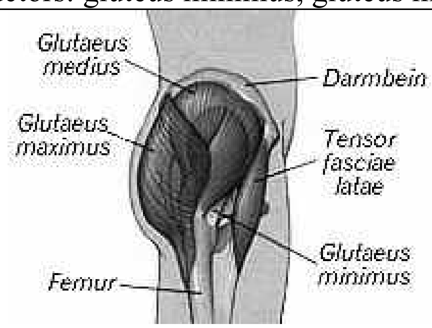


Upper Leg, Posterior



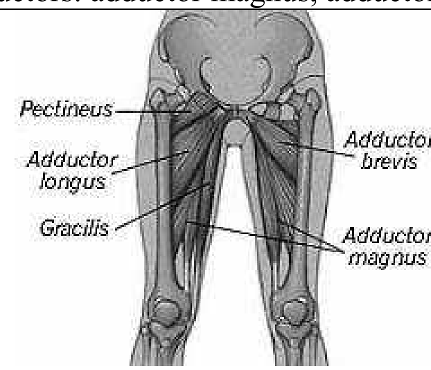
Upper Leg, Oblique

Leg abductors: gluteus minimus, gluteus medius



Upper Leg, Side

Leg adductors: adductor magnus, adductor longus

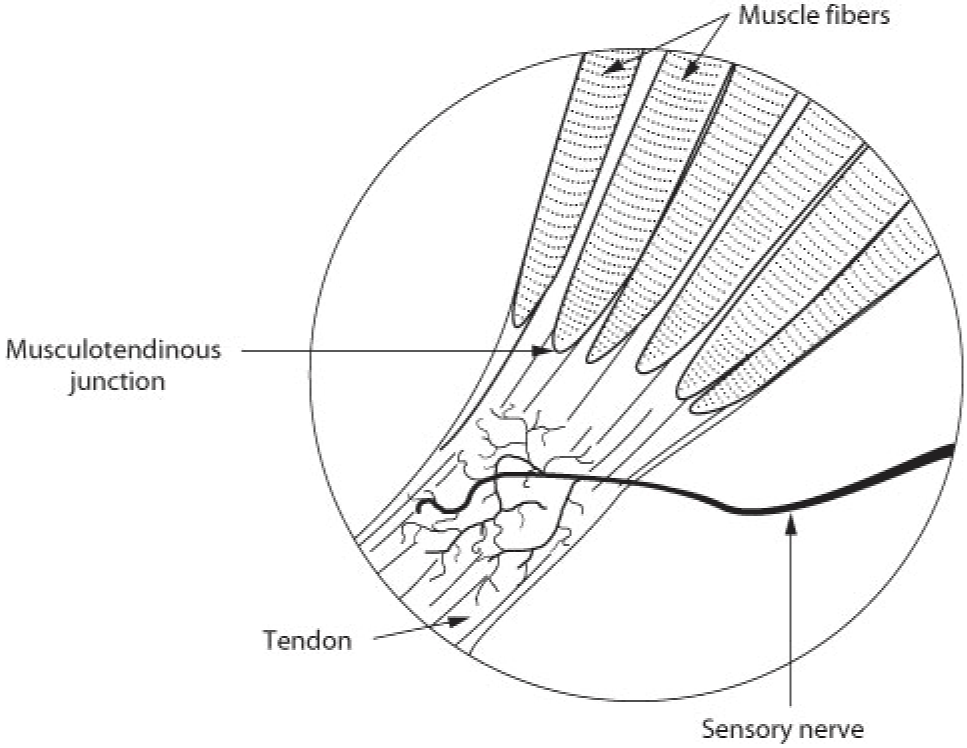


Upper Leg, Anterior

<u>Major Muscles Groups to move Major Joints</u> For more complete lists see Appendix C.

Joint	Direction	Muscles
knee	flexion	hamstrings, gastrocnemius
	extension	quadriceps femoris
hip	flexion	iliacus, psoas major
	extension	hamstrings, gluteus maximus
	abduction	abductors
	adduction	adductors
lumbar spine	flexion	rectus abdominus, obliques
	extension	erector spinae
elbow	flexion	brachialis, biceps brachii
	extension	triceps brachii

Bartel et al. (2006)

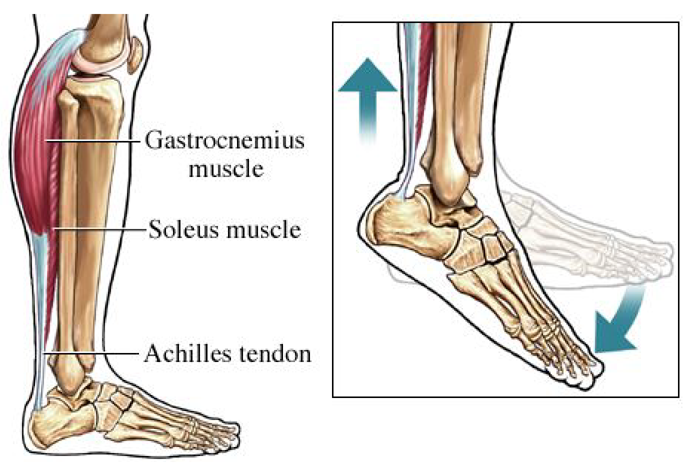


Most Skeletal Muscles connect to Bone via Tendons

Hamilton et al. (2008)

Four Functions of Tendons (Dr. Chleboun, presentation)

1. Transfer of force from muscle to bone.



2. Storage of elastic strain energy for walking, running, jumping, and other motions.

3. Adjusting muscle tension in a feedback loop with the Golgi tendon organ as the sensor.

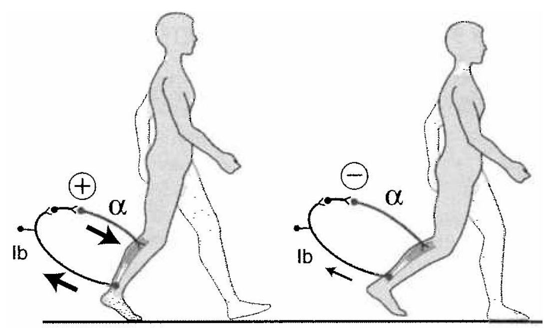
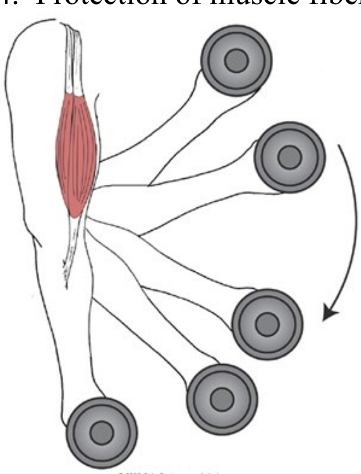
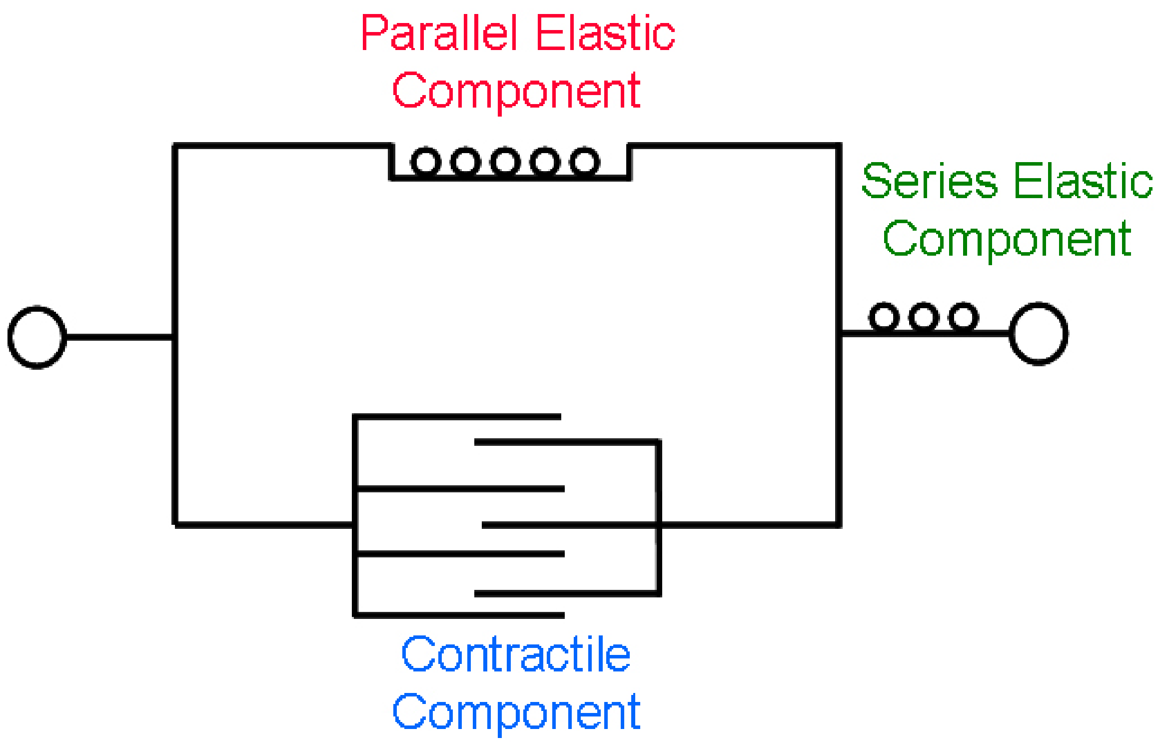


FIGURE 9-14
Golgi tendon organ. Stretch of a tendon activates type Ib afferents that synapse with interneurons. Depending upon the task, GTO input facilitates or inhibits LMN firing. For example, during stance phase of gait, GTO input facilitates LMNs to lower limb extensor muscles. During swing phase, GTO input inhibits LMNs to the same muscles.

4. Protection of muscle fibers from damage.



Muscle Mathematical Model



Mechanical Muscle Model

Hall (2007)

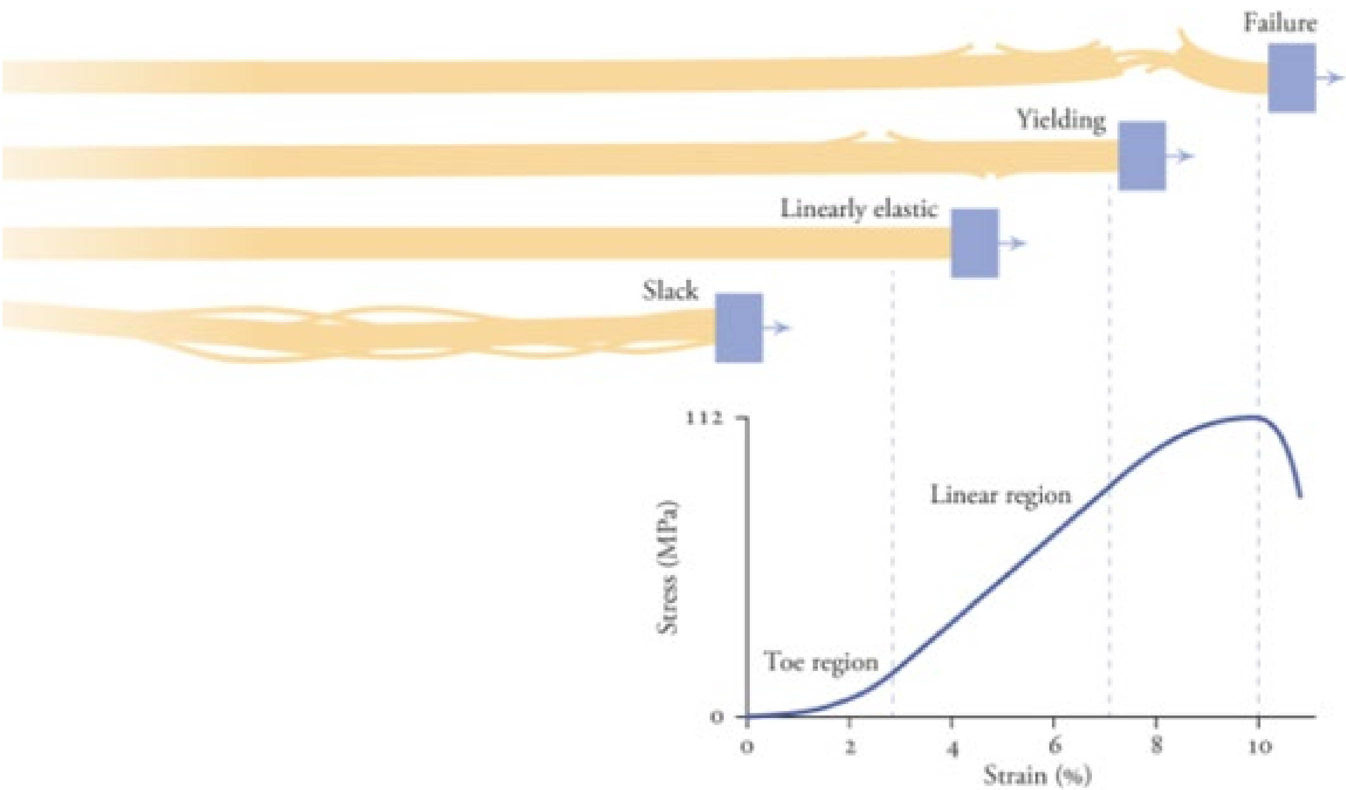
The elastic muscle behavior can be modeled with the figure above. The **parallel elastic component**, provided by the muscle membranes (the epimysium, perimysium, and endomysium fascia), gives spring resistance when passively stretched. The **series elastic component** represents the tendons, storing elastic energy when a muscle is stretched, and releasing some of this potential energy for subsequent motions. The tension-generating **contractile** effect of the sarcomeres is in parallel with the fascia membranes and in series with the tendons.

A detailed vibrational/dynamic mathematical model for human skeletal muscle is presented in Section 3.3 of the on-line 4670/5670 Supplement.

Tendon Stress-Strain Relationship

The tendon stress-strain curve shown below has the same meaning as those for engineering materials. Tendons are modeled as nonlinear springs. The tendon slack length l_S is the nominal length at which no force is generated. Stress in a tendon is defined as the force divided by the tendon cross-sectional area. The tendon strain ε (change in length compared to the slack length, expressed in percentage) equation is:

$$\varepsilon = \frac{l - l_S}{l_S} \times 100\%$$



Tendon Stress-Strain Curve

Uchida and Delp (2021)

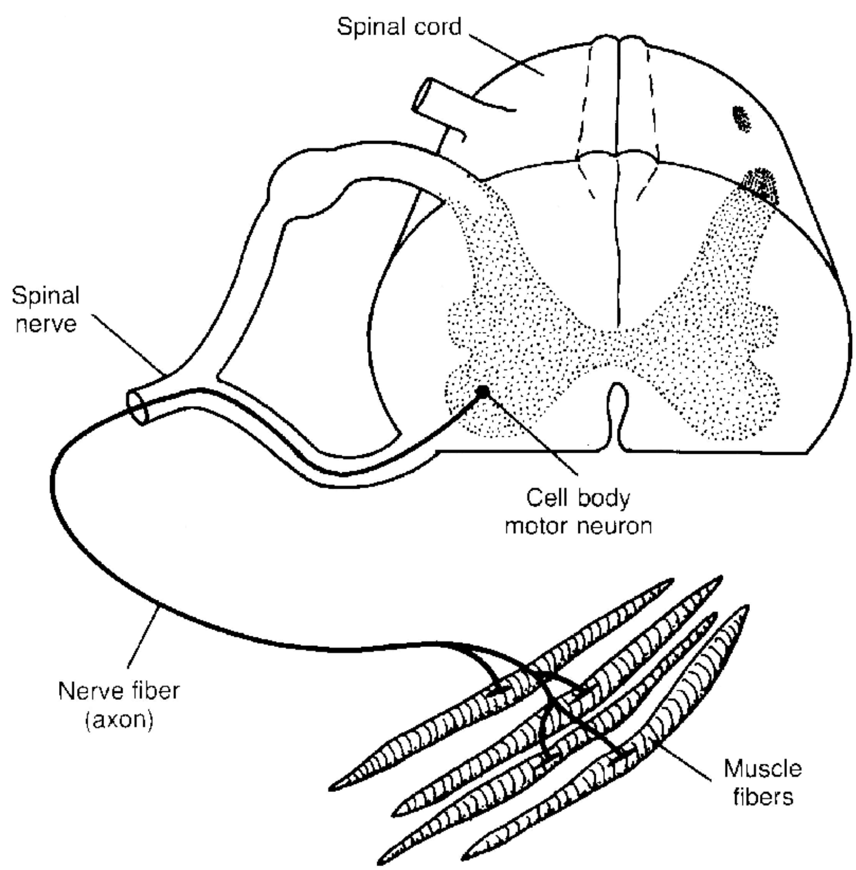
<u>**Neural Skeletal Muscle Control**</u>

- Voluntary skeletal muscle is controlled by the **central nervous system** (**CNS**).

- Each skeletal muscle is innervated by an α–**motor neuron** (**axon**). The axon branches so each muscle fiber is activated by one branch; several muscle fibers can be activated by one axon.

- A **motor unit** consists of the axon and all muscle fibers innervated by it. The motor unit is the functional contractile unit. All muscle cells within the unit synchronously contract as the motor nerve fires.

- Smaller motor units (i.e. less muscle fibers per axon) can be controlled more precisely than large motor units (i.e. more muscle fibers per axon). For instance the eye movement is much finer than spine posture control. Also, the hand allows much finer motor control than the large muscles of the legs. **Innervation ratio** is the number of muscle fibers per axon.



Motor Unit

Hamilton et al. (2008)

- When a voluntary movement is initiated, events in the brain and spinal cord generate **action potentials** in the **axons**. Each of these axons branches to send action potentials to many muscle fibers.

- Voluntary movement action potentials are generated in the primary motor cortex and the pre-motor cortex of the human brain. The left side of the brain controls the right side of the body and vice versa.

- Action potentials for involuntary motions, such as reflexes and pulling away after accidentally touching a hot stove, are generated in the spinal cord, too fast for the brain to send.

- There are 4000 nerves per hand. There are about 93 touch sensors per square cm in the hand (this is 100 times the number of touch sensors on the back of the leg). The most dense heat sensors are in the fingertips, nose, and, oddly enough, the elbows.

- Nerve signals travel at 400 kph (249 mph, Mach 0.32). Some nerve cells are longer than 1 m.

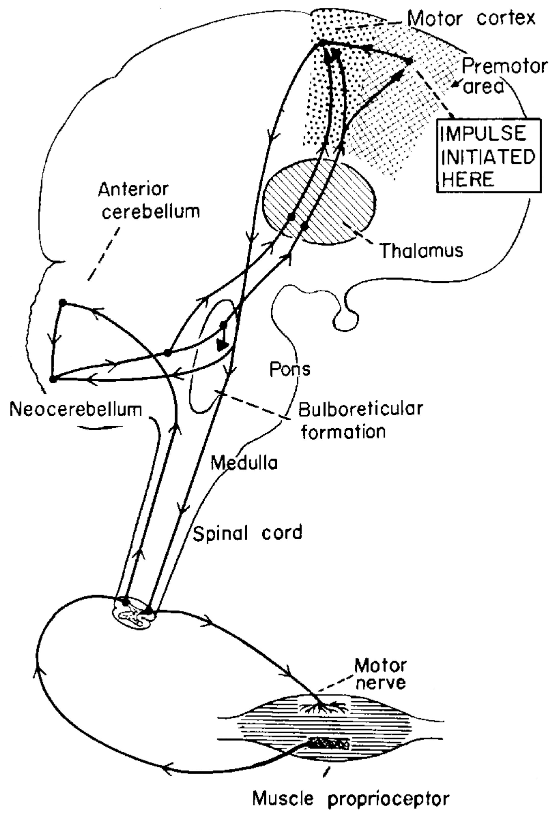
- At the nerve terminals of each axon branch (the **neuromuscular junction**) **acetylcholine** is liberated by the arriving action potential, and this combines with receptors on the membrane of the muscle fiber, causing it to contract due to the action potential.

- This action potential spreads over the whole surface of the fiber and also down an extensive network of fine **T-tubules**. Here a message passes from the T-tubule to the **sarcoplasmic reticulum (SR)**, causing some calcium to leak out into the interior of the muscle fiber.

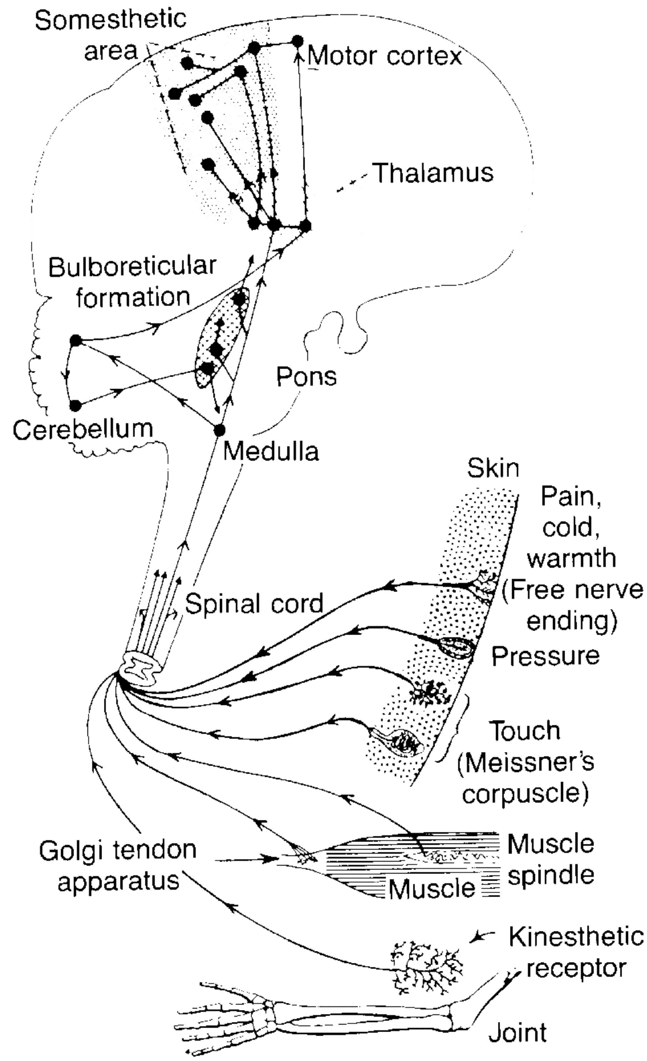
- In addition to **actin**, the thin filaments in the **myofibrils** contain two more proteins, **troponin** and **tropomyosin**. The calcium which leaks from the **SR** is able to interact for a brief period with the troponin molecule of the thin filament. Through movements of the tropomyosin molecules, this alters the thin filament so that the actin molecules are available to be joined by the **crossbridges**, starting the process of **contraction**.

- As soon as calcium escapes from the SR the process begins to collect it back again. There are calcium pumps in the membranes of the SR, which are able to move the calcium back inside, thus bringing to an end the short period of muscle activity (a **muscle twitch**). More sustained periods of activity are the norm and they require a sequence of action potentials to be sent to the muscle at 30 per second. The contractions produced in this way are stronger than a twitch.

The left figure below shows the closed-loop control system for neural muscle control of voluntary skeletal muscles. The right figure below shows some proprioceptors for muscle control feedback.



Closed-Loop Skeletal Muscle Neural Control System



Proprioceptors for Muscle Neural Control

Hamilton et al. (2008)

Proprioceptor – A nerve ending that functions as a sensory receptor in muscles, tendons, joints and the inner ear; they respond to movement and position.

en.wiktionary.org

The right figure above shows the **proprioceptors**, the sensors for neural muscle control feedback. The three classes of proprioceptors are listed below.

Muscle Proprioceptors

- **Muscle spindles** – when the spindle is stretched, a signal is sent to the CNS to cause more contraction (excitatory input). Muscle spindles are responsive to both length and rate of length change.

- **Golgi tendon organs** – as opposed to the muscle spindles, when stretched, these send signals to relax the muscle (inhibitory input) rather than contract.

Joint and Skin Proprioceptors

- **Pacinian corpuscles** – under the skin, concentrated around joints, activated by rapid joint angle changes and by pressure, for brief periods of time. They are not for constant pressure awareness.

- **Ruffini endings** – also activated by mechanical deformation, but can signal constant pressure. Located in deep skin layers and throughout joint capsules.

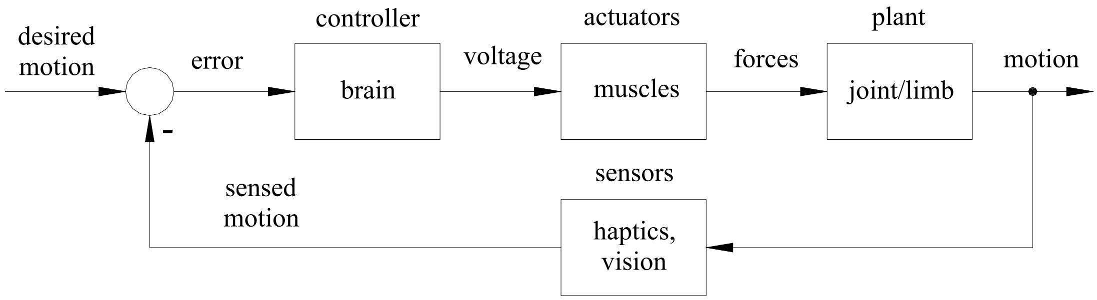
- **Cutaneous receptors** – touch (Meissner's corpuscles), pressure (pacinian corpuscles), and pain/temperature (free nerve endings). These exteroceptors serve as proprioceptors when showing sensitivity to texture, hardness, softness, and shape. They also serve as proprioceptors when they provide the extensor thrust reflex, pain, or flexion withdrawal reflex.

Labyrinthine and Neck Proprioceptors

- **Labyrinths of the inner ear** – provide sense of balance, orientation, and movements of the head.

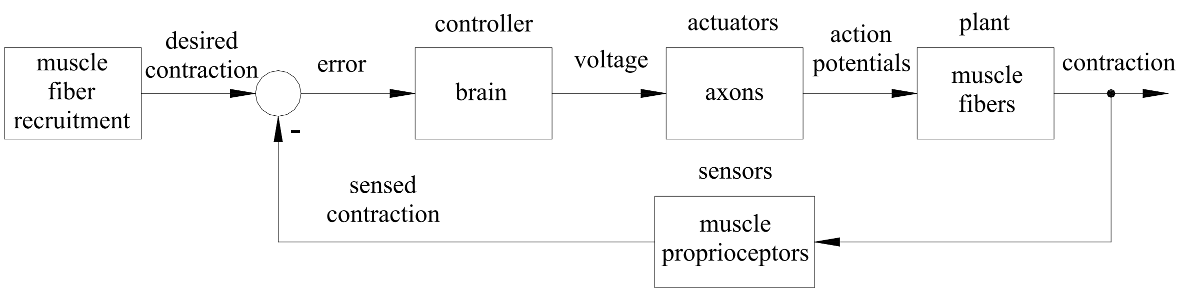
- **Neck Proprioceptors** – provide orientation of the head relative to the body. If the body orientation is changed with respect to gravity, the neck receptors do not counteract the labyrinthine receptors and equilibrium change is sensed.

The following figure shows a high-level controls block diagram for a general human voluntary motion controlled by the CNS.



High-Level Motion Control Block Diagram *Dr. Bob*

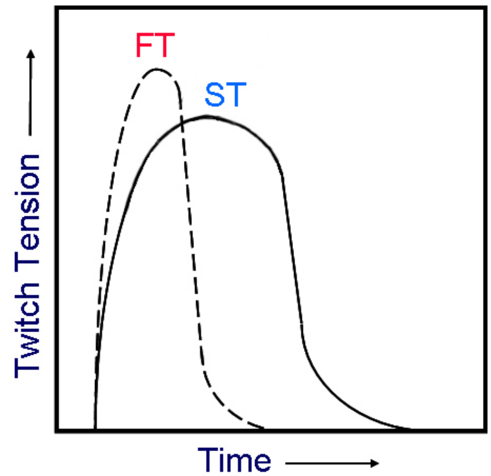
The following figure shows a low-level controls block diagram for control of muscle fibers recruited as part of a general human voluntary motion.



Low-Level Muscle Fiber Control Block Diagram *Dr. Bob*

<u>Skeletal Muscle Fiber Types</u>

The fibers of some motor units reach maximum tension more quickly than others. Muscle fibers are divided into **slow twitch** (**ST**, **Type I**, dark, distance runners) and **fast twitch** (**FT**, **Type II**; IIA light, sprinters and IIB grey, power lifters). As shown in the figure below, fast twitch muscles also relax more quickly and reach a higher tension than slow twitch muscles. This classification only applies to skeletal muscles, not cardiac nor smooth visceral muscles.



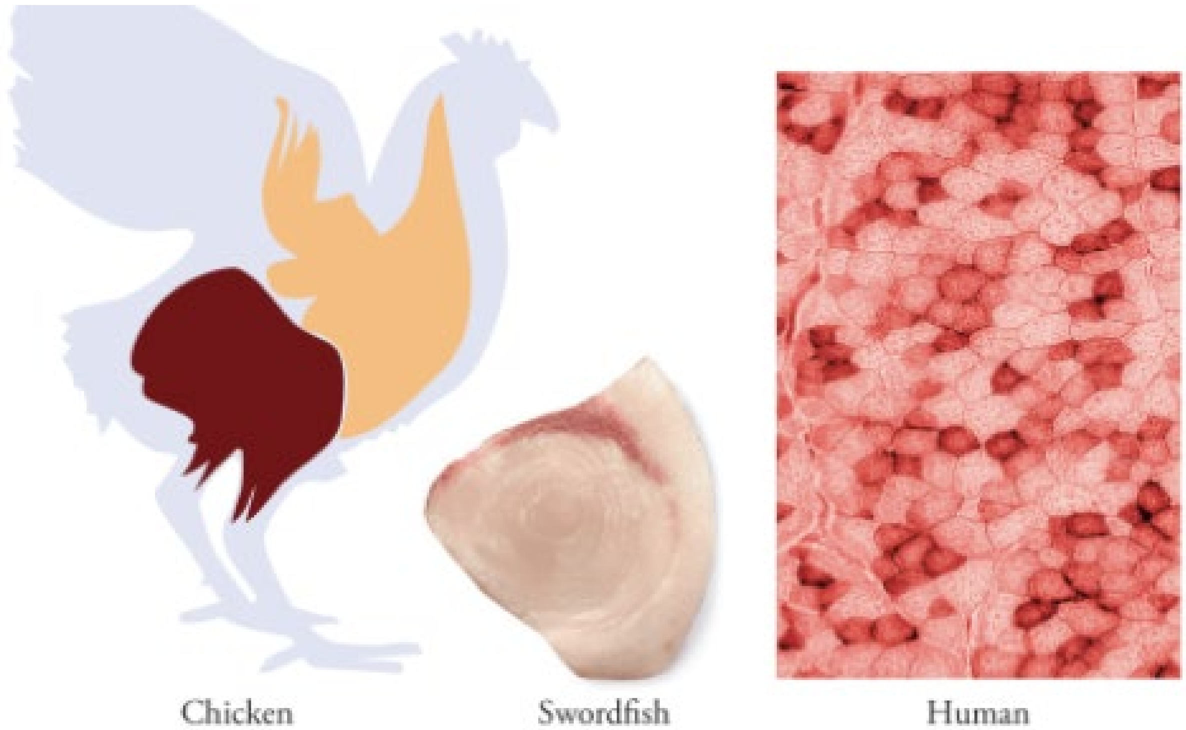
Fast-twitch (FT) and Slow-twitch (ST) Activation

adapted from Hall (2007)

FT and ST Muscle Characteristics *Hall (2007)*

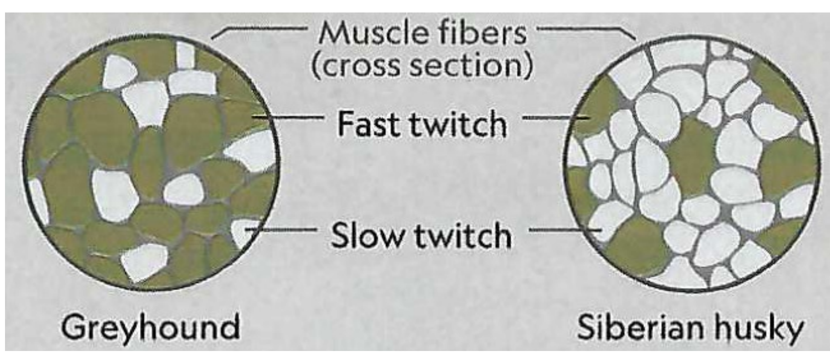
Characteristic	Type I ST Oxidative (SO)	Type IIA FT Oxidative Glycolytic (FOG)	Type IIB FT Glycolytic (FG)
Contraction speed	slow	fast	fast
Fatigue rate	slow	intermediate	fast
Diameter	small	intermediate	large
ATPase concentration	low	high	high
Mitochondrial concentration	high	high	low
Glycolytic enzyme concentration	low	intermediate	high

Some muscles in chicken and fish primarily have slow-twitch fibers (fatigue-resistant, dark meat), and other muscles are made of fast-twitch fibers (fatigable, white meat). As seen in the figure below, humans (and other mammals) have muscles with both fiber types interspersed within each muscle.



Chicken / Fish / Human Slow- and Fast-Twitch Muscle Fibers
Uchida and Delp (2021)

Like humans, dog thigh muscles have both fast- and slow-twitch fiber types interspersed. In the figure below, compare the greyhound muscle (bred and trained for running speed, up to 45 mph) with the Siberian husky muscle (bred and trained for running endurance).

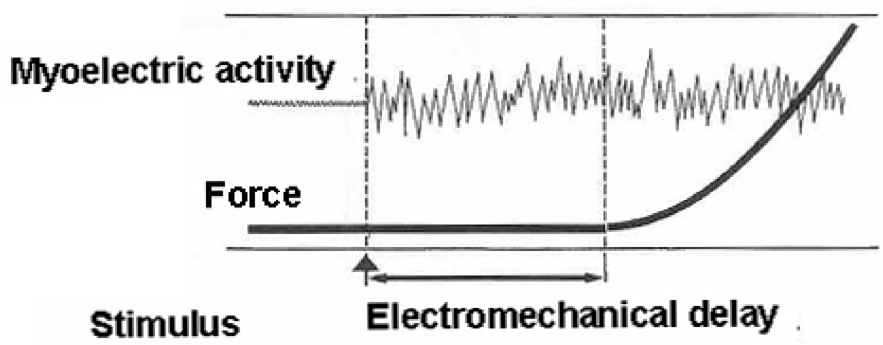


Greyhound vs. Siberian husky Slow- and Fast-Twitch Muscle Fibers
National Geographic Magazine (March 2021)

Incidentally, Florida has now banned betting on greyhound races, effectively killing the industry. Only three states now allow betting on greyhound races, Arkansas, Iowa, and West Virginia.

Muscle Fiber Activation

Individual muscle fibers are binary – that is, there is a threshold of activation and they are either off or on. If the voltage threshold is exceeded, the muscle fiber is contracted, if not, it remains at rest. **Electromechanical delay** is the time between arrival of a neural stimulus and tension development by the muscle, as shown below. This time delay is generally between 20-100 msec.



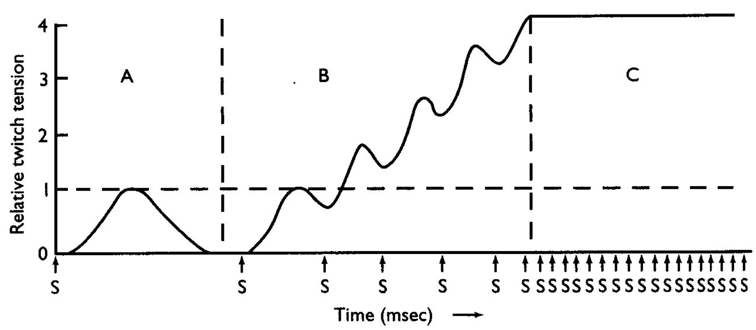
Hall (2007)

Modulation of Muscle Contraction Force

A single action potential contracts a muscle fiber. This so-called twitch peaks at around 100 msec but then disappears after a very short duration as the calcium is returned to the **SR** (see figure below, **A**). There are two ways to increase muscle output, **spatial** and **temporal summation**.

- **Spatial summation** – the simplest way to increase contraction force is to recruit more muscle fibers. **Size Principle of Motor Unit Recruitment**: motor units are recruited in the order of their size, from smallest (Type I) to largest (Type IIB) depending on the force they must produce.

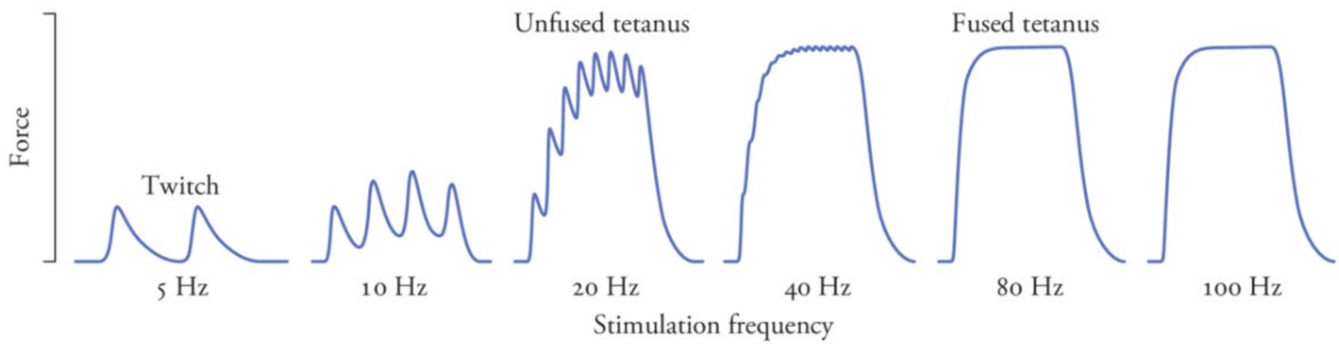
- **Temporal summation** – if the muscle fiber is re-stimulated before it fully relaxes, the contraction force increases (see figure below, **B**). If the frequency of action potentials increases, the force continues to rise, and the state of **tetanus** is reached (see figure below, **C**). On the independent axis in the figure below, **S** represents the arrival of an action potential in time.



Muscle Contraction Force Generation

Hall (2007)

As the firing rate (frequency) of the action potentials increases, the generated force increases as well. This peak force plateaus at the tetanic frequency. A muscle operating at or above the tetanic frequency is in a state of fused tetanus (see figure below).



Muscle Force Generation vs. Stimulation Frequency

Uchida and Delp (2021)

Electromyography (EMG)

EMG is a technique to record electrical activity produced by skeletal muscles during contractions. An EMG detects electric potential (voltage) generated by muscle cells, whether electrically or neurologically activated. The resulting signals can be used to find medical abnormalities, activation level, motor units recruitment order, and/or to analyze the biomechanics of animal/human motion.

Surface EMG records muscle activity for muscles operating below the skin. More than one electrode is required since the results measure relative voltage difference. Surface EMG is limited because it only measures activity of superficial muscles, it is affected by the depth of subcutaneous tissue, and it cannot reliably differentiate between adjacent muscles.

Intramuscular EMG is an invasive but safe procedure, requiring needle electrodes to be inserted into the muscle. These needle electrodes are so small they can distinguish individual muscle fibers (cells).

Surface EMG

electromyography_thumb.jpg (healthline.com)

Intramuscular EMG

patient-nerves-testing-using-electromyography-emg-picture

<u>Spatial and Temporal Summation Details (Dr. Brian Clark, personal communication)</u>

As stated earlier, there are two ways to increase muscle output, spatial and temporal summation. **Spatial summation** simply means recruiting more muscle fibers by activating more motor units (axons). **Temporal summation** means activating the same muscle fibers again before they have a chance to relax. This page presents more details regarding these two concepts.

- The biophysical basis of motor unit recruitment is based on Ohm's Law $V = iR$, where V is the voltage potential difference in the neuron (~70 mV), i is the (constant) current from the brain, and R is the resistance. Smaller motor units have higher resistance and so these fire first, with the constant current, since V is higher than in larger motor units.

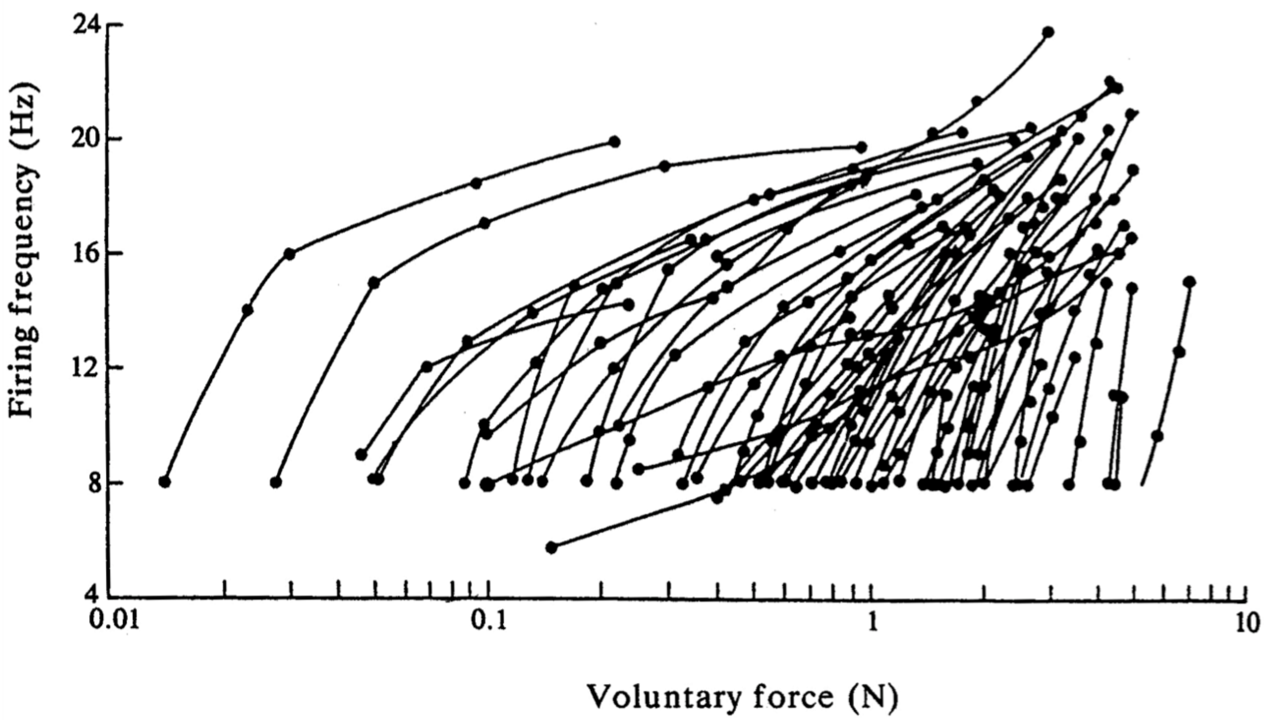
- Summation of individual motor unit action potentials to contribute to the voluntary EMG signals.

- This is a combination of motor unit recruitment and motor unit discharge rate that primarily contributes to increased force production from muscle.

- Both **Spatial** and **Temporal Summation** increase force production.

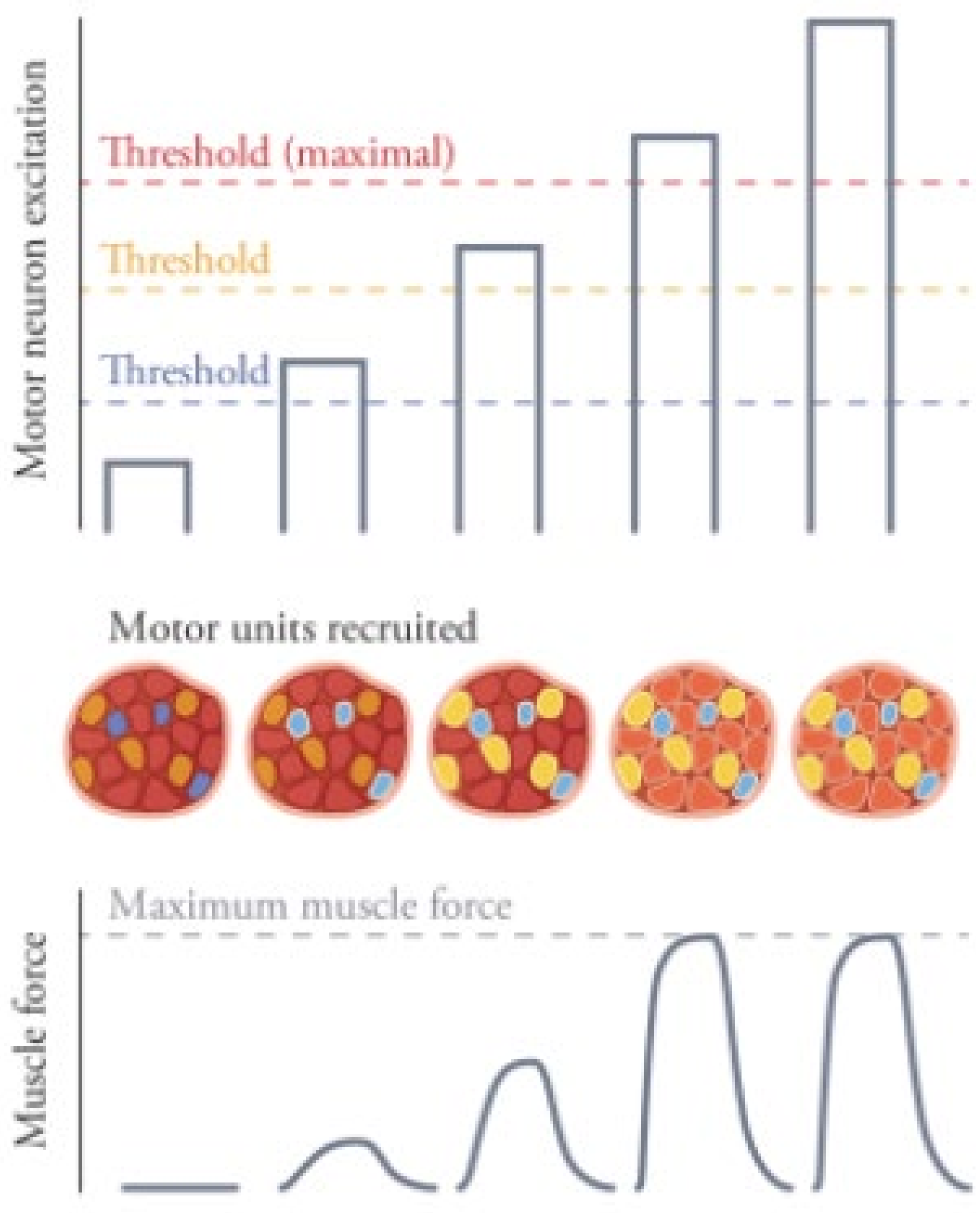
- If one were to say which comes first, it would be Spatial Summation recruitment (as a motor unit has to first be recruited and then it starts discharging faster).

- In some muscles virtually all motor units have been recruited by the time 50-70% of its max force has been reached (e.g., intrinsic hand muscles), whereas in other muscles motor units can be recruited (and discharged faster) up through maximal force capacity.



Relationship among motor unit recruitment, discharge rate, and force production

Each curve in the above plot represents a single motor unit.



Motor Unit Recruitment

Uchida and Delp (2021)

The human central nervous system regulates muscle force by changing the number of motor units recruited.

Hill Model (Force Development and Muscle Velocity)

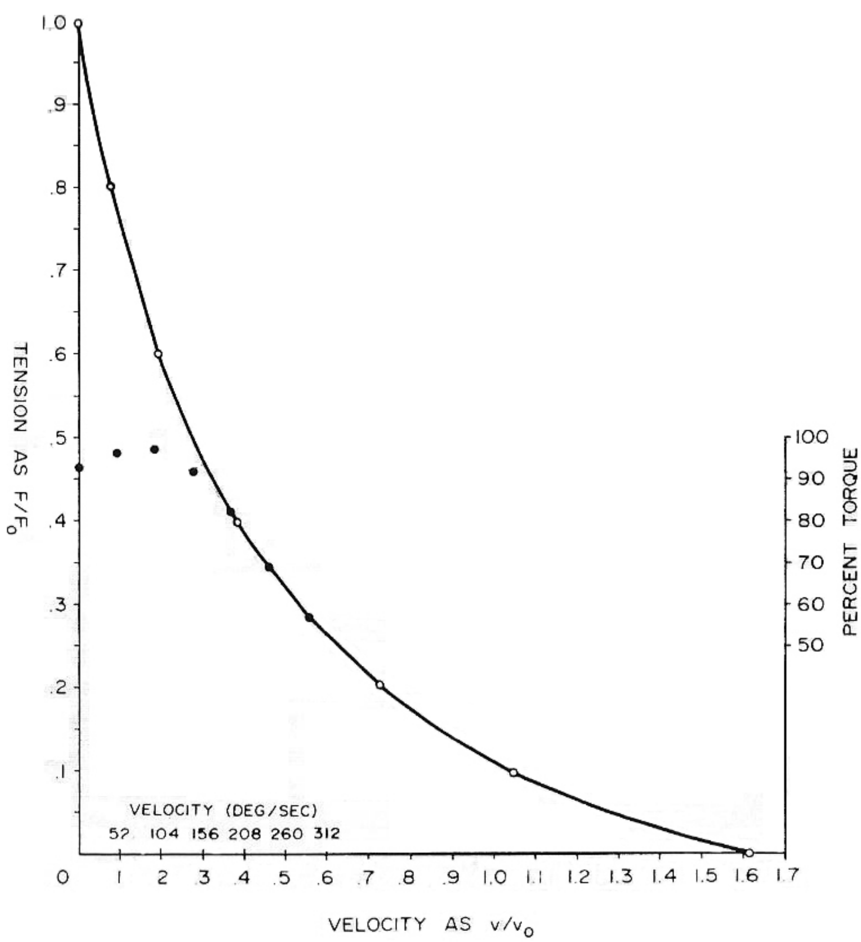
The most widely used muscle force/velocity model was developed by Hill in 1938 and augmented by the same Hill in 1964 (Jones et al., 1986). The **Hill mathematical muscle model** is:

$$F = \frac{b\left(F_0 + a\right)}{v + b} - a$$

F	muscle force developed	a	muscle cross-section constant
F_0	maximal isometric force	b	muscle length constant
v	muscle velocity		

Basically, **the developed force is inversely proportional to the muscle velocity**, as shown in the figure below. Both curves in the following plot are from experimental data. The clear dots are from in vitro testing of a frog quadriceps femoris muscle, clearly showing the inverse force/velocity muscle relationship. The solid dots are from a human cyclist tested in vivo, using the velocity and percent torque scales.

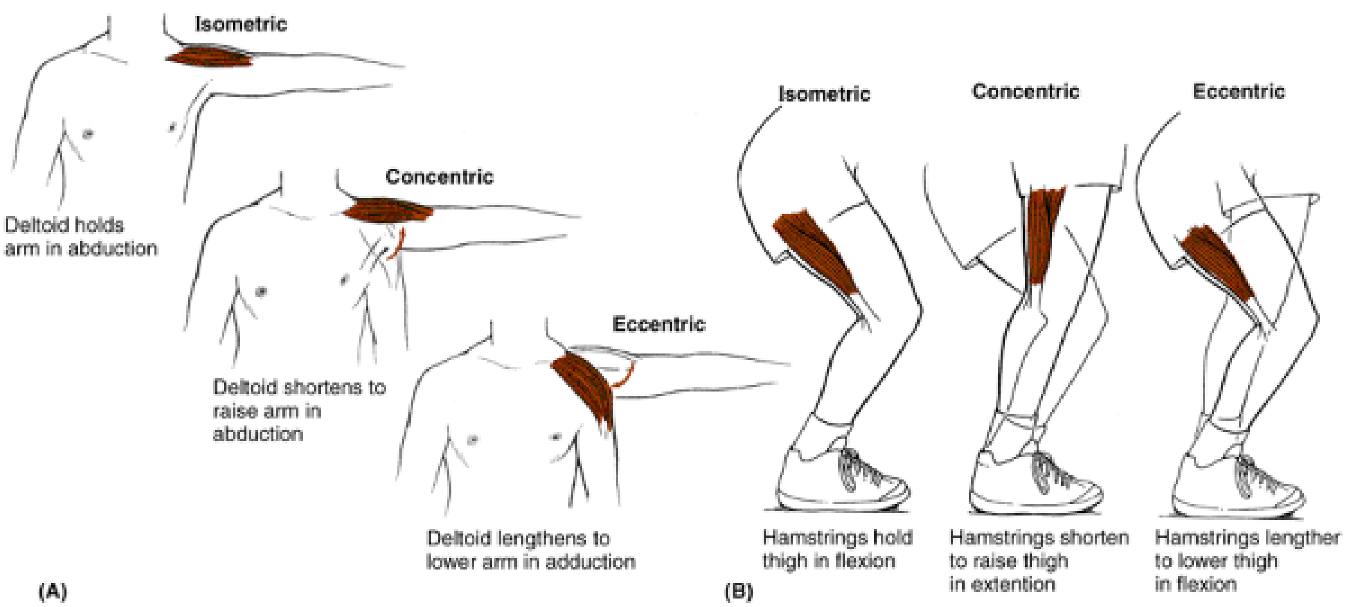
Using scaling F/F_0 and v/v_0 (where v_0 is the maximal contraction velocity), we see that the human and frog data correspond for a range in this plot. The constants a and b vary by muscle type. Note that F_0 and v_0 do not correspond to each other, poor notation indeed!



Hill Model Results *Jones et al. (1986)*

Tension Development and Muscle Length Changes *Hall (2007)*

When a muscle shortens as it is being stimulated to develop tension the contraction is **concentric**. When a muscle develops tension with no change in length, the contraction is **isometric**. When a muscle lengthens as it is being stimulated to develop tension the contraction is **eccentric**.

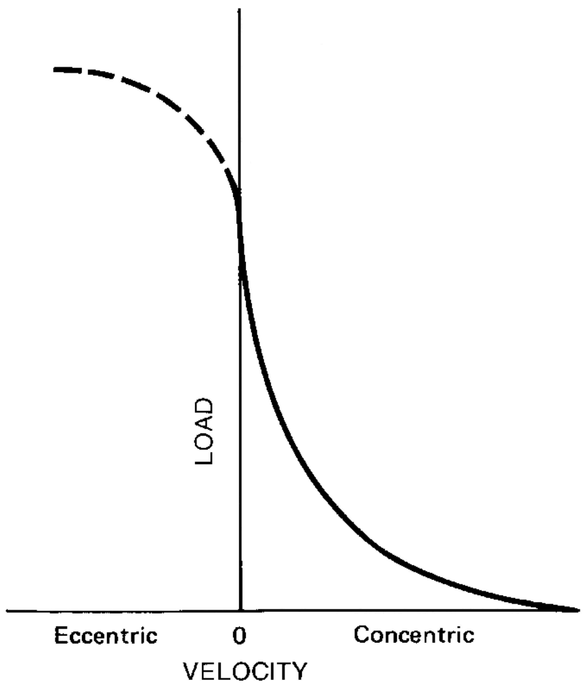


Isometric, Concentric, and Eccentric Contractions Examples

https://cpb-us-east-1-juc1ugur1qwqqqo4.stackpathdns.com/sites.marjon.ac.uk/dist/6/184/files/2016/03/Plyo-phases-ttdoyd.png

The figure below shows the muscle **LOAD** (force) vs. contraction **VELOCITY** curve for **concentric** and **eccentric** contractions (where is the **isometric** condition located on the graph below?). We see the Hill muscle model appear again in the concentric region.

Both **concentric** and **eccentric** contractions are generally **isotonic** (constant-force) contractions.



Muscle Force (Load) vs. Contraction Velocity *adapted from Hamilton et al. (2008)*

When a muscle contracts to move a body part with concentric contraction, it is the **agonist**. Muscles with opposite actions, developing tension with eccentric contraction, are called **antagonists**.

A muscle acting to support a body part against another force is a **stabilizer**. Muscles that stabilize one joint so that the desired movement can occur at another joint are known as **stabilizer muscles**. For example, during the biceps curl exercise, the pectoralis major and latissimus dorsi muscles contract isometrically to stabilize the shoulder joint so that controlled movement can occur at the elbow.

leeapperson.com

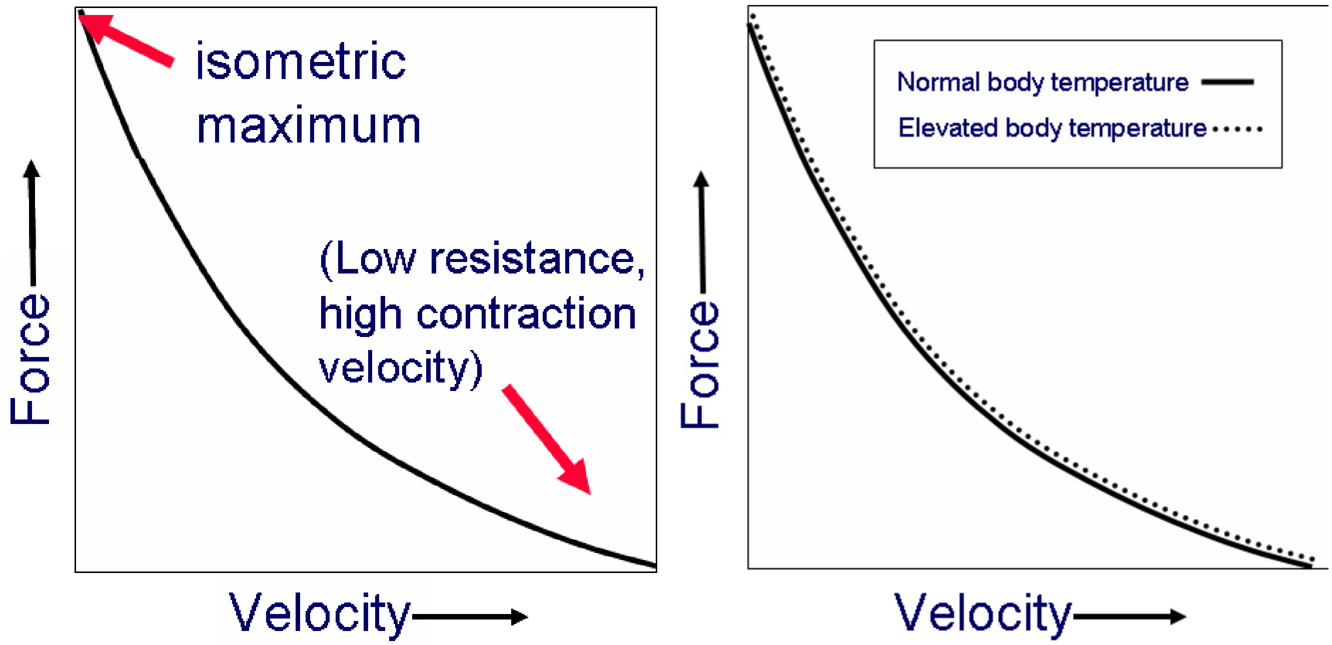
A **neutralizer** is a muscle acting to eliminate an unwanted action produced by an agonist. **Neutralizer muscles** prevent unwanted accessory actions that normally occur when an agonist develops concentric tension. The biceps brachii flexes the elbow and supinates the forearm. For pure elbow flexion, supination must be constrained by the pronator teres counteracting the biceps.

bsu.edu/web/ykwon

A **synergist** is a muscle that has the same action at the same joint as another muscle. A **fixator** is a muscle that acts as a stabilizer of one part of the body during movement of another part.

<u>The Force-Velocity Relationship for Muscle Tissue</u>

- As the force increases, concentric contraction velocity slows to zero at isometric maximum.
- When the force is negligible, muscle contracts with maximal velocity.



Muscle Force vs. Velocity *Hall (2007)*

The force-velocity relationship is inverse and nonlinear for human skeletal muscle.

<u>Muscle Force-Velocity and Temperature</u>

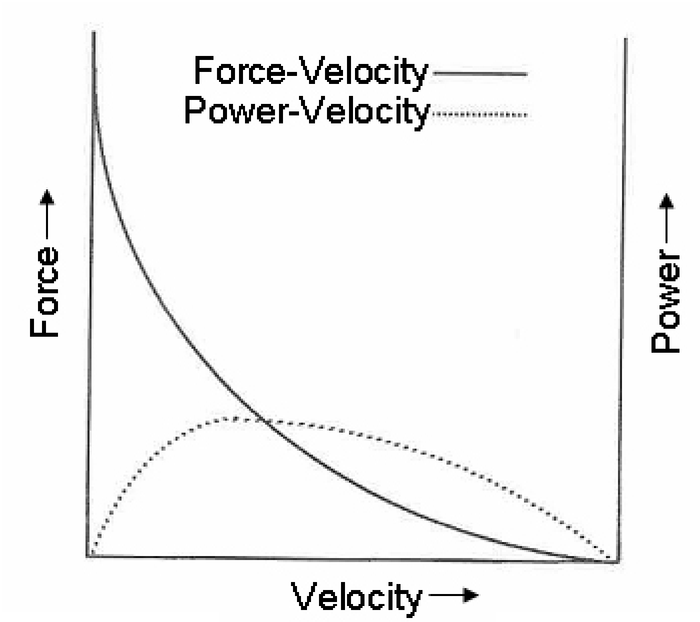
As body temperature increases, there is a shift to the right in the muscle force-velocity curve, with higher maximum isometric tension and higher maximum velocity of contraction possible at a given load. This is one reason you should always warm up prior to exercise.

<u>Muscle Power</u>

Recall the following definitions and SI units.

Work	$W = \underline{F} \bullet \Delta \underline{s}$	Nm, Joule
Power	$P = \frac{W}{\Delta t} = \underline{F} \bullet \frac{\Delta \underline{s}}{\Delta t}$	Nm/sec, Joule/sec, Watt
So muscle power is	$P = \underline{F} \bullet \underline{v}$	Nm/sec, Joule/sec, Watt

The figure below shows that there is no muscle power in isometric contractions and there is also no muscle power at zero force (at the highest velocity). Maximum muscle power occurs between these extremes, closer to lower velocities.

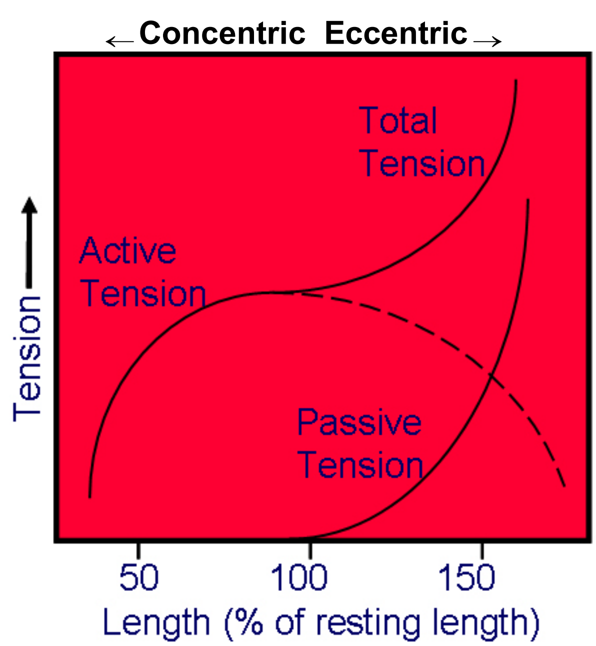


Muscle Force and Power vs. Contraction Velocity

Hall (2007)

The Muscle Length-Force Relationship

Force (tension) present in a stretched muscle is the sum of the active tension provided by the muscle fibers and the passive tension provided by the tendons and membranes (protective fascia epimysium, perimysium, and endomysium), as shown in the figure below. Passive tension only exists for a muscle under eccentric contractions.

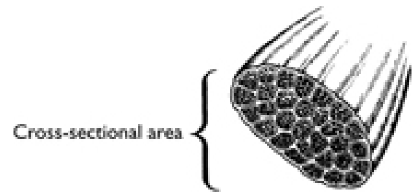


Muscle Force (Tension) vs. Length *Hall (2007)*

Muscle Tension vs. Cross-Sectional Area

The overall force-generating capacity F_f of human skeletal muscle is linearly related to the muscle cross-sectional area A:

$$F_f = \text{slope } A$$

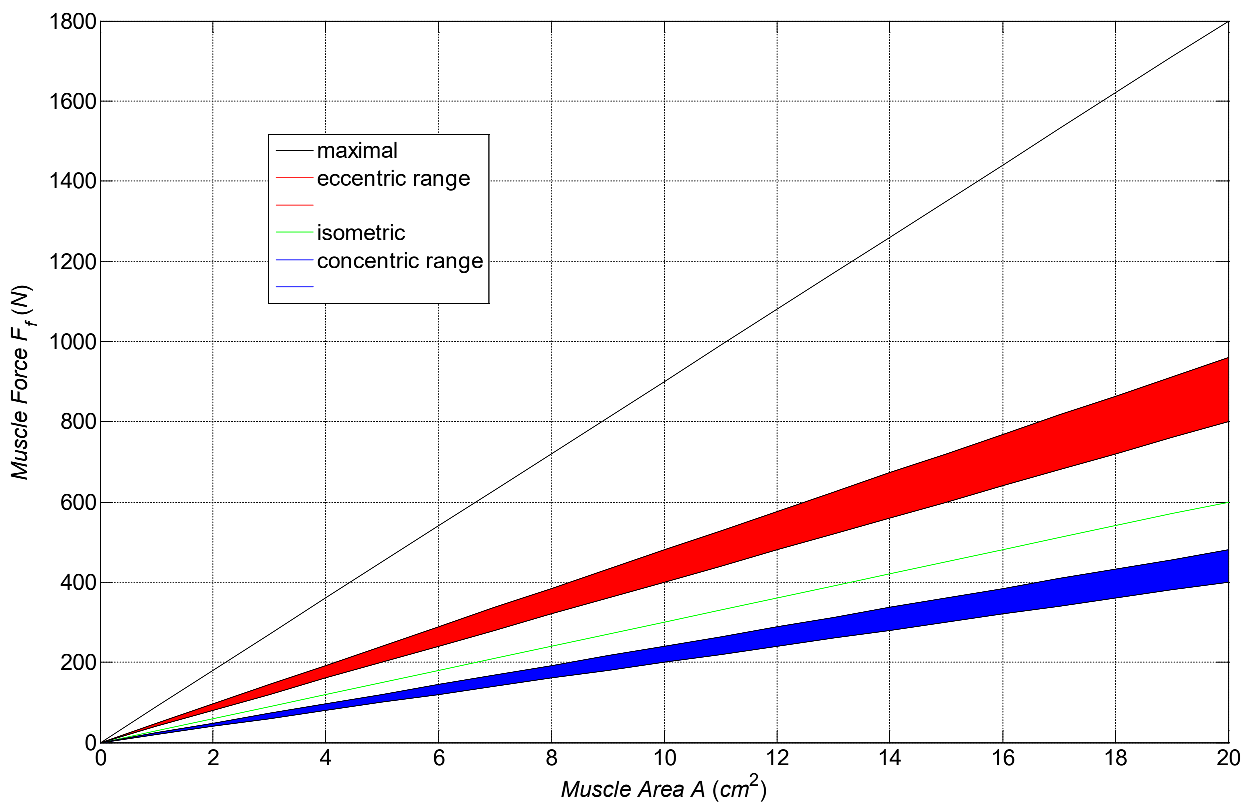


Hall (2007)

The theoretical maximum force/area slope is 90 N/cm^2, if all muscle fibers fire at once. Generally only 1/3 of all possible muscle fibers fire at once, even in strenuous activities. Below are listed the approximate force/area slopes for the different human skeletal muscle contraction types (Eric Snively, 2010, personal communication). Interestingly, these linear relationships and even the specific force/area slope numbers hold true over a wide range of animal species. The slope numbers below are averages; in general, the shorter the fascicles, the higher the muscle force generation capacity.

- maximal contractions 90 N/cm^2
- eccentric contractions 40 – 48 N/cm^2
- isometric contractions 30 N/cm^2
- concentric contractions 20 – 24 N/cm^2

The plot below summarizes the force-generating capacity F_f for muscle areas up to 20 cm^2, for the different types of contractions. A range is shown for both eccentric and concentric contractions.



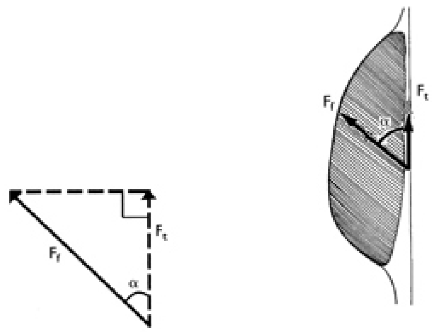
Muscle force vs. cross-sectional area

Effect of Pennation Angle on Tendon Force

The pennation angle α (see figure below) is the angle between the direction of the muscle fibers, where the forces of the previous page are being generated, and the tendon axis. The force of the muscle is applied to human motion through the tendons. This force is reduced by the pennation angle α.

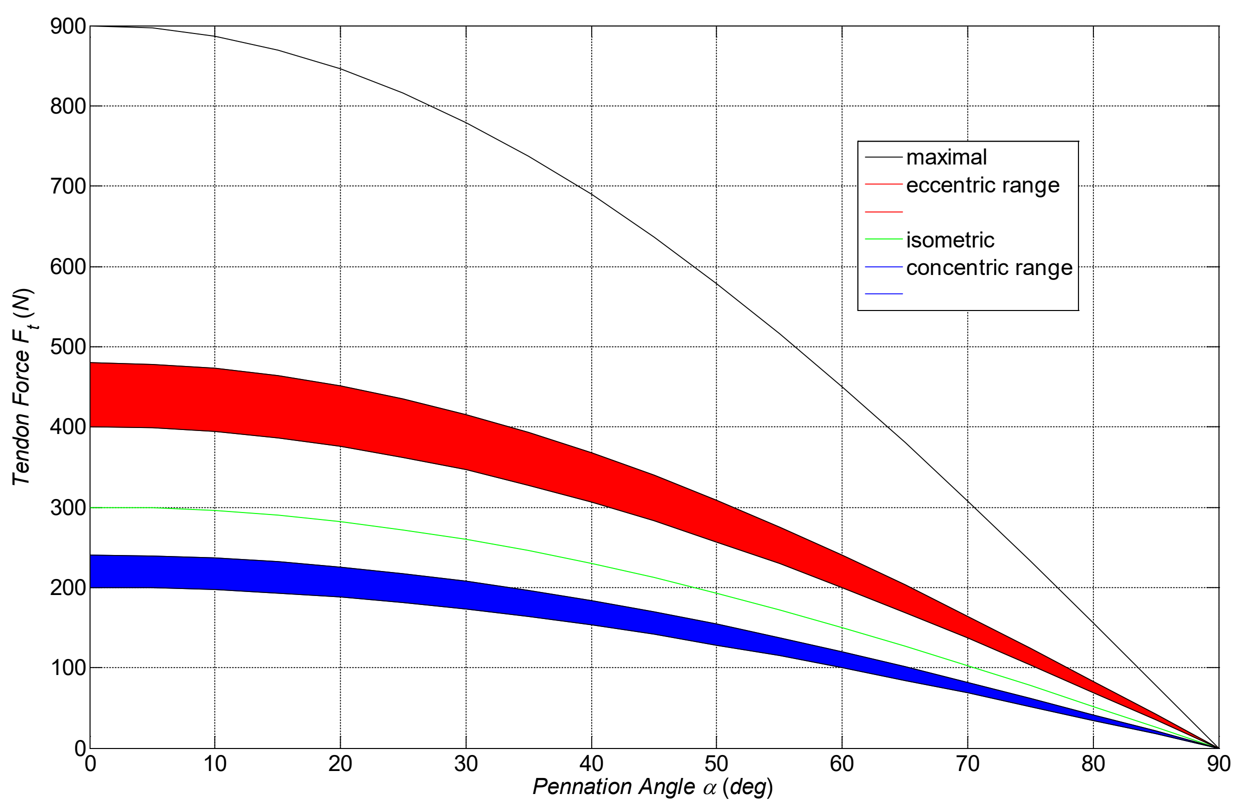
$$F_t = F_f \cos\alpha$$

where F_f is the total muscle force along the direction of the muscle fibers, F_t is the resulting force along the direction of the tendon, and α is the pennation angle.

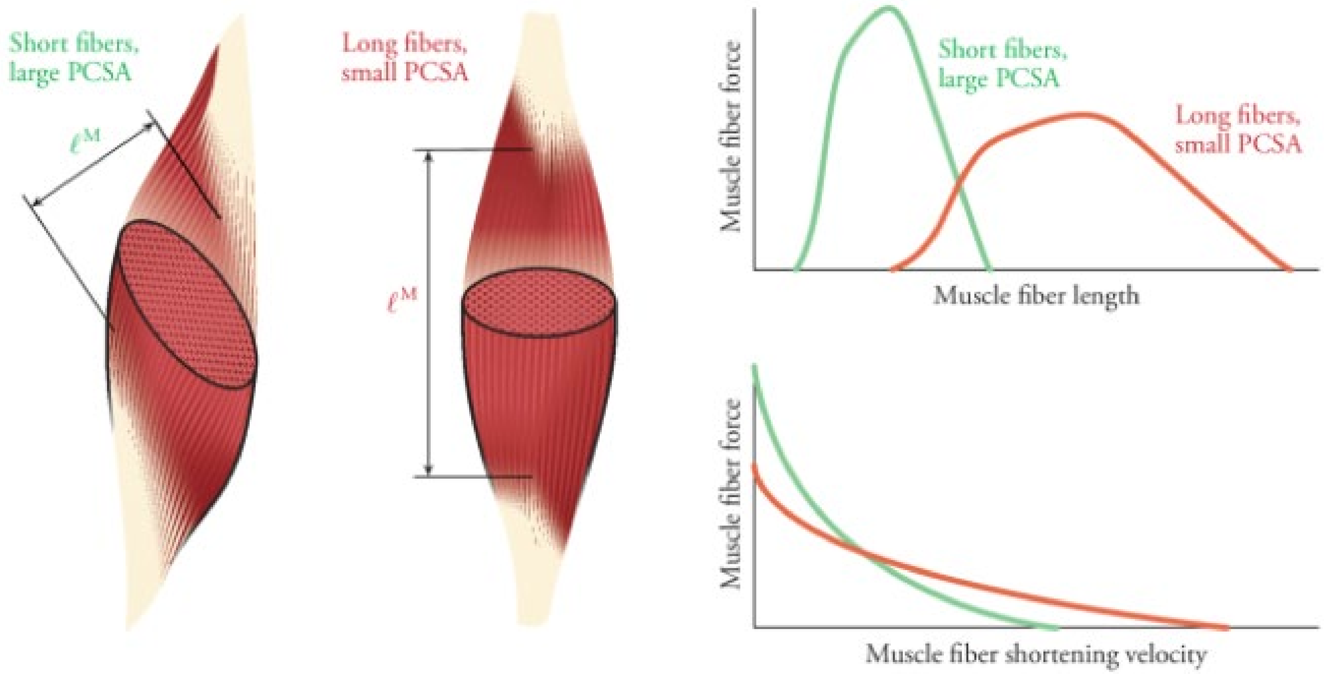


adapted from Hall (2007)

The plot below summarizes the tendon forces generated for various pennation angles. This plot assumes A = 10 cm^2 from the previous example and again shows results for the different types of contractions. Again, a range is shown for both concentric and eccentric contractions. No real human muscle pennation angles are close to 90°, otherwise their force transmission would go to zero.



Tendon force vs. pennation angle



Muscle PCSA

Uchida and Delp (2021)

Muscle **<u>P</u>**hysiological **<u>C</u>**ross-**<u>S</u>**ectional **<u>A</u>**rea (**PCSA**) is the area of a muscle perpendicular to its fibers. In the figure above, the left muscle has the same volume, but different PCSA, optimal fiber lengths, and pennation angles than the muscle on the right. This more-pennated muscle can generate a higher active force but it has shorter fibers. Therefore, its active force-length and force-velocity curves are higher but also narrower.

Muscle Physiology Notes (Dr. Clark)

- Overall, human strength is weakly correlated to muscle size. Innervation ratio is more important (how many motor units there are per muscle).

- Neural training of muscles is far more significant than physical strength training. For instance, when subjects physically trained one leg only (the second was immobilized for a significant period of time), the untrained leg strength increased along with the trained leg strength.

- Just thinking about contracting your muscles (e.g. forearm) significantly increases the voltage signal in the electromyography without actual contractions! ME 4670 / 5670 observed this in Dr. Clark's lab in Spring 2009, performed by one of our own as subject. Her brain was stimulated via external electromagnetic paddle, non-invasive and painless.

- According to the size principle of motor recruitment, the smallest motor units are recruited first, followed sequentially by more motor units according to size (largest last).

- Muscles are activated by excitatory neurons (axons). Muscles at rest are controlled by inhibitory neurons (axons).

- Generally a motor unit has one axon that controls 10-100 muscle fibers. For fine control in the hand, a single axon innervates 10-15 muscle fibers (cells). But in the quadriceps, a single axon innervates up to 1000 muscle fibers; this loses the ability for fine control but provides large force.

- The human body activation of muscles involves series of actual electrical circuits with voltage and current.

- There are about 50,000 synaptic inputs in the brain for each axon.

- 200,000 acetylcholine reactions are required to pass on a single action potential.

- In patients with stroke or cerebral palsy, **muscle spasticity** often results from the interruption of inhibitory inputs

- **Malignant hyperthermia** is a rare potentially fatal condition, often arising in allergic reactions to anesthesia, in which all muscles are simultaneously contracted and the body temperature rises alarmingly.

Muscle Soreness and Injury (Dr. Howell)

- Muscle **soreness** and **injury** is worse under exercises and motion with eccentric contractions (muscle lengthening while developing tension).

- This is due to mechanical damage to the muscle at the sarcomere level – sarcomere 'popping'.

- The human body can adapt – more sarcomeres are laid down in areas of previous damage, so you will be stronger the next time the same eccentric exercise is performed.

- Serious cyclists experience only concentric contractions during their exercise. If they then must walk downhill with eccentric contractions, they are often easily injured. The remedy for this is cross-training with hiking or running.

Other Muscle Physiology Topics

- Muscle **fatigue** is the effect of a set of mechanisms which ensure that muscle is not made active when there is not enough energy available for the activity. If that were to happen, theoretically the muscle could go into **rigor mortis**, and could fail to retain the large amount of potassium it contains, with dire consequences for the body as a whole.

- The body contains several different varieties of skeletal muscle fiber, which can be seen as specialized for different purposes. The 'slower' muscles are more economical at holding up loads, such as maintaining **posture** of the body itself, and probably also more efficient at producing external work. Related to their lower energy use they are less easily fatigued.

- Faster muscle fibers, however, can produce faster movements and higher power outputs, and are essential for such tasks as jumping or throwing.

- The way different muscles are constructed allows function specialization. Muscle with shorter fibers hold forces more economically, muscle with longer fibers produce faster movements.

- A pennate arrangement allows muscles to be built with many short fibers, increasing the force they can exert, whereas long fibers, running almost parallel to the axis of the muscle, give the fastest movements.

- Some people have more **muscular strength** than others; they can exert larger forces, do external work more rapidly, or move faster. To a large extent this is because the stronger individuals have larger muscles, but there seem to be other factors at work as well.

- Training can change the properties of muscle. **Strength training** consists in using the muscles to make just a few very strong contractions each day. Over months and years this leads to an increase in the force that can be exerted and in increase in the size of the muscles.

- Force increase often precedes size increase. Endurance training consists of using the muscles less intensely but for longer periods.

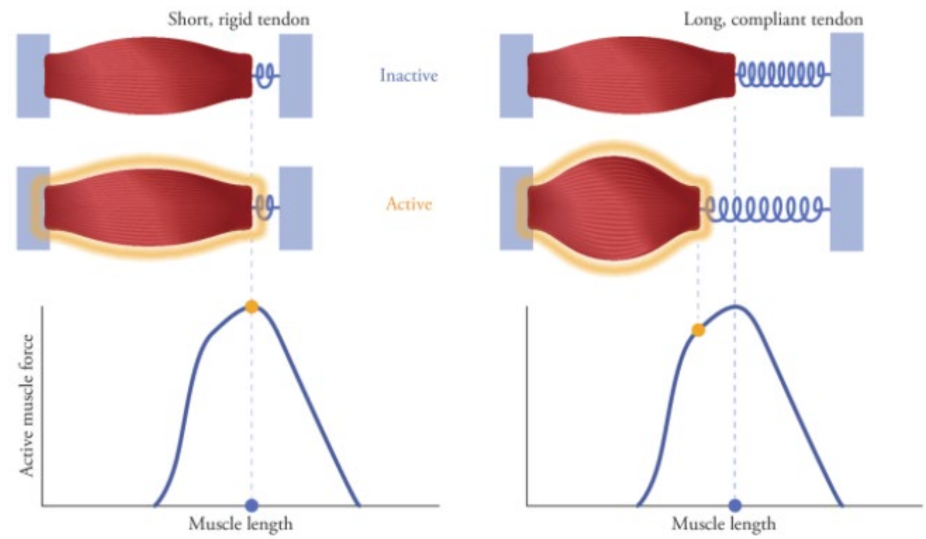
- Again, over months of training the ability of the muscles to get energy through the oxidation of carbohydrate and fat is raised. The supply of blood to the muscle is also increased through changes in the blood vessels and also in the heart.

- Training can also lead to changes in the fatigue resistance of muscle fibers, and perhaps cause them to change into a slower type of fiber.

answers.com

Muscle Force and Tendon Compliance

A parallel-fibered (zero pennation angle) muscle is at its optimal length when the muscle is inactive (zero force generated). If the tendon is short and relatively rigid (left muscle in the figure below) there is little fiber length change upon muscle activation. On the other hand, if the tendon is long and relatively compliant (right muscle in the figure below), the tendon stretches upon muscle activation, which causes the muscle fibers to shorten and hence reduce the generated force.

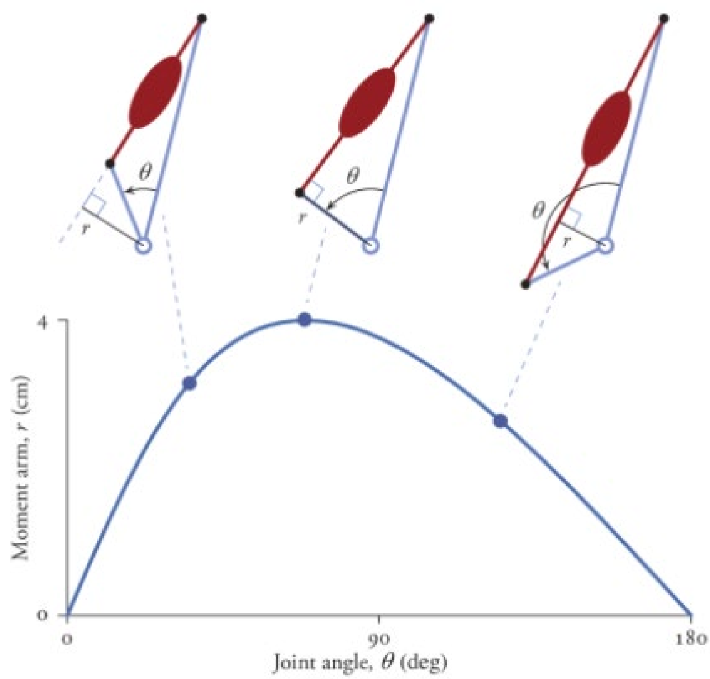


Tendon Compliance Affects Muscle Force Generation

Uchida and Delp (2021)

Muscle Moment Arm and Joint Angle

The effective muscle moment arm r depends non-linearly on the joint angle θ, as shown.

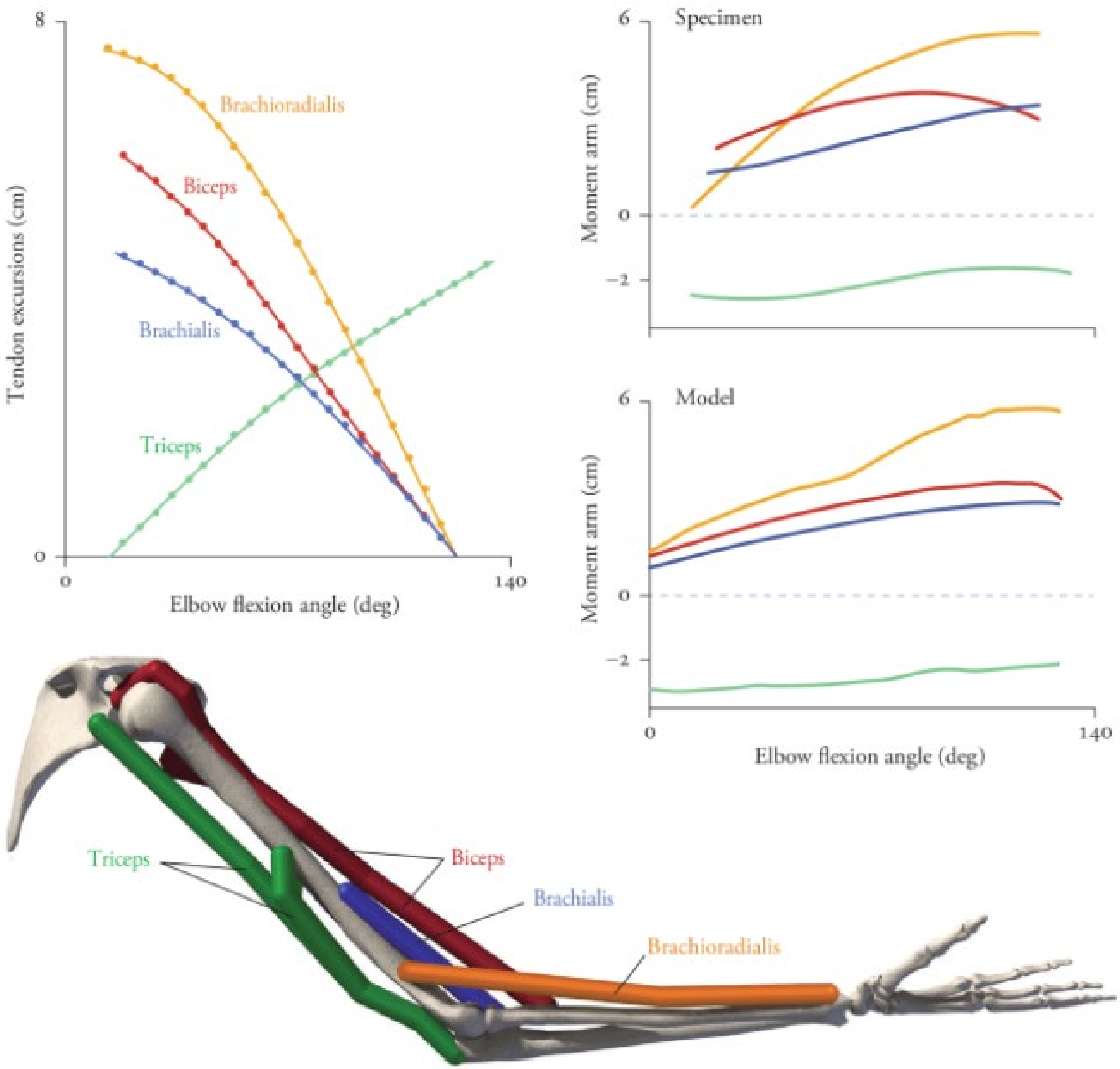


Muscle Moment Arm r vs. Joint angle θ

Uchida and Delp (2021)

Tendon Excursion

Tendon Excursion occurs when a muscle contracts and the joint rotates. A pulley-type constraint keeps the tendon close to the bone when the tendon crosses a joint. The magnitude of the tendon excursion depends on the joint angle. The plots below are from experimental data (compared to models of the same).

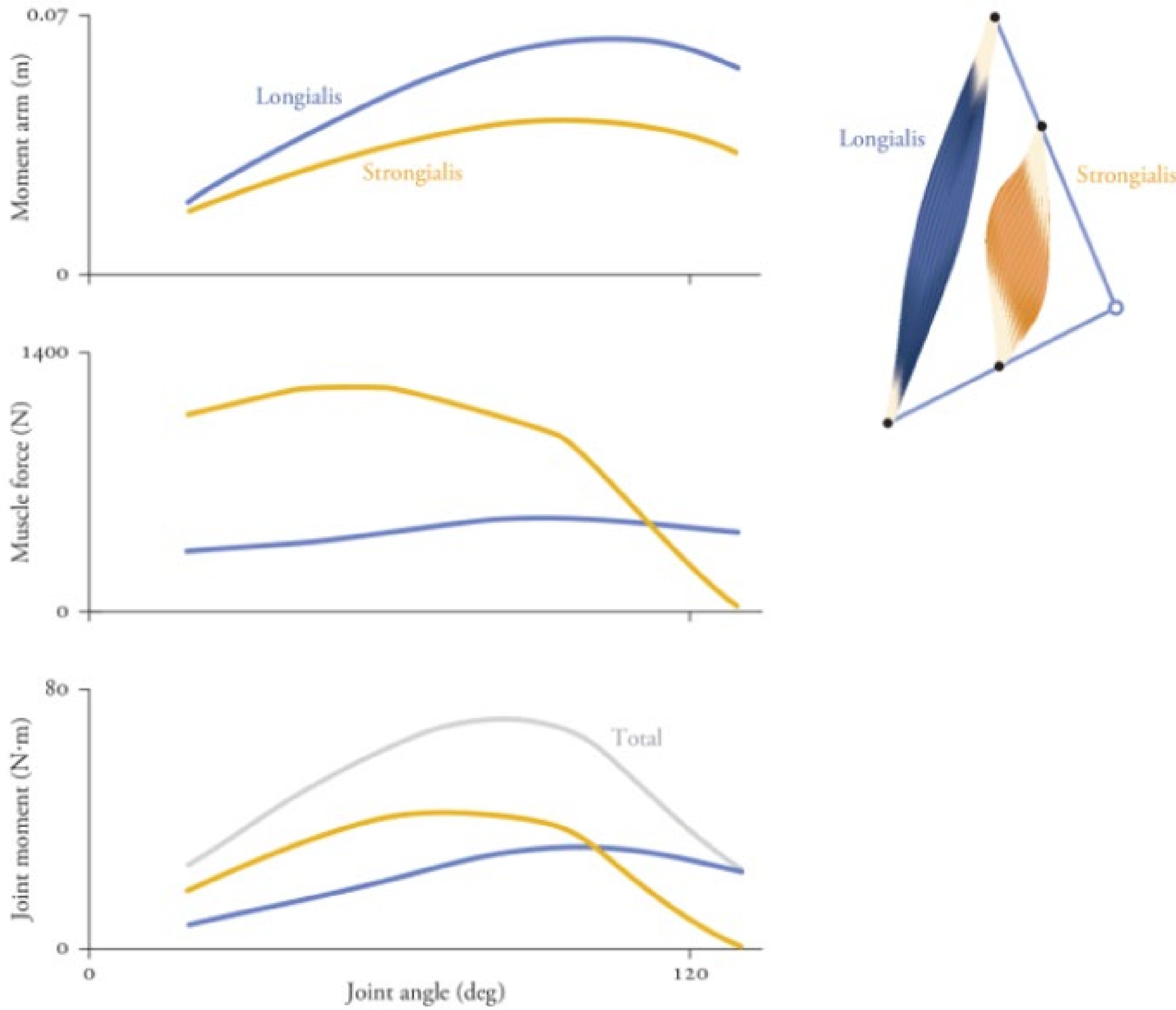


Tendon Excursion and Moment Arms, Major Elbow Muscles

Uchida and Delp (2021)

Maximum Muscle Moment

The **Maximum Muscle Moment** generated by muscle force acting through a moment arm occurs at neither maximum muscle force nor maximum moment arm, as shown in the graphs below. (The **Longalis** and **Strongalis** are, of course, made-up muscles.) The total muscle moment is simply the sum of all ***r* x *f*** moments from each active muscle in a given motion.



Muscle Force, Moment Arm, and Moment

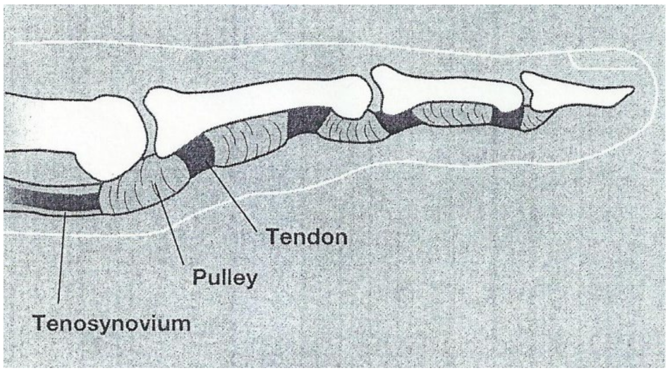
Uchida and Delp (2021)

Tendon Pathologies

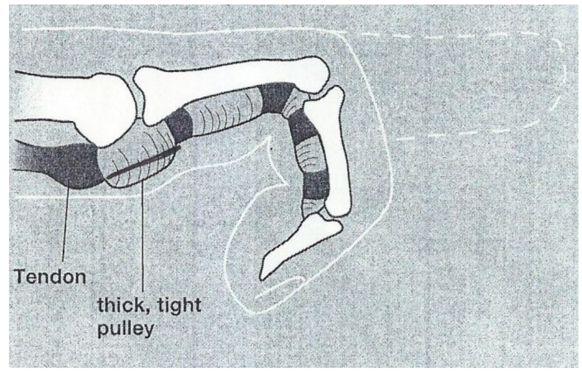
Trigger Finger

Trigger Finger (stenosing tenosynovitis) is a painful and disruptive condition in the human finger, generally caused by overuse. Osteoarthritis and diabetes can also lead to trigger finger.

The left figure below shows a normal healthy human finger where the actuating tendon slides freely within its tendon sheaths (pulleys).



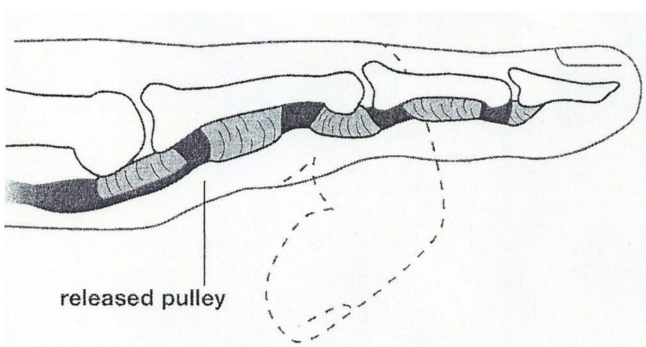
Normal Human Finger



Trigger Finger Condition

The right figure above shows a human finger with Trigger Finger, where the base pulley has become swollen and the tendon no longer slides freely within the pulley.

In some cases Trigger Finger can be treated with cortisone shots and/or physical therapy. Many cases require surgery, where the base pulley is slit, using a tool like a seam ripper in sewing, thus releasing the tendon for nearly normal usage. This is pictured in the figure below.

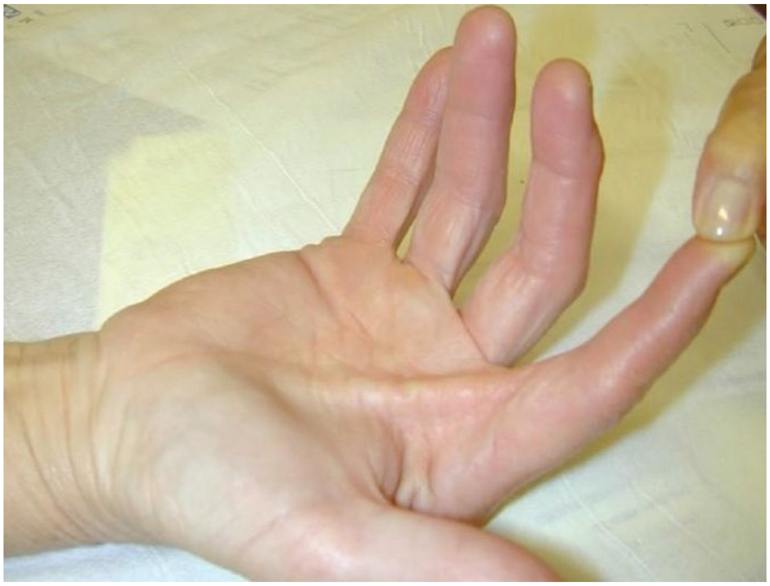


Trigger Finger Surgically Corrected

Climber's Finger

Climber's Finger is an overuse injury in rock climbers . Often occurring in the middle and/or ring fingers, it is generally due to the climber supporting their entire body weight on only one or two fingers with small holds.

The figure below shows a potential associated condition with Climber's Finger called 'bowstringing'.



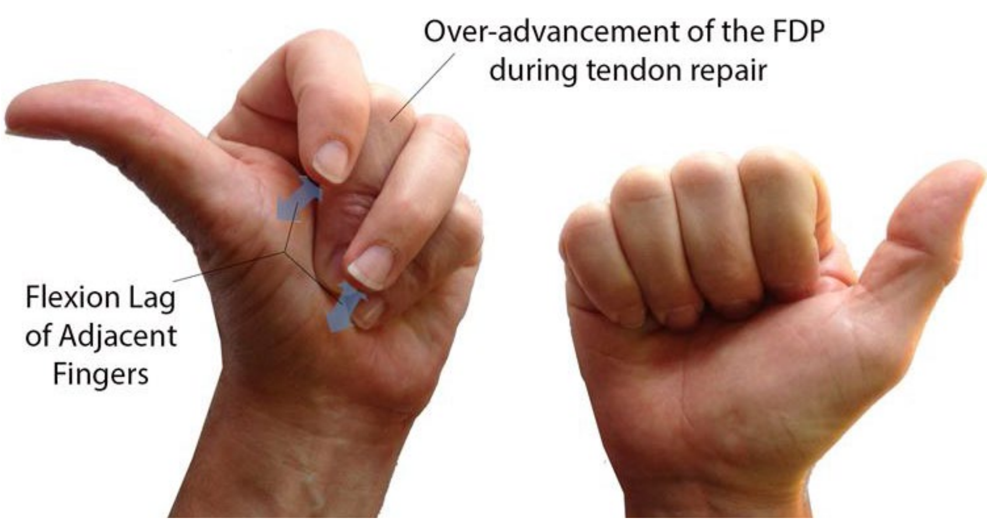
'Bowstringing' on the Human Right Index Finger

s3.amazonaws.com/classconnection/720/flashcards/1137720/jpg/bowstring_phenomenon_clinical_example

This figure, which Dr. Bob finds hard to look at, shows that one or more crucial tendon sheaths has failed.

The Quadriga Effect

Elite rock climbers may suffer from this condition due to injuries caused by hanging their entire body weight from one or two fingers (one-finger pocket hold). In the Quadriga Effect, the climber has an active flexion lag in the fingers next to a finger that previously suffered injury to the Flexor Digitorum Profundus (FDP) tendon.



Quadriga Effect **Normal**

Quadriga Effect - Hand - Orthobullets

4. Human Body Engineering Mechanics: Kinematics

4.1 Human Body Kinematics

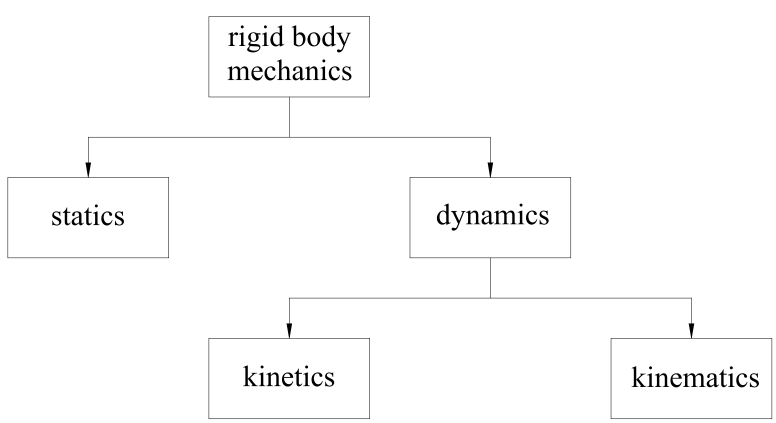
Hamilton et al. (2008)

- **Kinematics** is the study of motion without regard to **forces**/**moments**.

- We will study **kinematics** (position, velocity, and acceleration, both translational and rotational) for human body motion prior to **statics** and **dynamics**, both of which require force/moments plus kinematics information.

- **Statics** analysis requires **position** information, both **translational** and **angular**. **Dynamics** analysis requires **position**, **velocity**, and **acceleration** information, both **translational** and **rotational**.

- The mathematics of **robotics** (Craig, 2005) applies well to human body kinematics. The mathematics of **planar mechanisms** (Norton, 2008) can also apply.



4.1.1 Mobility

Mobility – the number of degrees-of-freedom (**dof**, number of parameters required to completely specify the location of a device) which a human or machine possesses. Recall there are about 244 degrees-of-freedom (dof, based on skeletal motion) in the adult human body.

How many dof for a human body segment in **planar motion**? Grubler's equation is used to calculate M_P, the planar mobility (the planar number of dof):

$$M_P = 3(N-1) - 2J_1$$

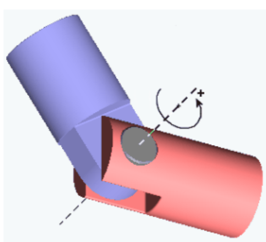
where:

M_P	is the planar mobility
N	is the total number of links, including ground
J_1	is the number of one-degree-of-freedom joints

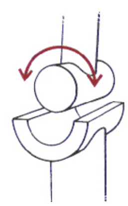
Caution Kutzbach's equation for **spatial mobility** M_S is different than the above equation for M_P.

Planar biological joints review

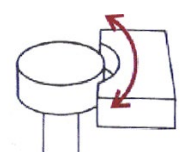
joint	dof	engineering name	biological name
J_1	1-dof	revolute (R)	hinge, pivot



1-dof Revolute Joint www.mathworks.com



Hinge



Pivot

The engineering 1-dof prismatic (P) joint and the 2-dof cam, gear, and slotted-pin joints have no biological equivalents. The 2-dof engineering plane joint (called gliding or plane joint in biology) exists in the human body, but only for 3D, not planar motion. So if you have a plane joint to include in planar motion, it must be modeled as a 1-dof prismatic (P) joint.

A general planar n-link serial human body segment has n+1 links (including the fixed ground link), connected by n active 1-dof joints (just count the number of active joints).

$$M_P = 3(n+1-1) - 2n = 3n - 2n = n \text{ dof}$$

Planar human arm 3R example (lock shoulder and wrist to 1-dof pitch joints).

In mechanisms and robotics we can generally count the number of 1-dof motors to determine the dof – this does not work in biology (i.e. counting active muscles) due to significant **actuation redundancy**.

How many dof does a human body segment in **spatial motion** have? Kutzbach's equation is used to calculate M_S, the spatial mobility (the number of spatial dof).

$$M_S = 6(N-1) - 5J_1 - 4J_2 - 3J_3$$

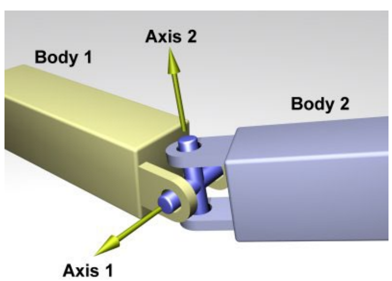
where:

M_S	is the spatial mobility
N	is the total number of links, including ground
J_1	is the number of one-degree-of-freedom joints
J_2	is the number of two-degree-of-freedom joints
J_3	is the number of three-degree-of-freedom joints

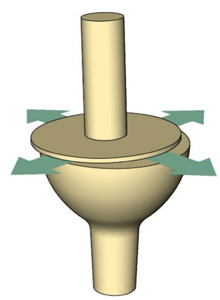
Spatial biological joints review

joint	dof	engineering name	biological name
J_1	1-dof	revolute (R)	hinge, pivot
J_2	2-dof	universal (U); plane	ellipsoidal, saddle; plane (gliding)
J_3	3-dof	spherical (S)	ball and socket

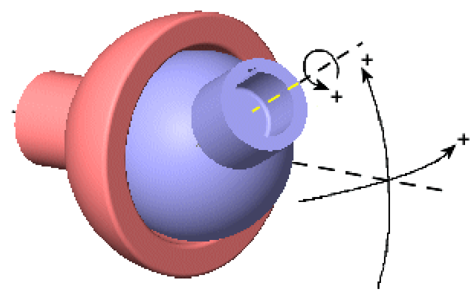
1-dof Revolute Joint – see previous page for images.



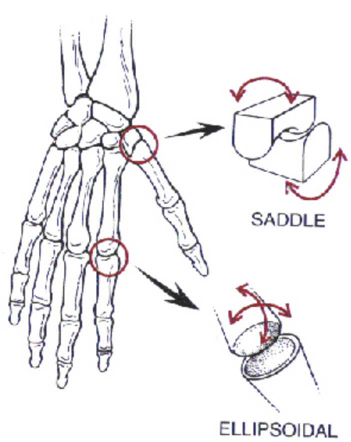
2-dof Universal Joint



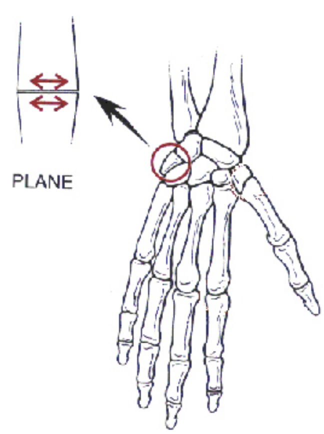
2-dof Plane Joint



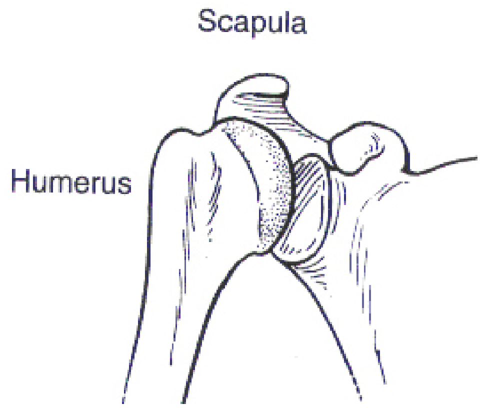
3-dof Spherical Joint



2-dof Ellipsoidal and Saddle Joints



2-dof Plane Joint



3-dof Ball-and-Socket Joint

The engineering 1-dof prismatic and screw joints, the 2-dof cylindrical joint, the 4-dof slotted spherical joint, and the 5-dof spatial cam have no biological equivalents.

A general spatial n-link serial human body segment has n+1 links (including the fixed ground link), connected by n active 1-dof joints (just count the number of active joints).

$$M_S = 6(n+1-1) - 5n - 4(0) - 3(0) = 6n - 5n = n \text{ dof}$$

__**Spatial human arm**__ S-R-S example (or 7R, since we can represent the ball and socket joint with three intersecting R joints). S-R-R-U is the most accurate biomechanical model for the human arm.

__**DO NOT**__ use Grubler's equation for planar mobility M_P to calculate the mobility M_S for spatial body parts. Or vice versa.

The human body has both **kinematic redundancy** (more joints than minimally necessary for doing motions) and **actuation redundancy** (more active muscles than joint dof).

4.1.2 Basic 1-dof Translational Kinematics Equations

Derivatives **Integrals**

Constant Velocity Case **Equations** **Plots**

Constant Acceleration Case **Equations** **Plots**

The same equations and plots also hold for 1-dof ***Rotational Kinematics***.

All basic physics kinematics equations can be derived from the above equations. It is essential to know the assumptions behind every case. If your problem has neither constant velocity nor constant acceleration, you must use the general derivative/integral equations given first above.

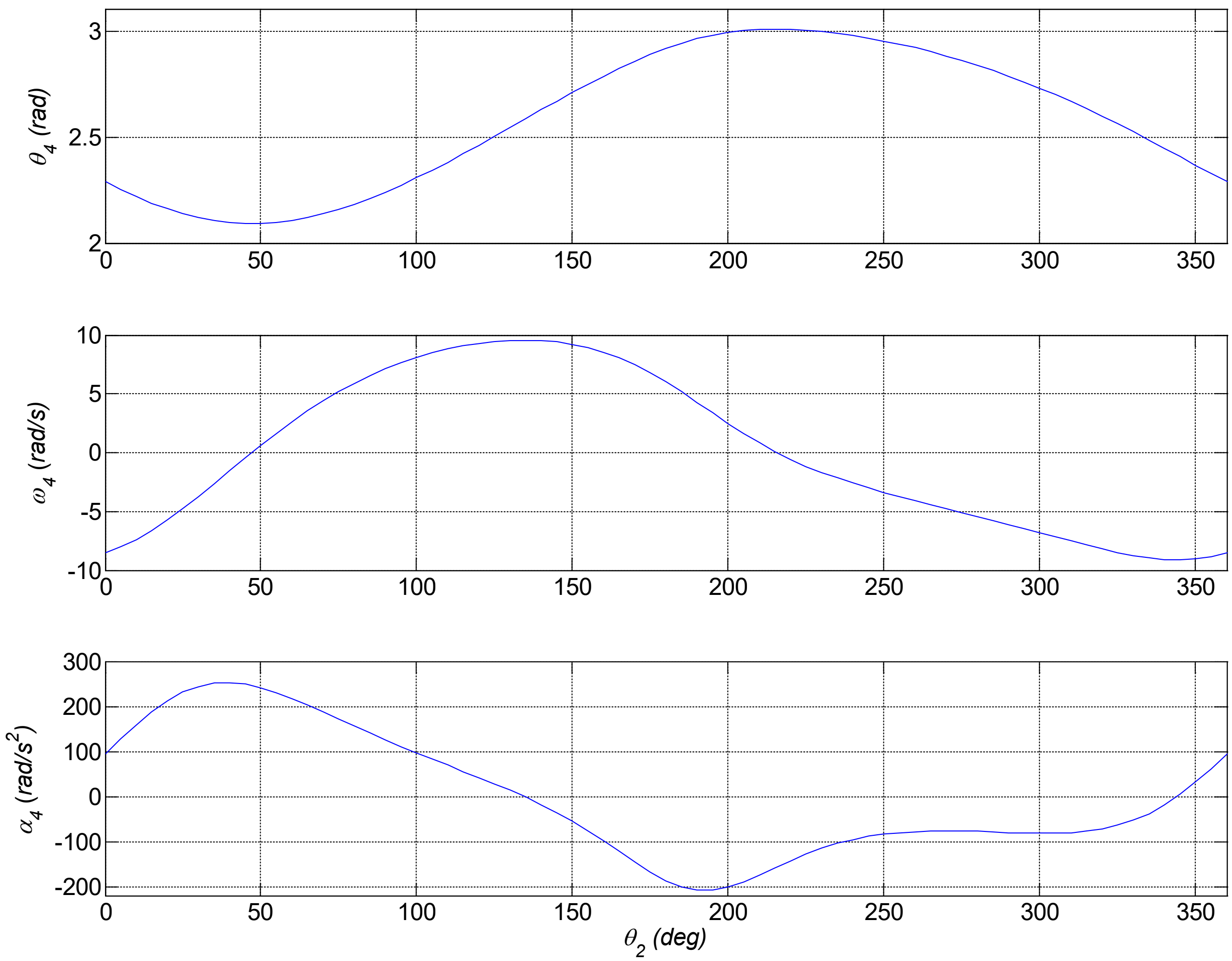
For example, dropping an object without air friction is subject to constant acceleration (g downward). In this case, starting from zero initial displacement and zero initial velocity we have $x(t)=at^2/2$ and $v(t)=at$, where a is g, the acceleration due to gravity, in this example. Eliminating explicit time t between these two equations yields a familiar equation, $v(t)=\sqrt{2ax(t)}$.

Derivative/Integral Kinematic Relationships

When one variable is the derivative of another, recall the relationships from calculus. For example (from a four-bar mechanism):

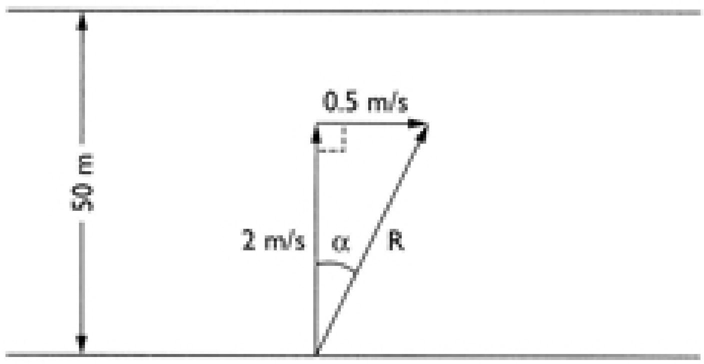
$$\omega_4 = \frac{d\theta_4}{dt} \qquad\qquad \omega_4 = \omega_{40} + \int \alpha_4 dt$$

$$\alpha_4 = \frac{d\omega_4}{dt} = \frac{d^2\theta_4}{dt^2} \qquad\qquad \theta_4 = \theta_{40} + \int \omega_4 dt = \theta_{40} + \omega_{40}t + \int\left(\int \alpha_4 dt\right)dt$$



<u>Example 1</u>

A swimmer attempts to swim perpendicular to the parallel banks of a river that is 50 m wide. If the swimmer's speed is 2 m/s and the downriver current is 0.5 m/s (both constant), what is her resultant velocity? What actual distance must she swim to reach the other side? How much time is required to reach the other side? How much time is required to reach the other side if there is no downstream current?



not to scale; *Hall (2007)*

Solution

The resultant velocity is found in the problem statement – if the downriver direction is $+X$ and the perpendicular to the banks is $+Y$, the resultant swimmer velocity is:

$$\mathbf{R} = \underline{v} = 0.5\hat{i} + 2\hat{j} \quad \text{m/s}$$

The polar coordinates (magnitude and direction) for this resultant velocity are:

$$|\mathbf{R}| = v = \sqrt{0.5^2 + 2^2} = 2.06 \quad \text{m/s}$$

$$\alpha = \tan^{-1}\left(\frac{0.5}{2}\right) = 14^\circ$$

The actual distance d (at the angle α) and time t swum to reach the other side is (the current does not affect the time):

$$d\cos\alpha = 50 \qquad\qquad d = \frac{50}{\cos 14^\circ} = 51.5 \quad \text{m}$$

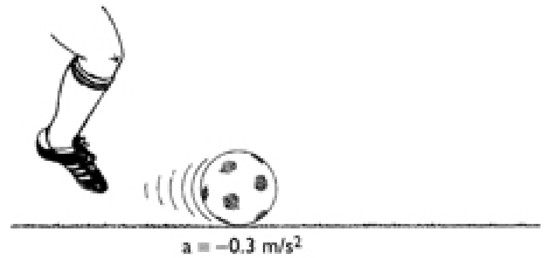
$$t = \frac{51.5}{2.06} = \frac{50}{2} = 25 \quad \text{sec}$$

And the (unwanted) downstream distance is:

$$D = d\sin\alpha = 12.5 \quad \text{m}$$

Example 2

A soccer ball is rolling down the field. If the ball has an initial velocity of 4 m/s and a constant deceleration of –0.3 m/s^2, calculate the time required for the ball to come to a stop. Also calculate the distance travelled by the ball during this time.



Hall (2007)

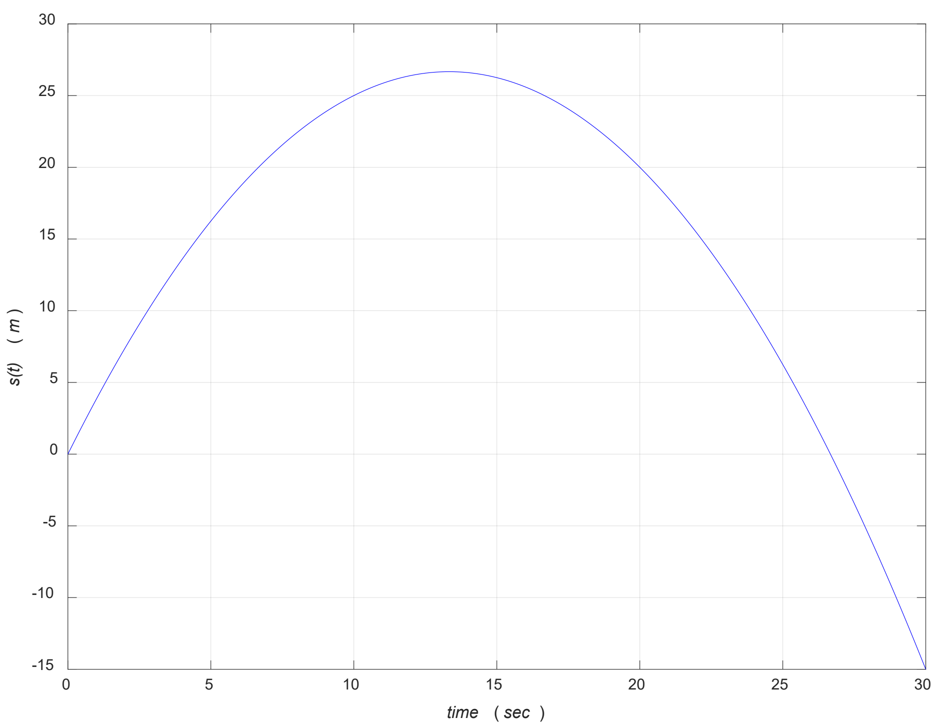
Solution

For constant acceleration a, the following applies

$$a = \frac{v_f - v_0}{t_f - t_0} \qquad -0.3 = \frac{0-4}{t_f - 0} \qquad t_f = \frac{-4}{-0.3} = 13.3 \quad \text{sec}$$

For constant acceleration a, the distance traveled is:

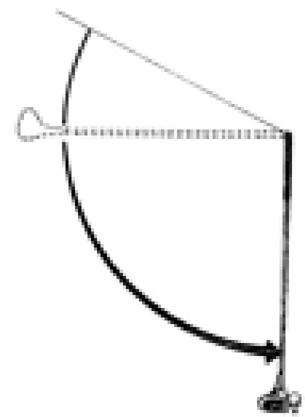
$$s(t) = s_0 + v_0 t + \frac{1}{2} a t^2 \qquad s(13.3) = 0 + 4(13.3) - \frac{0.3}{2}(13.3)^2 = 26.7 \quad \text{m}$$



As seen in the parabola plot above for soccer ball displacement $s(t)$, the distance starts at 0, with a positive slope (4 m/s initial velocity). Also, the ball stops at $t_f = 13.3$ sec, at a maximum distance of 26.7 m (flat slope, zero velocity). If the constant negative acceleration (deceleration) could continue, the displacement would actually reverse after that point in time.

Example 3

A golf club is swung from an initial angle from rest with a constant angular acceleration of $\alpha = 2.5$ rad/s^2. If the swing time is 1 sec (from start to contact), calculate the angular velocity of the club head as it contacts the ball. Assume the club is vertical (down) when it strikes the ball. What was the initial angle? Use a right-handed angle convention as shown, from the observer's perspective.



Hall (2007)

Solution

For constant acceleration α, the following applies.

$$\alpha = \frac{\omega_f - \omega_0}{t_f - t_0} \qquad 2.5 = \frac{\omega_f - 0}{1 - 0} \qquad \omega_f = 2.5(1) = 2.5 \quad \text{rad/s}$$

The given final angle where the club meets the ball is $3\pi/2$ rad (270°, vertically down). For constant acceleration α, the angular displacement equation is:

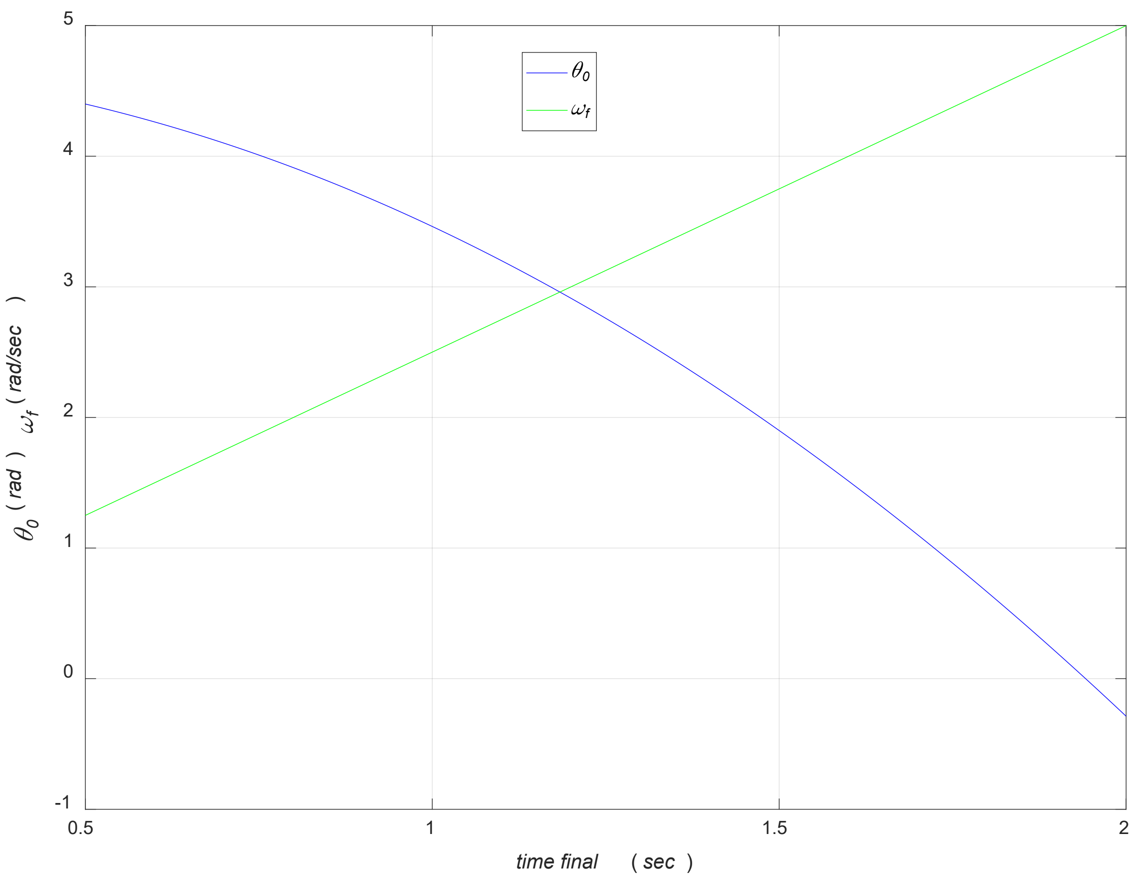
$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \qquad \theta(1) = \frac{3\pi}{2} = \theta_0 + 0(1) + \frac{1}{2}(2.5)(1)^2 \qquad \theta_0 = 3.46$$

Thus, the initial angle is $\theta_0 = 3.46$ rad ($\theta_0 = 198.4^\circ$), i.e. 18.4° down beyond horizontal to the right side of a right-handed golfer (to the left for the observer in the above figure).

The initial problem in Hall (not shown here) turned out to have a tiny initial angle (about 265°, or 5° away from 270°), so Dr. Bob re-designed this problem for a more-full golf swing. True golfers start their standard swing well beyond this near-horizontal location, beyond the upward vertical, say at an initial angle of 30° (0.52 rad). What is the required swing time and final club angular velocity for this more-realistic initial angle, assuming the same constant $\alpha = 2.5$ rad/s^2?

1.83 sec and **4.58 rad/s**

Assuming you can maintain directional control, a greater windup angle on your pre-swing will impart a greater energy to your drive via a greater club head speed $v_T = r\omega$.



As seen in the plot above for initial club angle θ_0, θ_0 increases parabolically (smaller angles further from the vertical down) as final time t_f increases (a longer swing requires a larger angle). Also, as seen in the plot above for final club angular velocity ω_f, ω_f increases linearly with increasing t_f. Both data points from the example may be clearly seen in the plots:

$t_f = 1$ sec	$\theta_0 = 3.46$ rad	$\omega_f = 2.5$ rad/sec
$t_f = 1.83$ sec	$\theta_0 = 0.52$ rad	$\omega_f = 4.58$ rad/sec

4.1.3 Planar Kinematics

Planar Motion Description

We need to describe the planar position and orientation (**pose**) of human body segments and points. We attach a separate, independent right-handed Cartesian coordinate frame to each moving body segment of interest. These frames are fixed to the moving body segment and are moved relative to each other via active joints. Here is a figure of moving frame $\{B\}$ relative to reference frame $\{A\}$.

${}^{A}\mathbf{P}_{B}$ and ϕ are required to describe the position (translations) and orientation (rotation) of $\{B\}$ with respect to $\{A\}$ – together this is called the **pose**.

pose	$X = \{x \quad y \quad \phi\}^T$	
velocity	$\dot{X} = \{\dot{x} \quad \dot{y} \quad \dot{\phi}\}^T$	What is the next time derivative?
acceleration	$\ddot{X} = \{\ddot{x} \quad \ddot{y} \quad \ddot{\phi}\}^T$	

Parabolic Motion Horizontal (constant velocity); Vertical (constant acceleration ***g***).

top of Half Dome, about 5,000 feet above Yosemite Valley

Body Segment in Pure Rotation

This derivation requires the use of the product and chain rules of time differentiation. Here is a brief review of these principles, with examples.

Product rule

$$\frac{d}{dt}(x(t)y(t)) = \frac{dx(t)}{dt}y(t) + x(t)\frac{dy(t)}{dt}$$

x, y are both functions of time t.

Chain rule

$$\frac{d}{dt}(f(x(t))) = \frac{df(x(t))}{dx(t)}\frac{dx(t)}{dt}$$

f is a function of *x*, which is a function of *t*.

Velocity Examples

$$\begin{aligned}\frac{d}{dt}(L\cos\theta) &= \dot{L}\cos\theta + L\frac{d}{dt}(\cos\theta)\\ &= \dot{L}\cos\theta + L\frac{d\cos\theta}{d\theta}\frac{d\theta}{dt}\\ &= \dot{L}\cos\theta + L(-\sin\theta)\dot{\theta}\\ &= 0 \quad -L\omega\sin\theta\end{aligned}$$

$$\begin{aligned}\frac{d}{dt}(L\sin\theta) &= \dot{L}\sin\theta + L\frac{d}{dt}(\sin\theta)\\ &= \dot{L}\sin\theta + L\frac{d\sin\theta}{d\theta}\frac{d\theta}{dt}\\ &= \dot{L}\sin\theta + L(\cos\theta)\dot{\theta}\\ &= 0 \quad +L\omega\cos\theta\end{aligned}$$

where $\dot{L} = \frac{dL}{dt}$ $\qquad \omega = \dot{\theta} = \frac{d\theta}{dt}$

Note in all human body segments, since there are no 1-dof prismatic joints, segment length L is a constant and thus $\dot{L}(t) = 0$. But note, in 3D motions with 2-dof planar joints, there will be sliding velocities.

Acceleration Examples

$$\begin{aligned}\frac{d}{dt}(-L\omega\sin\theta) &= -L\alpha\sin\theta - L\omega\frac{d}{dt}(\sin\theta)\\ &= -L\alpha\sin\theta - L\omega\frac{d\sin\theta}{d\theta}\frac{d\theta}{dt}\\ &= -L\alpha\sin\theta - L\omega(\cos\theta)\omega\\ &= -L\alpha\sin\theta - L\omega^2\cos\theta\end{aligned}$$

$$\begin{aligned}\frac{d}{dt}(L\omega\cos\theta) &= L\alpha\cos\theta + L\omega\frac{d}{dt}(\cos\theta)\\ &= L\alpha\cos\theta + L\omega\frac{d\cos\theta}{d\theta}\frac{d\theta}{dt}\\ &= L\alpha\cos\theta - L\omega(\sin\theta)\omega\\ &= L\alpha\cos\theta - L\omega^2\sin\theta\end{aligned}$$

where

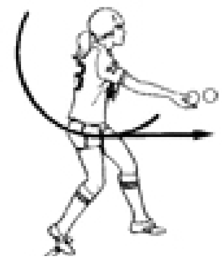
$$\begin{aligned}\omega &= \dot{\theta} = \frac{d\theta}{dt}\\ \alpha &= \dot{\omega} = \frac{d\omega}{dt} = \ddot{\theta} = \frac{d^2\theta}{dt^2}\end{aligned}$$

and θ, ω, and α are all functions of time.

Body Segment in Pure Rotation position, velocity, and acceleration derivation:

<u>Example 4</u>

A fast-pitch, windmill-style softball pitcher performs a pitch in 0.65 sec. If her arm is 0.7 m long and the tangential velocity of the softball is 20 m/s (horizontal) at release, what are the vector tangential and radial accelerations of her hand at the instant of release? Also find the magnitude and direction of the total acceleration. Assume she starts the pitch from rest and there is constant angular acceleration.



Hall (2007)

Solution

Tangential acceleration is horizontal (to the right) at the instant of release; assuming the initial tangential velocity is zero, the magnitude of the tangential acceleration at this instant is:

$$a_T = \frac{v_2 - v_1}{\Delta t} = \frac{20-0}{0.65} = 30.8 \quad \frac{\text{m}}{\text{s}^2}$$

Note that another expression for tangential acceleration is $a_T = r\alpha$, where r is the arm length and α is the angular acceleration at the instant considered. So $\alpha = 44.0$ rad/sec^2 in this problem.

The expression for radial, or centripetal acceleration is $a_R = r\omega^2$, directed from the ball inwards towards the center of rotation (the shoulder joint), where r is again the arm length and ω is the angular velocity at the instant considered. Since the expression for tangential velocity is $v_T = r\omega$, the radial acceleration magnitude may be calculated as follows.

$$a_R = r\omega^2 = r\left(\frac{v_T}{r}\right)^2 = \frac{v_T^2}{r} = \frac{20^2}{0.7} = 571.4 \quad \frac{\text{m}}{\text{s}^2}$$

This radial acceleration vector is vertical (up) at the instant of release.

The total absolute acceleration of the softball at the instant of release is the vector sum of the tangential and radial acceleration vectors. This result is calculated below in polar coordinates.

magnitude

$$a = \sqrt{a_T^2 + a_R^2} = \sqrt{30.8^2 + 571.4^2} = 572.2 \quad \frac{\text{m}}{\text{s}^2}$$

direction (up from the horizontal)

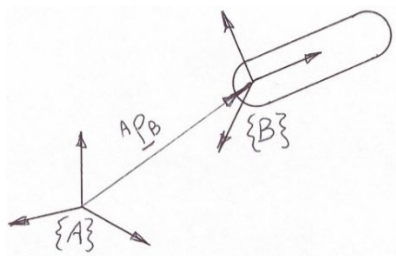
$$\phi = \tan^{-1}\left(\frac{a_R}{a_T}\right) = \tan^{-1}\left(\frac{571.4}{30.8}\right) = 86.9^\circ$$

In this example adapted from Hall (2007), the acceleration is in excess of 58*g*! Can the human arm withstand this level of acceleration? The centripetal acceleration is what leads to this high acceleration, but the tangential velocity is reasonable (20 m/s is about 45 mph; the fast-pitch record is about 72 mph). This centripetal acceleration is independent of the pitch time. For a general discussion on human tolerance of acceleration, see Section 4.1.5.

4.1.4 Spatial Kinematics

Motion Description

We need to describe the spatial position & orientation (pose) of human body segments and points. We attach a separate, independent right-handed Cartesian coordinate frame to each moving body segment of interest. These frames are fixed to the moving body segment and are moved relative to each other via active joints. Here is a figure of moving frame $\{B\}$ relative to reference frame $\{A\}$.



$^{A}P_{B}$ and $^{A}_{B}R$ are required to describe the pose (position and orientation) of $\{B\}$ with respect to $\{A\}$.

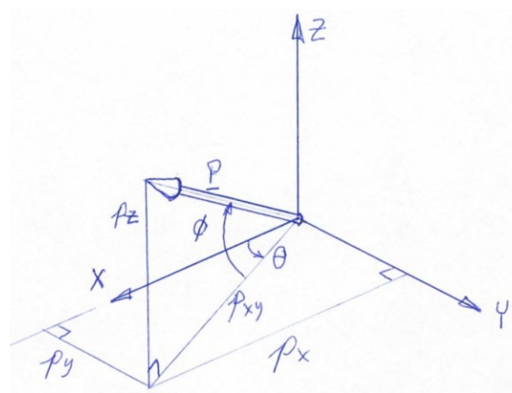
Pose $X = \{x \quad y \quad z \quad yaw \quad pitch \quad roll\}^T$

Velocity $\dot{X} = \{\dot{x} \quad \dot{y} \quad \dot{z} \quad \omega_x \quad \omega_y \quad \omega_z\}^T$

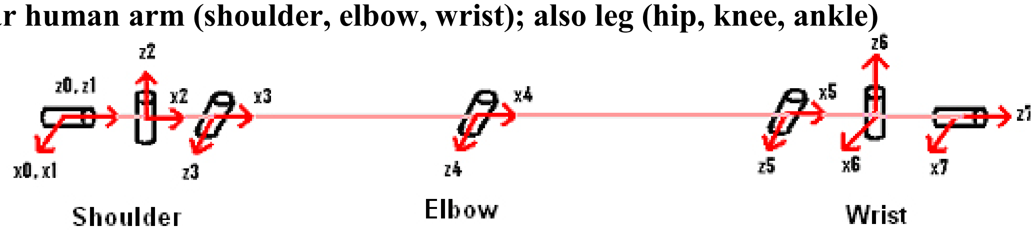
Acceleration $\ddot{X} = \{\ddot{x} \quad \ddot{y} \quad \ddot{z} \quad \alpha_x \quad \alpha_y \quad \alpha_z\}^T$

Velocity and acceleration are vector representations, both position and orientation. The 3D representation of orientation (rotations) cannot be expressed as a vector. Instead we can use orthonormal rotation matrices (as in EE/ME 4290/5290 Robotic Manipulators).

3D Projection of a vector onto a coordinate system

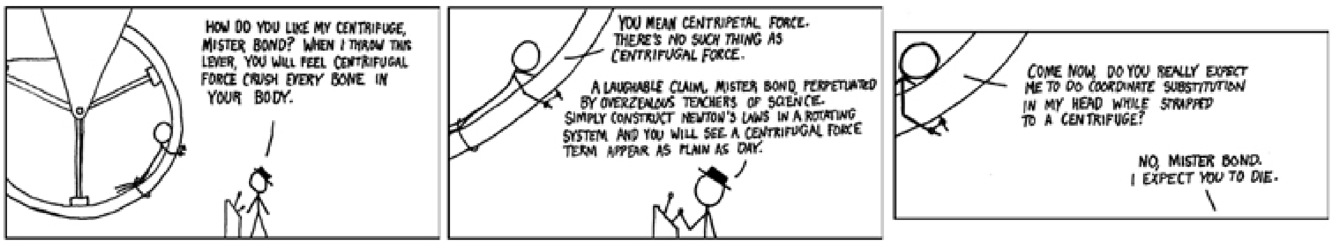


7-dof planar human arm (shoulder, elbow, wrist); also leg (hip, knee, ankle)



The shoulder and wrist joints can be modeled as spherical, i.e. no offsets between the three R joints.

4.1.5 Acceleration Tolerance by Humans



Physics Humor from xkcd

xkcd.com

- Humans do not sense velocity (except by sight).

- Humans are very sensitive to acceleration (change in velocity) via semicircular canals in the inner ear.

- In a commercial jet at a constant speed of 500 mph, there is no sensation of motion; however, changes in velocity (accelerations) due to turbulence are immediately sensed.

- Even with 9 derivatives of position (velocity, acceleration, jerk, snap, crackle, pop, . . .) set to zero at the beginning and end of an elevator ride (in a skyscraper, not Stocker Center), the human can sense acceleration. Going up, you are first heavier, and then lighter, with the acceleration and deceleration, respectively.

- Acceleration sensations are familiar in driving a car or riding a bicycle – cornering, rapid ramping up and down from constant velocities.

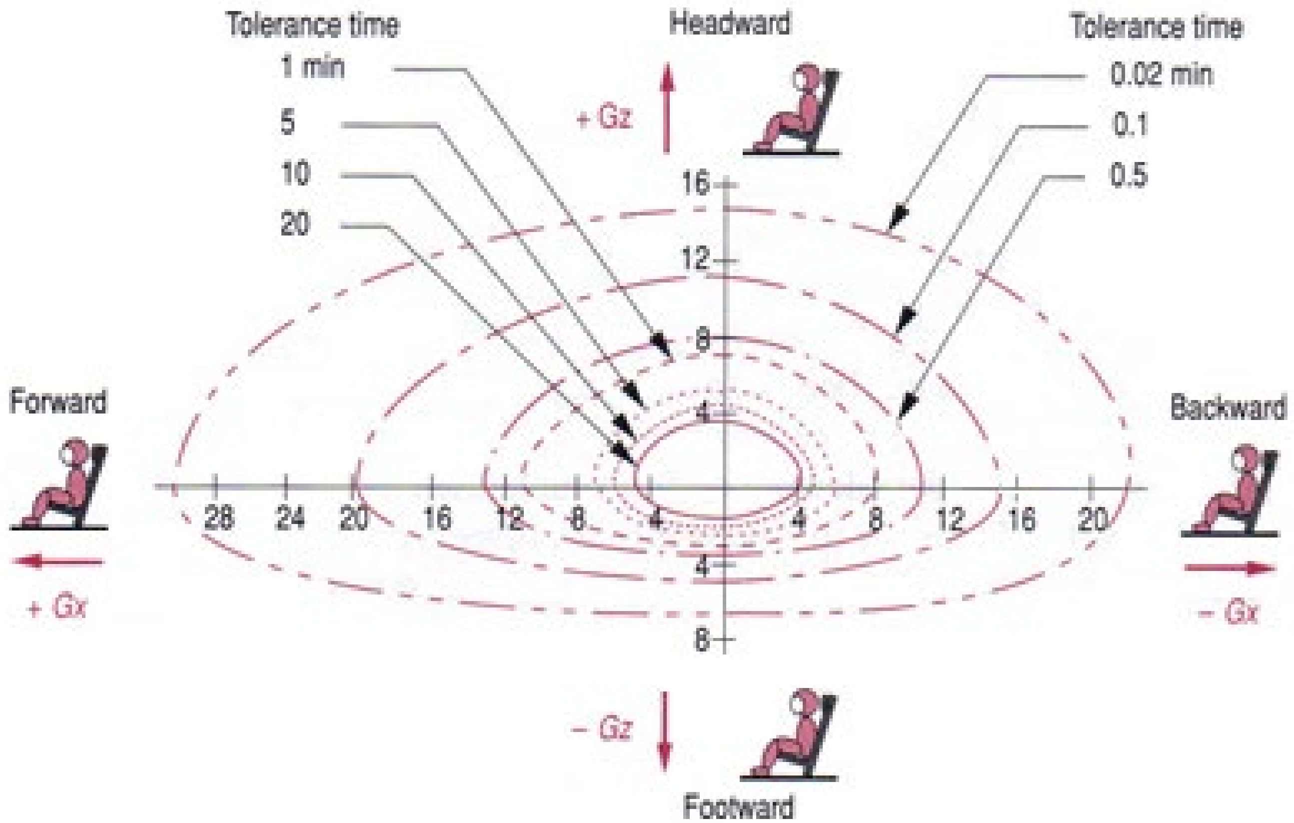
- Some state laws limit roller coaster accelerations to 5.5*g* (5.5 times the acceleration due to gravity; weight is 1*g*).

Norton (2008)

Dynamic forces ($\boldsymbol{F} = m\boldsymbol{A}$) due to excessive or prolonged accelerations $\boldsymbol{A}$ may be harmful – the human body is not rigid but a loosely-coupled bag of water and tissue that is internally mobile.

The military and NASA have done research to determine the limits of voluntary exposure to high-*g* environments. The figure below shows some measured *g*-limits in different directions for various exposure times.

Jet fighter pilots wear a gravity suit to ensure adequate blood supply to the brain continuously. Zero-*g* is not good for humans either; astronauts exercise 2 hours per day just to minimize muscle loss due to lack of gravity. This is the subject of much biomechanical research, the response of humans to microgravity environments.



Average levels of linear acceleration, in different directions that can be tolerated on a voluntary basis for specified periods. Each curve shows the average *G* load that can be tolerated for the time indicated. The data points obtained were actually those on the axes; the lines as such are extrapolated from the data points to form the concentric figures.

Typical Accelerations in Human Activities

Activity	Acceleration (*g*)
gentle car acceleration	+0.1
commercial jet takeoff	+0.3
hard car acceleration	+0.5
panic car stop	–0.7
sports car cornering	+0.9
formula 1 race car	+2.0, –4.0
roller coasters	±3.5 to ±6.5
space shuttle launch	+4.0
dragster with drogue chute	±4.5
military jet fighter	±9.0

Norton (2008)

By contrast, machines without humans are subject to *g*-limitations only constrained by their geometry, materials, actuators, etc. For example, the piston acceleration in a four-cylinder economy car is about 40*g* at idle, 700*g* at highway speeds, and 2000*g* at 6000 rpm (top engine speed). Engines are designed to last a long time at extremely high number of fatigue cycles in spite of these high accelerations.

4.1.6 Kinematics Problems

Kinematics of the human body is concerned with formulating and solving for the translational and rotational position, velocity, and acceleration analysis problems for each human body segment of interest, for various real-world motions.

Forward pose kinematics calculates the pose (position and orientation) of each human body segment of interest given the joint angles. **Inverse pose kinematics** calculates the required joint angles given the current human body (or portion thereof) pose.

Statics requires the positions and angles of each segment for static free-body diagrams.

Dynamics requires the translational and rotational position, velocity, and acceleration variables for each human body segment, plus the CG translational accelerations, for dynamic free body diagrams.

Inverse dynamics calculates the joint torques given all translational and rotational kinematics terms through acceleration. The joint torques must then be resolved into muscle forces. **Forward dynamics**, a much harder mathematical problem, calculates the unknown kinematics terms given the muscle forces/joint torques; this requires the solution of coupled nonlinear differential equations.

The next section presents the kinematics problems and solutions for a simplified model of the human arm, which also serves as a simplified model for the human leg (with different joint limits).

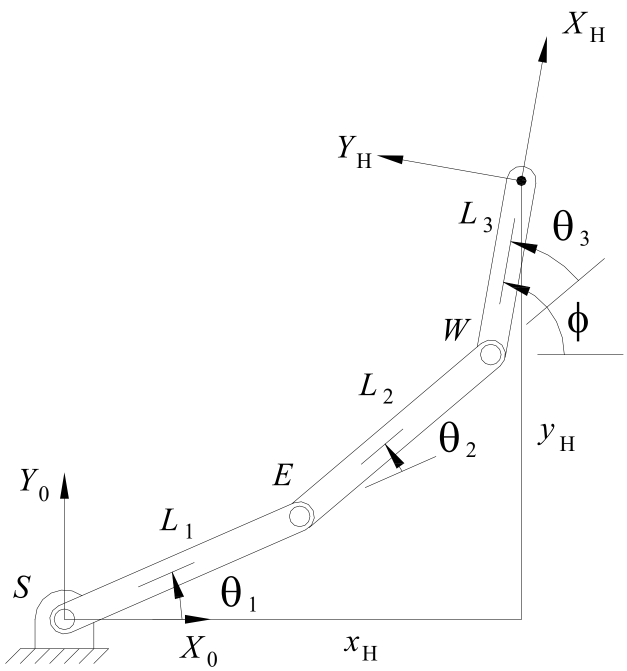
4.2 Human Arm Kinematics

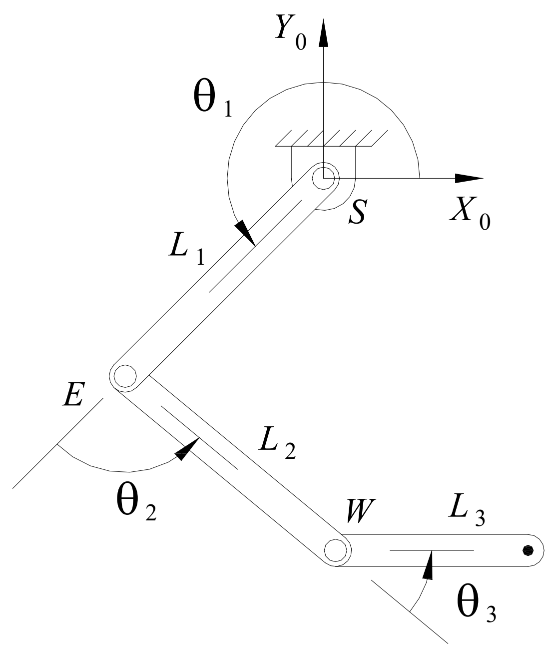
This section presents a simplified model for the kinematics (the study of motion without regard to forces) of the planar 3-dof human arm, with biceps and triceps at the elbow. We will use this simplified model for kinematics, statics, and dynamics examples. The methods are applicable to a wide range of human joints/motions/muscles.

Simplified Human Arm Model

The figures below show the simplified 3-dof planar human arm model with an R joint at the shoulder, elbow, and wrist pitch (flexion/extension) motions. This also serves as a simplified leg model, with the hip, knee, ankle joints, and leg lengths and joint limits substituted for the arm parameters.

The arm constants are: L_1 is the upper arm length, L_2 is the forearm length, and L_3 is the hand length. The joint variables are: θ_1 is the absolute shoulder pitch angle, θ_2 is the relative elbow pitch angle, and θ_3 is the relative wrist pitch angle. The Cartesian variables are x_H, y_H, and ϕ.





4.2.1 Forward Pose Kinematics (FPK) Solution

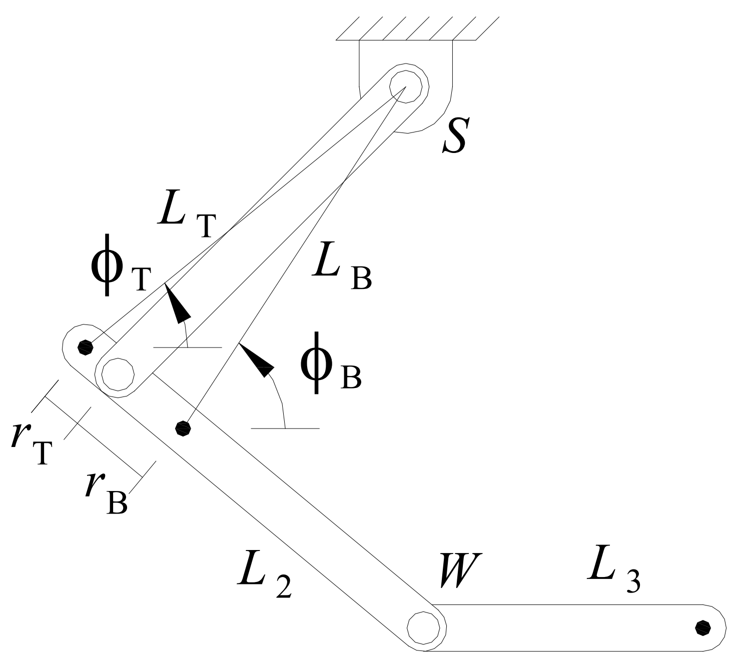
FPK Problem Statement

Given:

Find:

Here is the human arm FPK solution, using geometry and trigonometry.

Add simple straight-line models for the biceps and triceps muscles to actuate the elbow (we are not yet concerned with forces, just muscle insertion positions, and muscle angles and lengths).



The kinematics solutions for the biceps and triceps muscle lengths and angles are now presented.

biceps diagram

$$\underline{L}_1 + \underline{r}_B + \underline{L}_B = \underline{0}$$

$$\underline{L}_B = -\underline{L}_1 - \underline{r}_B$$

$$\underline{L}_B = \begin{Bmatrix} -L_1c_1 - r_Bc_{12} \\ -L_1s_1 - r_Bs_{12} \end{Bmatrix} = \begin{Bmatrix} L_{Bx} \\ L_{By} \end{Bmatrix}$$

$$L_B = \sqrt{L_{Bx}^2 + L_{By}^2}$$

$$L_B = \sqrt{L_1^2 + r_B^2 + 2L_1r_B\cos\theta_2}$$

alternate: **`LB = norm([LBx; LBy]);`**

$$\phi_B = \tan^{-1}\left(\frac{-L_1s_1 - r_Bs_{12}}{-L_1c_1 - r_Bc_{12}}\right)$$

$$\phi_B = \text{atan2}\left(-L_1s_1 - r_Bs_{12}, -L_1c_1 - r_Bc_{12}\right)$$

triceps diagram

$$\underline{L}_1 + \underline{r}_T + \underline{L}_T = \underline{0}$$

$$\underline{L}_T = -\underline{L}_1 - \underline{r}_T$$

$$\underline{L}_T = \begin{Bmatrix} -L_1c_1 - r_Tc(\theta_{12} + \pi) \\ -L_1s_1 - r_Ts(\theta_{12} + \pi) \end{Bmatrix}$$

$$\underline{L}_T = \begin{Bmatrix} -L_1c_1 + r_Tc_{12} \\ -L_1s_1 + r_Ts_{12} \end{Bmatrix} = \begin{Bmatrix} L_{Tx} \\ L_{Ty} \end{Bmatrix}$$

$$L_T = \sqrt{L_{Tx}^2 + L_{Ty}^2}$$

$$L_T = \sqrt{L_1^2 + r_T^2 - 2L_1r_T\cos\theta_2}$$

alternate: **`LT = norm([LTx; LTy]);`**

$$\phi_T = \tan^{-1}\left(\frac{-L_1s_1 + r_Ts_{12}}{-L_1c_1 + r_Tc_{12}}\right)$$

$$\phi_T = \text{atan2}\left(-L_1s_1 + r_Ts_{12}, -L_1c_1 + r_Tc_{12}\right)$$

where:

$$c_1 = \cos\theta_1$$

$$s_1 = \sin\theta_1$$

$$c_{12} = \cos(\theta_1 + \theta_2)$$

$$s_{12} = \sin(\theta_1 + \theta_2)$$

MATLAB function **`atan2`** returns the correct quadrant-specific inverse tangent angle considering the ***signs*** of the numerator and denominator.

Human Arm FPK Examples

From Appendix D, we use the following segment lengths (m).

Subject	L_1 (upper arm)	L_2 (forearm)	L_3 (hand)	r_B	r_T
adult male	0.315	0.287	0.105	0.057	0.029
adult female	0.272	0.252	0.091	0.050	0.025

Snapshot FPK Examples

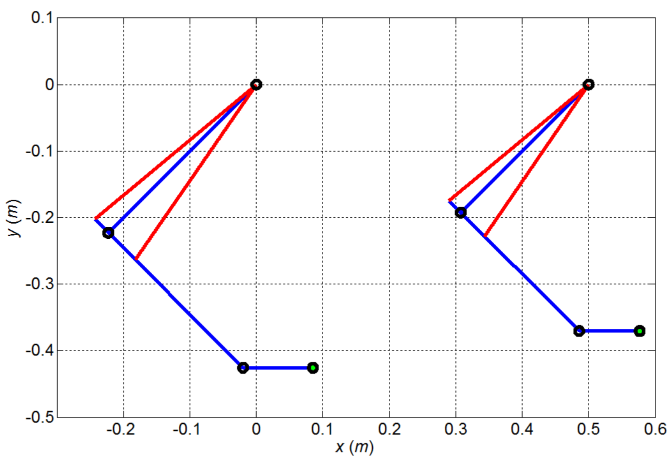
Given $\theta_1 = -135^\circ$, $\theta_2 = 90^\circ$, $\theta_3 = 45^\circ$, the unique FPK solutions are

$$X_{\text{male}} = \begin{bmatrix} x_H & y_H & \phi \end{bmatrix} = \begin{bmatrix} 0.085 & -0.426 & 0^\circ \end{bmatrix}$$
$$X_{\text{female}} = \begin{bmatrix} x_H & y_H & \phi \end{bmatrix} = \begin{bmatrix} 0.077 & -0.371 & 0^\circ \end{bmatrix} \quad (\text{m})$$

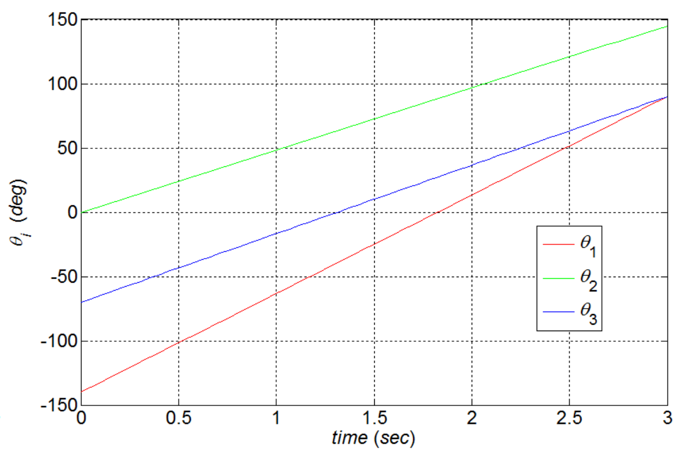
The associated muscle lengths and angles are (given r_{Bm}=0.057, r_{Tm}=0.029, r_{Bf}=0.050, r_{Tf}=0.025 m)

$$L_{\text{male}} = \begin{bmatrix} L_{Bm} & L_{Tm} \end{bmatrix} = \begin{bmatrix} 0.320 & 0.316 \end{bmatrix}$$
$$L_{\text{female}} = \begin{bmatrix} L_{Bf} & L_{Tf} \end{bmatrix} = \begin{bmatrix} 0.276 & 0.273 \end{bmatrix} \quad (m)$$

$$\phi_{\text{male}} = \begin{bmatrix} \phi_{Bm} & \phi_{Tm} \end{bmatrix} = \begin{bmatrix} 55.3^\circ & 39.8^\circ \end{bmatrix}$$
$$\phi_{\text{female}} = \begin{bmatrix} \phi_{Bf} & \phi_{Tf} \end{bmatrix} = \begin{bmatrix} 55.5^\circ & 39.7^\circ \end{bmatrix}$$



FPK Snapshot Example Poses (male left, female right)



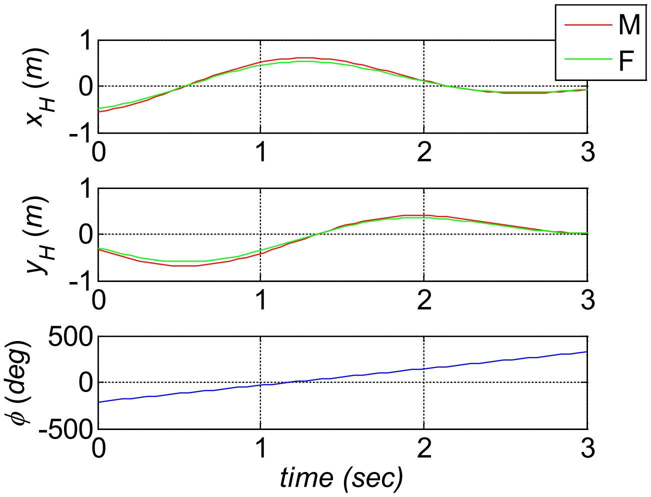
Trajectory Input Angles

Trajectory FPK Examples

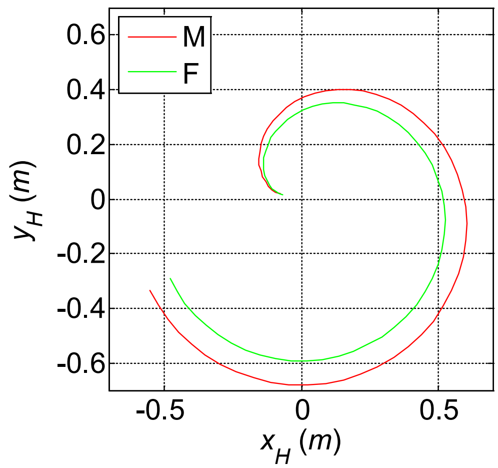
We simulate FPK for the entire range of motion from the minimum to maximum angle on each joint simultaneously. From Appendix B we use the following angle limits (for both male and female). The FPK input angles are given in the right plot above.

Angle	Min	Max
θ_1	-140°	90°
θ_2	0°	145°
θ_3	-70°	90°

The results of this human arms FPK trajectory motion simulation are given in the next six plots.

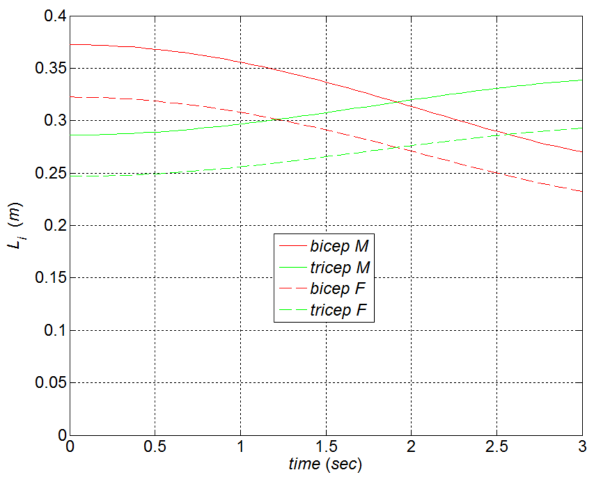


FPK Results (x_H, y_H, ϕ vs. t)

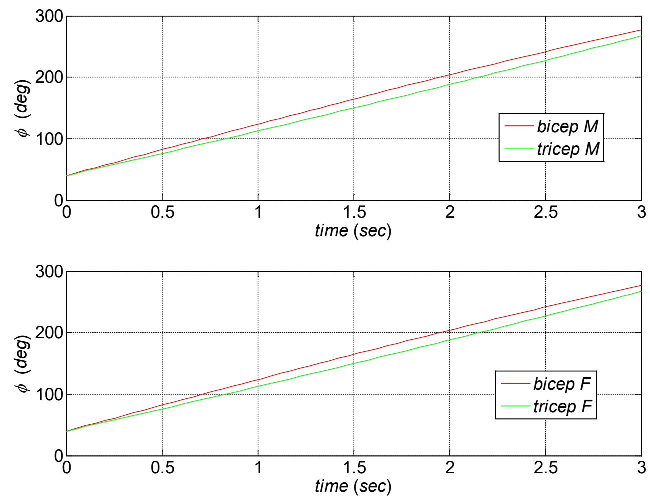


FPK Results (y_H vs. x_H)

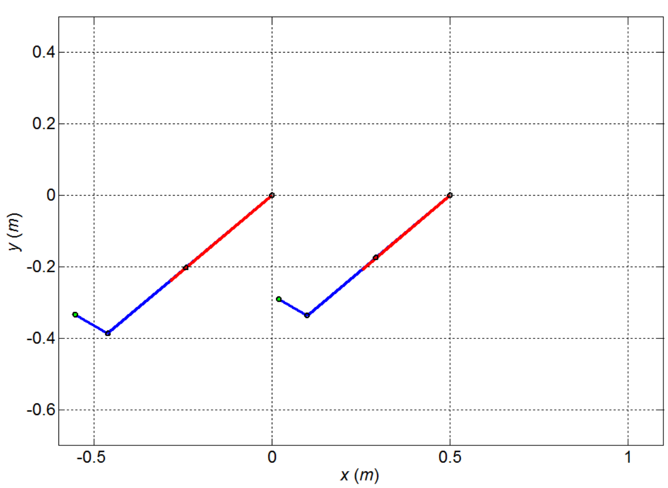
any resemblance to the Fibonacci spiral is purely coincidental (or is it?)



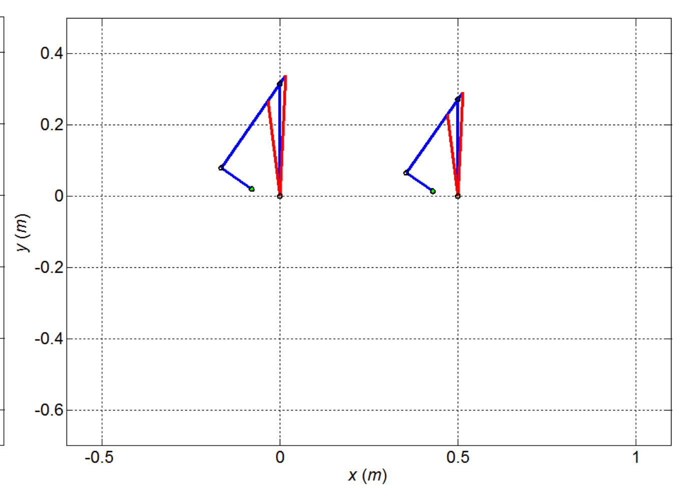
Simulated Muscle Lengths



Simulated Muscle Angles



Initial Poses (male left, female right)

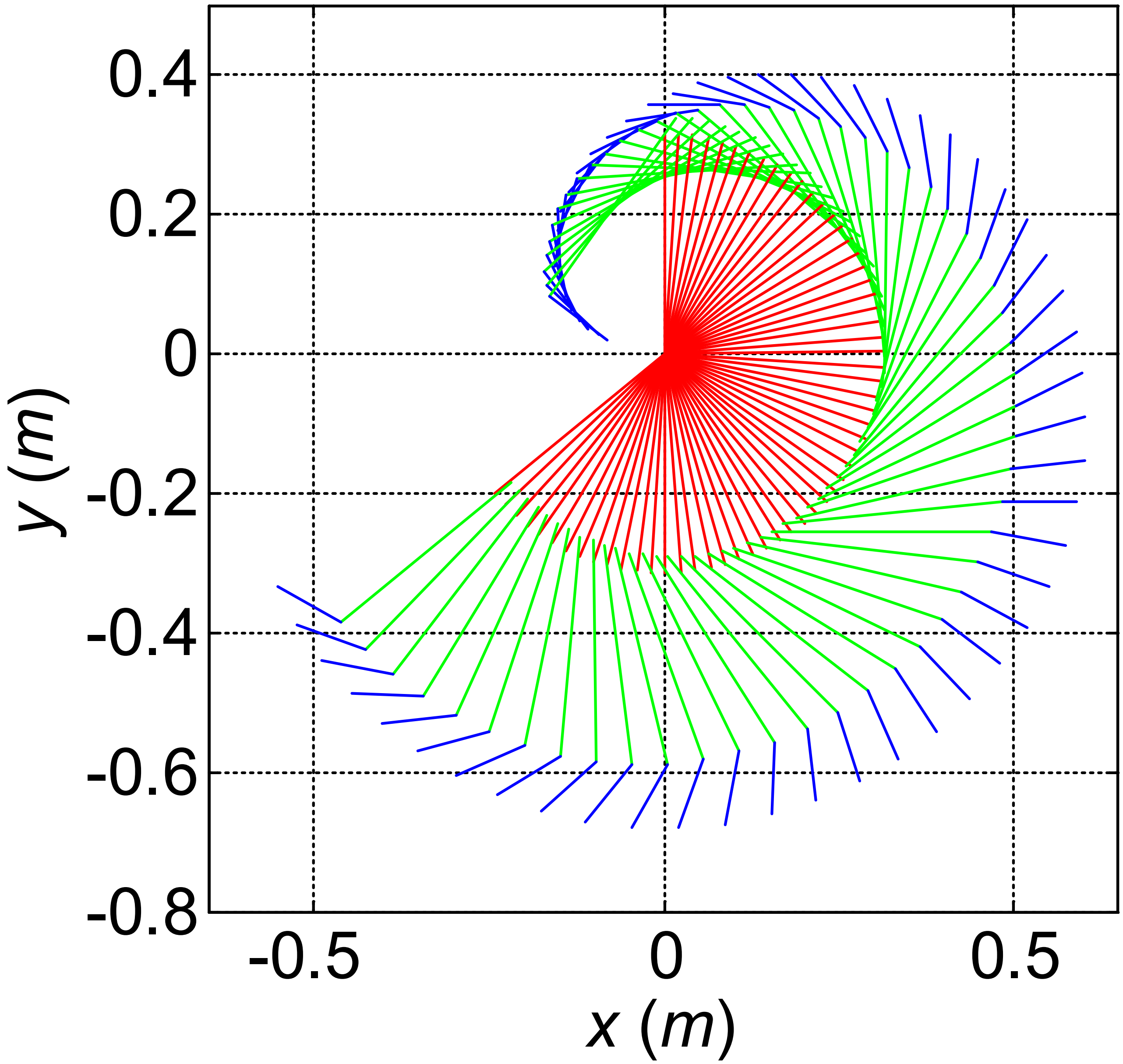


Final Poses (male left, female right)

The MATLAB program **`FPK.m`** that was used to generate the male FPK results and animate the arm to the screen is given on the next two pages.

```
%---------------------------------------------------------------------
%   FPK.m
%       Forward pose kinematics trajectory for planar 3R
%           adult male human arm shoulder, elbow, wrist
%                 ME 4670 / 5670, Dr. Bob
%---------------------------------------------------------------------
clc; clear;
%  Constants
DR = pi/180;
L = [0.315 0.287 0.105];  L1 = L(1); L2 = L(2); L3 = L(3); % male arm lengths

rB =  ((3/8)/(1+7/8))*L2;  %  biceps insertion length - scaled from a figure
rT = ((3/16)/(1+7/8))*L2;  % triceps insertion length - scaled from a figure

th1lim = [-140  90]*DR;    % shoulder pitch joint absolute angle limits
th2lim = [   0 145]*DR;    %  elbow   pitch joint relative angle limits
th3lim = [ -70  90]*DR;    %  wrist   pitch joint relative angle limits

th10 = th1lim(1);  th1f = th1lim(2);
th20 = th2lim(1);  th2f = th2lim(2);
th30 = th3lim(1);  th3f = th3lim(2);

N = 60;
dth1 = (th1f-th10)/(N-1);
dth2 = (th2f-th20)/(N-1);
dth3 = (th3f-th30)/(N-1);

th1 = [th10:dth1:th1f];
th2 = [th20:dth2:th2f];
th3 = [th30:dth3:th3f];

t0 = 0;  tf = 3;    % Artificial time vector
dt = (tf-t0)/(N-1);
t  = [t0:dt:tf];

%
%  Loop for FPK motion simulation
%
figure;
for i = 1:N,
    %
    %  Forward Pose Kinematics
    %
    c1   = cos(th1(i));
    s1   = sin(th1(i));
    c12  = cos(th1(i)+th2(i));
    s12  = sin(th1(i)+th2(i));
    c123 = cos(th1(i)+th2(i)+th3(i));
    s123 = sin(th1(i)+th2(i)+th3(i));

    xH(i) = L1*c1 + L2*c12 + L3*c123;     % FPK solution
    yH(i) = L1*s1 + L2*s12 + L3*s123;
    phi(i)  = th1(i)+th2(i)+th3(i);

    xb(i) = L1*c1 + rB*c12;               % biceps insertion point
    yb(i) = L1*s1 + rB*s12;
    xt(i) = L1*c1 - rT*c12;               % triceps insertion point
    yt(i) = L1*s1 - rT*s12;
```

```
    %  biceps and triceps muscle lengths
    Lb(i) = norm([xb(i) yb(i)]);  Lt(i) = norm([xt(i) yt(i)]);

    %  biceps and triceps muscle angles
    phib(i) = atan2(-yb(i),-xb(i));  phit(i) = atan2(-yt(i),-xt(i));

    %
    %  Animate human arm
    x1 = [0                          L1*c1];
    y1 = [0                          L1*s1];
    x2 = [xt(i)             L1*c1 + L2*c12];
    y2 = [yt(i)             L1*s1 + L2*s12];
    x3 = [L1*c1 + L2*c12             xH(i)];
    y3 = [L1*s1 + L2*s12             yH(i)];
    xb = [0                          xb(i)];
    yb = [0                          yb(i)];
    xt = [0                          xt(i)];
    yt = [0                          yt(i)];

    plot(x1,y1,'b',x2,y2,'b',x3,y3,'b',xb,yb,'r',xt,yt,'r'); grid;
    axis('square'); axis([-0.7 0.7 -0.7 0.7]);
    set(gca,'FontSize',18); xlabel('{\itx} ({\itm})'); ylabel('{\ity} ({\itm})');
    pause(dt);
    if i==1
        pause;
    end
end
%
%  Plots
figure;                 %  input angles trajectory plot
plot(t,th1/DR,'r',t,th2/DR,'g',t,th3/DR,'b'); grid;
set(gca,'FontSize',18);
xlabel('{\ittime} ({\itsec})'); ylabel('{\it\theta_i}  ({\itdeg})');
legend('{\it\theta}_1','{\it\theta}_2','{\it\theta}_3');

figure;                 %  Cartesian FPK results x, y, phi plots
subplot(311)
plot(t,xH,'b'); grid;
set(gca,'FontSize',18); ylabel('{\itx} ({\itm})');
subplot(312)
plot(t,yH,'b'); grid;
set(gca,'FontSize',18); ylabel('{\ity} ({\itm})');
subplot(313)
plot(t,phi/DR,'b'); grid; set(gca,'FontSize',18);
xlabel('\ittime ({\itsec})'); ylabel('{\it\phi} ({\itdeg})');

figure;                 %  Cartesian FPK hand tip trajectory
plot(xH,yH,'b'); grid;
axis('square'); axis([-0.7 0.7 -0.7 0.7]);
set(gca,'FontSize',18); xlabel('{\itx} ({\itm})'); ylabel('{\ity} ({\itm})');

figure;                 %  muscle lengths plots (angle plots code not shown)
plot(t,Lb,'r',t,Lt,'g'); grid; set(gca,'FontSize',18);
xlabel('{\ittime} ({\itsec})'); ylabel('{\itL_i}  ({\itm})');
legend('\itbicep','\ittricep');
```



Adult Male Human Arm FPK Simulation with all Segments, 60 steps

Upper Arm, Forearm, Hand

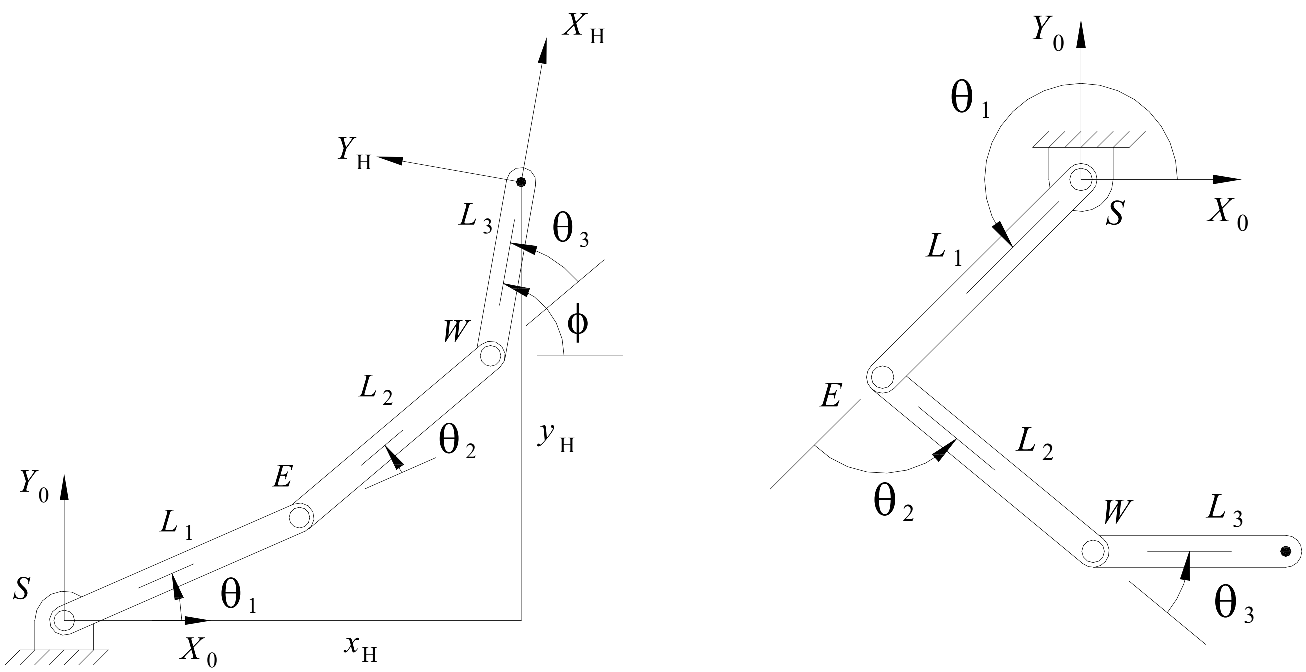
Biceps and triceps muscles omitted for clarity

4.2.2 Inverse Pose Kinematics (IPK) Solution

Simplified Human Arm Model

The figures below show the simplified 3-dof planar human arm model with an R joint at the shoulder, elbow, and wrist pitch (flexion/extension) motions. This also serves as a simplified leg model, with the hip, knee, ankle joints, and leg lengths and joint limits substituted for the arm parameters.

The arm constants are: L_1 is the upper arm length, L_2 is the forearm length, and L_3 is the hand length. The joint variables are: θ_1 is the absolute shoulder pitch angle, θ_2 is the relative elbow pitch angle, and θ_3 is the relative wrist pitch angle. The Cartesian variables are x_H, y_H, and ϕ.



IPK Problem Statement

Given:

Find:

This is a much harder problem than Forward Pose Kinematics (FPK) since it requires the solution of coupled nonlinear (transcendental) equations.

We now present two separate IPK solution techniques:

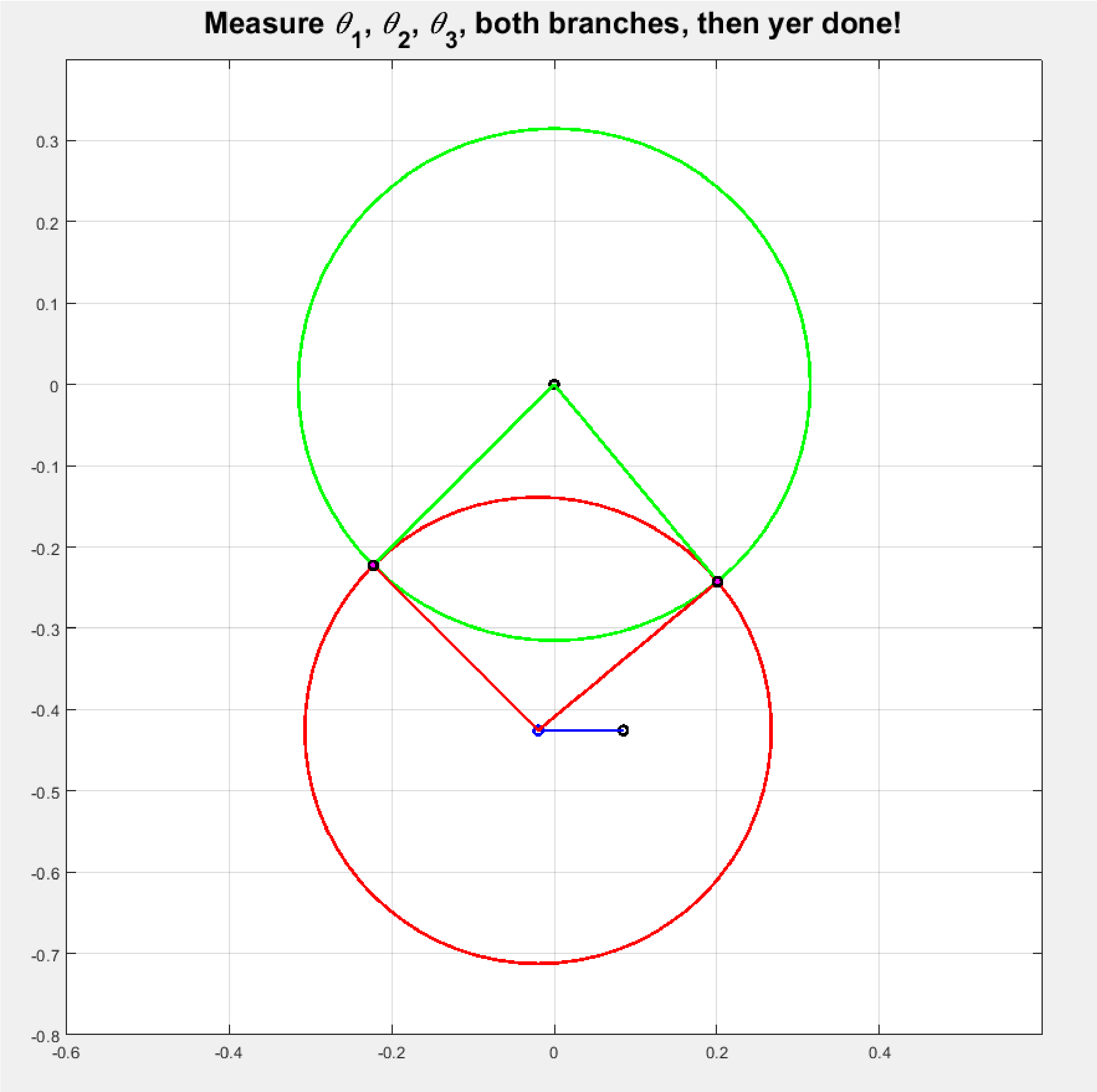
- Graphical Solution
- Analytical Solution

Graphical solution

The human arm IPK solution may be achieved *graphically*, by drawing the arm and measuring the angular answers. This is an excellent method to validate your computer results at a given snapshot.

- Draw the known fixed ground point at (0,0) – this is the shoulder point *S*.
- Draw the given hand link 3, tip *H* at (x_H, y_H), length L_3 at the given angle ϕ to yield the wrist point *W*. You must reverse angle ϕ, i.e. $\phi+180^\circ$, to get from *H* to *W*.
- Draw a circle of radius L_1, centered at point *S*.
- Draw a circle of radius L_2, centered at point *W*.
- These two circles intersect in general in two places to yield two possible elbow points *E*.
- Connect the two human arm solution branches (elbow up and elbow down) and measure the unknown angles θ_1, θ_2, and θ_3 for each.
- Only one solution is valid for the human, the other is an oucher due to elbow joint limits.
- In the valid solution branch, draw lines representing the biceps and triceps muscles (based on their known origins, i.e. the shoulder point *S*, and their known insertion points on the forearm) and measure their unknown lengths and angles.

Graphical Solution Figure



MATLAB Graphical Solution Results

`IPKGraphical.m`

<u>Another Validation Method</u> – after the analytical IPK solution, use a circular check with FPK.

Analytical 3-dof planar human arm IPK solution

The arm IPK solution starts from the FPK expressions.

$$\begin{aligned} x_H &= L_1 c_1 + L_2 c_{12} + L_3 c_{123} \\ y_H &= L_1 s_1 + L_2 s_{12} + L_3 s_{123} \\ \phi &= \theta_1 + \theta_2 + \theta_3 \end{aligned}$$

where:

$$c_1 = \cos\theta_1 \qquad c_{12} = \cos(\theta_1 + \theta_2) \qquad c_{123} = \cos(\theta_1 + \theta_2 + \theta_3)$$
$$s_1 = \sin\theta_1 \qquad s_{12} = \sin(\theta_1 + \theta_2) \qquad s_{123} = \sin(\theta_1 + \theta_2 + \theta_3)$$

To simplify, use the wrist center $\{W\}$ instead of the hand point $\{H\}$ as the IPK input. First calculate the position of the wrist center $\{W\}$ from given Cartesian information (equivalent to the second bullet on the previous page).

$$\begin{aligned} x_W &= x_H - L_3 c\phi \\ y_W &= y_H - L_3 s\phi \end{aligned}$$

Then solve the simplified FPK equations for θ_1 and θ_2 (the ϕ equation is the same as above – use it last to find θ_3).

$$\begin{aligned} x_W &= L_1 c_1 + L_2 c_{12} \\ y_W &= L_1 s_1 + L_2 s_{12} \end{aligned}$$

The human arm IPK solution is derived below.

$$\begin{aligned} E &= -2 x_W L_1 \\ F &= -2 y_W L_1 \\ G &= x_W^2 + y_W^2 + L_1^2 - L_2^2 \end{aligned}$$

Tangent half-angle substitution If $t = \tan\frac{\theta_1}{2}$ then $\cos\theta_1 = \frac{1-t^2}{1+t^2}$ and $\sin\theta_1 = \frac{2t}{1+t^2}$

The special mathematical form $(G+E)(G-E) = G^2 - E^2$ is called the **difference of two squares**.

Now that θ_1 is known, return to the *XY* equations that were prepared for squaring-and-adding, and use them in a different way:

$$L_2 c_{12} = x_W - L_1 c_1$$
$$L_2 s_{12} = y_W - L_1 s_1$$

Finally, the θ_3 solution is extremely straight-forward:

There are two overall solution sets to the inverse pose kinematics problem for the S-E-W 3R (hinges) human arm in the sagittal plane. Be sure to keep the solutions together, i.e. $(\theta_1, \theta_2, \theta_3)_1$ and $(\theta_1, \theta_2, \theta_3)_2$. You cannot mix-and-match!

The only pattern evident between the two solution sets is $\theta_{2\text{Elbow Down}} = -\theta_{2\text{Elbow Up}}$. Make a clear table of your multiple-solution results:

i	***t*** **±**	**branch**	**comfort**	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_3}$
1	+	elbow up	OUCH	θ_{1_1}	θ_{2_1}	θ_{3_1}
2	–	elbow down	OK	θ_{1_2}	$\theta_{2_2} = -\theta_{2_1}$	θ_{3_2}

Validation: Be sure to use the circular check to prove both sets of solution angles satisfy the originally-given Cartesian pose. That is, use your FPK solution with the answers of the IPK problem as inputs.

Also, visual validation is useful, that is why we ask MATLAB to draw the human arm model to the screen.

Human Arm IPK Examples

For the same male and female human arm models as the FPK examples, we now simulate the IPK solution for a given snapshot and Cartesian motion trajectory.

Snapshot IPK Examples

Given $X_0 = \begin{bmatrix} x_H & y_H & \phi \end{bmatrix} = \begin{bmatrix} 0.085 & -0.426 & 0 \end{bmatrix}$, the admissible (elbow down) IPK solutions are:

$$\Theta_{\text{male}} = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 \end{bmatrix} = \begin{bmatrix} -135.0^\circ & 90.0^\circ & 45.0^\circ \end{bmatrix}$$

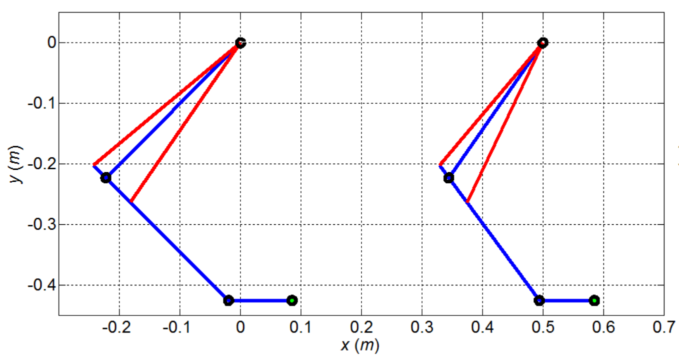
$$\Theta_{\text{female}} = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 \end{bmatrix} = \begin{bmatrix} -124.9^\circ & 71.4^\circ & 53.5^\circ \end{bmatrix}$$

The associated muscle lengths and angles are (given r_{Bm}=0.057, r_{Tm}=0.029, r_{Bf}=0.050, r_{Tf}=0.025 m):

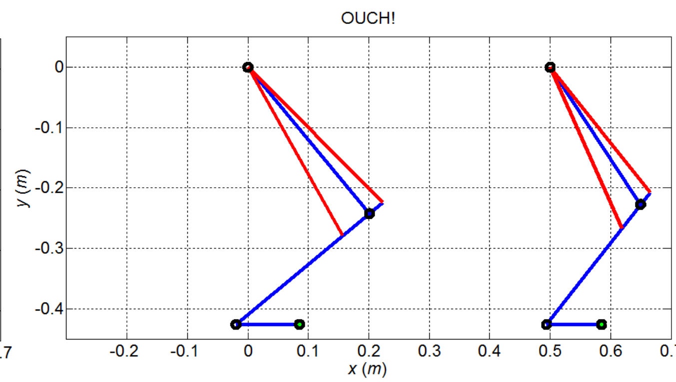
$$\begin{matrix} L_{\text{male}} = \begin{bmatrix} L_{Bm} & L_{Tm} \end{bmatrix} = \begin{bmatrix} 0.320 & 0.316 \end{bmatrix} \\ L_{\text{female}} = \begin{bmatrix} L_{Bf} & L_{Tf} \end{bmatrix} = \begin{bmatrix} 0.292 & 0.265 \end{bmatrix} \end{matrix} \quad (m)$$

$$\phi_{\text{male}} = \begin{bmatrix} \phi_{Bm} & \phi_{Tm} \end{bmatrix} = \begin{bmatrix} 55.3^\circ & 39.8^\circ \end{bmatrix}$$

$$\phi_{\text{female}} = \begin{bmatrix} \phi_{Bf} & \phi_{Tf} \end{bmatrix} = \begin{bmatrix} 64.5^\circ & 49.9^\circ \end{bmatrix}$$



Admissible IPK Snapshot Example Poses (male left, female right)



Impossible IPK Snapshot Example Poses (male left, female right)

For the same example

Given $X_0 = \begin{bmatrix} x_H & y_H & \phi \end{bmatrix} = \begin{bmatrix} 0.085 & -0.426 & 0 \end{bmatrix}$, the impossible (elbow up) IPK solutions are

$$\Theta_{\text{male}} = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 \end{bmatrix} = \begin{bmatrix} -50.3^\circ & -90.0^\circ & 140.3^\circ \end{bmatrix}$$

$$\Theta_{\text{female}} = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 \end{bmatrix} = \begin{bmatrix} -56.7^\circ & -71.4^\circ & 128.0^\circ \end{bmatrix}$$

The associated muscle lengths, though meaningless, are identical to those lengths from the admissible solution branch. The muscle angles for the impossible solution branch are also given.

$$\begin{matrix} L_{\text{male}} = \begin{bmatrix} L_{Bm} & L_{Tm} \end{bmatrix} = \begin{bmatrix} 0.320 & 0.316 \end{bmatrix} \\ L_{\text{female}} = \begin{bmatrix} L_{Bf} & L_{Tf} \end{bmatrix} = \begin{bmatrix} 0.292 & 0.265 \end{bmatrix} \end{matrix} \quad (m)$$

$$\phi_{\text{male}} = \begin{bmatrix} \phi_{Bm} & \phi_{Tm} \end{bmatrix} = \begin{bmatrix} 119.3^\circ & 134.9^\circ \end{bmatrix}$$

$$\phi_{\text{female}} = \begin{bmatrix} \phi_{Bf} & \phi_{Tf} \end{bmatrix} = \begin{bmatrix} 113.9^\circ & 128.5^\circ \end{bmatrix}$$

Trajectory IPK Examples

In this case we will use the same Cartesian trajectory for both male and female arms, whereas in the FPK examples we used the same joint angles for both male and female arms.

Starting from the initial Cartesian pose X_0 given above, we add the following Cartesian offsets at each time step.

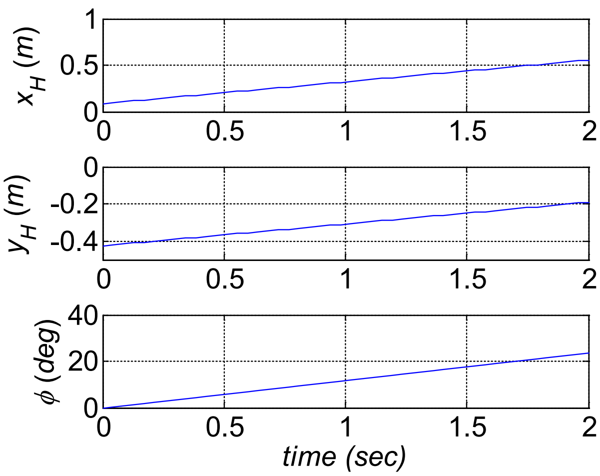
$$\begin{aligned} \Delta x_H &= 0.010 \\ \Delta y_H &= 0.005 \text{ m} \\ \Delta\phi &= 0.5^\circ \end{aligned}$$

The muscle length and angle calculations and the arm animations are done in the exact same manner as previously for the FPK simulation.

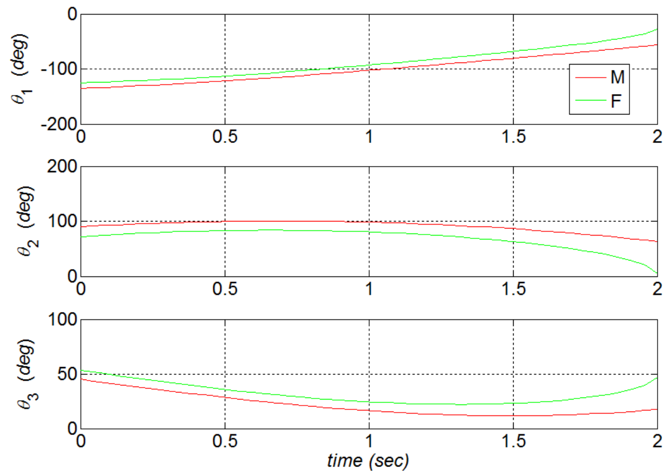
The results of this human arms IPK trajectory motion simulation are given in the next six plots. Exactly $N = 48$ steps were simulated with the given Cartesian offsets and these were artificially mapped to a time range from 0 to 2 sec ($\Delta t = t_f / (N-1) = 0.0426$ sec; step 1 is at time = 0, corresponding to the initial Cartesian pose). If the simulation proceeds to 49 steps, the simulated female will have violated her elbow joint limit and gone through an oucher of a singularity.

This human arm inverse pose kinematics example demonstrates the kinematic problem of **singularity** that did not exist for the FPK joint control simulation. Towards the end of motion, the female arm just reaches a singularity where the elbow angle is 0 (the forearm is straight out from the upper arm). In the case of this singularity, further correct Cartesian motion is impossible under IPK motion simulation.

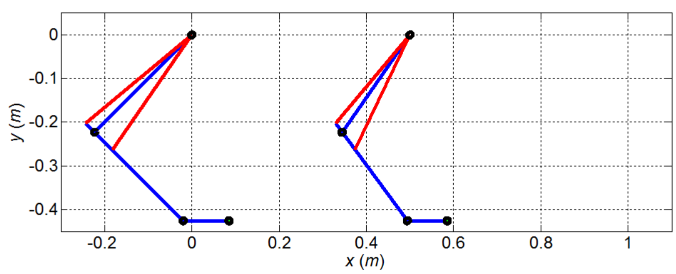
In robotics **singularities** are associated with infinite joint rates to accomplish a finite Cartesian velocity (see the female arm joint angles whose slopes become steep at the end of the IPK trajectory simulation), which is **bad**. However, singularities are also associated with infinite Cartesian force for finite joint torques (see the following **Statics** section), so singularities are **good** to resist large Cartesian forces, with the human skeletal structure instead of large muscle forces.



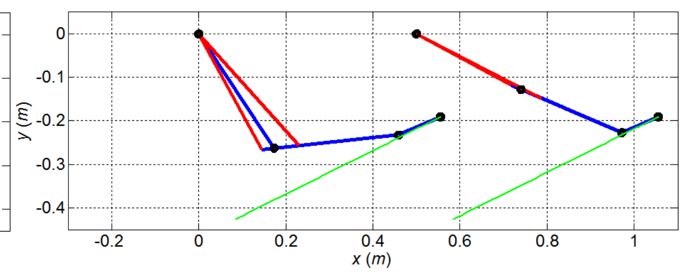
Input Cartesian Pose (x_H, y_H, ϕ vs. t)



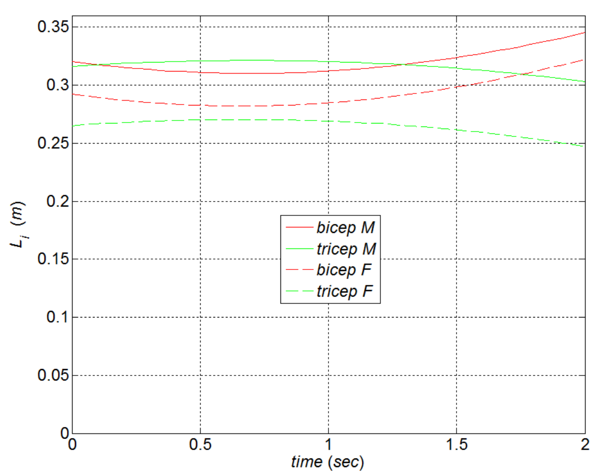
IPK Results (θ_1, θ_2, θ_3 vs. t)



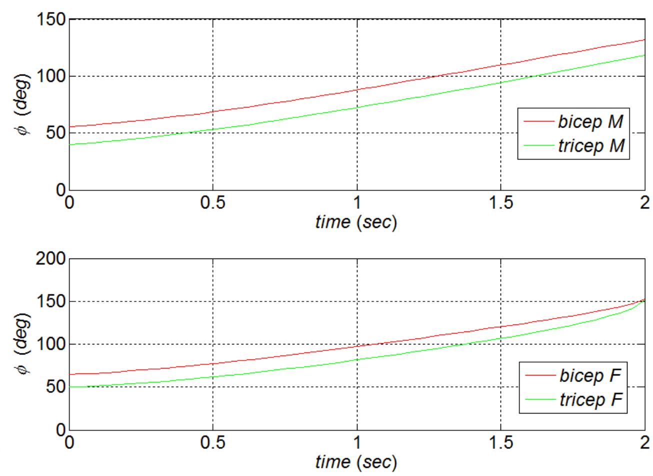
Initial Poses (male left, female right)



Final Poses (male left, female right)



Simulated Muscle Lengths



Simulated Muscle Angles

5. Human Body Engineering Mechanics: Statics

5.1 Human Body Statics

This chapter presents conditions of **static equilibrium** for human body segments, with examples. **Statics** is the study of force/moment balance with no motion. We must draw free-body diagrams (FBDs) for each segment of interest. The most difficult part of **statics** is **kinematics** (the pose of each segment and associated muscles), especially for 3D statics. Weight can be important in statics, $W = mg$, a vector in the direction of gravity, located at each segment CG. We will treat muscles simply as massless 1D cables with tension. The muscles' weight can be lumped into the segment weight.

force review

moment review

General Human Body Segment FBD

Static Equilibrium Equations

Pseudostatics

Pseudostatics involves using conditions of static equilibrium for human body segments under motion, where the velocities and accelerations are small enough to ignore their dynamic effects.

Center of Gravity

The **center of gravity** (**CG**) is the point through which the sum of gravitational forces on a body act. It is the same as the **center of mass**, which is the point at which the body's mass behaves as if it were concentrated. If one supports the body at the CG, it will be balanced with respect to gravity.

The general formula for CG is
$$\underline{P}_{CG} = \frac{\int_{body} \underline{r}dm}{\int_{body} dm}$$

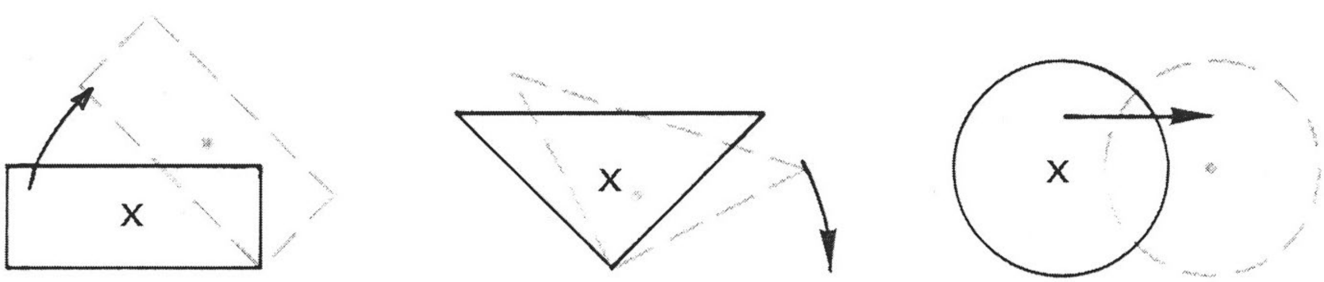
In Cartesian components
$$X_{CG} = \overline{X} = \frac{\int_x xdm}{\int dm} \qquad Y_{CG} = \overline{Y} = \frac{\int_y ydm}{\int dm}$$

To calculate the CG of the overall human body (or a portion of the human body) composed of multiple segments, we must discretely combine the CGs of all segments as follows.

$$\underline{P}_{CG_T} = \begin{Bmatrix} \overline{X}_T \\ \overline{Y}_T \end{Bmatrix} = \begin{Bmatrix} \dfrac{m_1\overline{x}_1 + m_2\overline{x}_2 + \cdots + m_n\overline{x}_n}{m_T} \\ \dfrac{m_1\overline{y}_1 + m_2\overline{y}_2 + \cdots + m_n\overline{y}_n}{m_T} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\sum_{i=1}^{n} m_i\overline{x}_i}{m_T} \\ \dfrac{\sum_{i=1}^{n} m_i\overline{y}_i}{m_T} \end{Bmatrix}$$

where the total mass is $m_T = \sum_{i=1}^{n} m_i$.

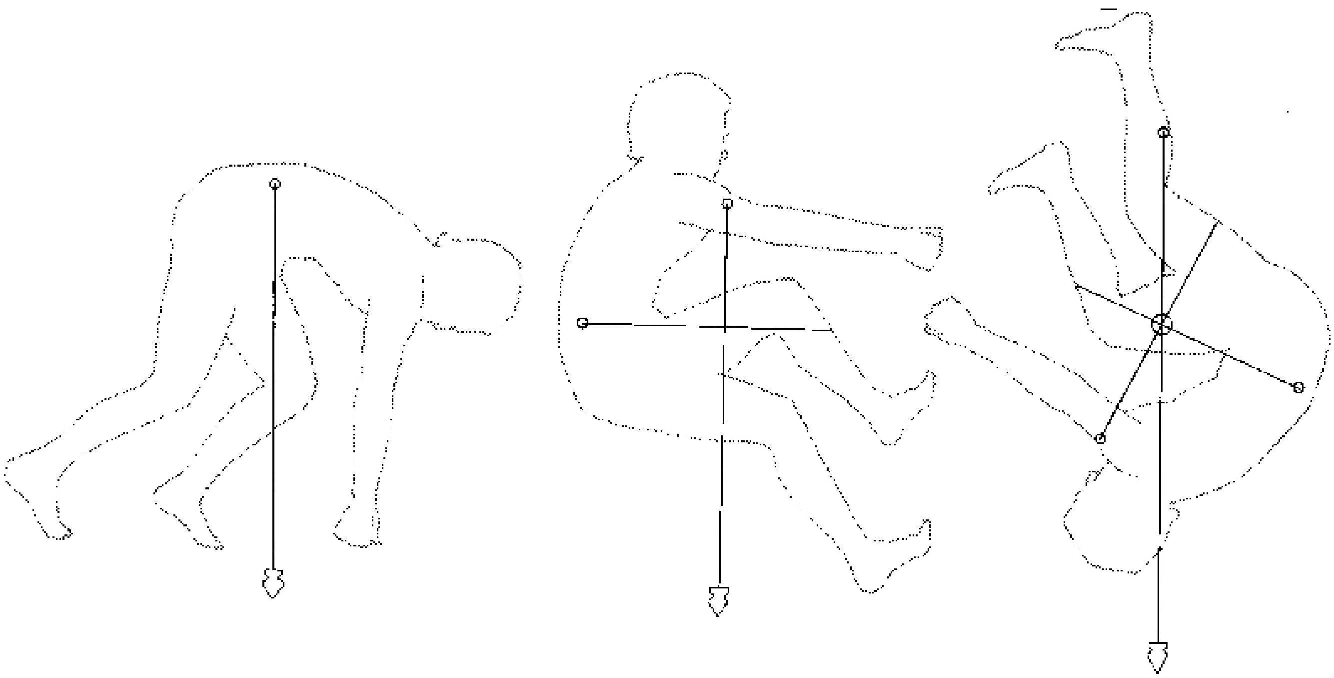
Static/pseudostatic equilibrium can be stable, unstable, or neutral, as shown below. For bodies in motion with significant velocities/accelerations, **dynamic equilibrium** must account for inertial forces and moments, in addition to static equilibrium.



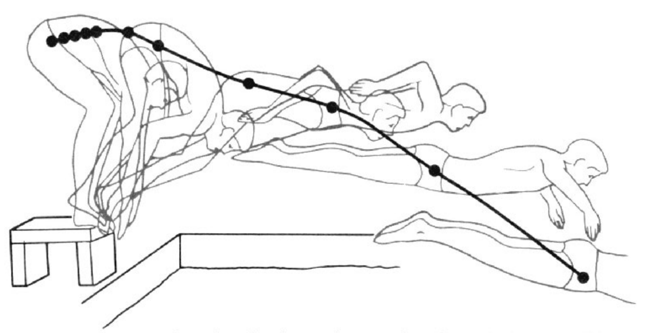
Stable Equilibrium **Unstable Equilibrium** **Neutral Equilibrium**
adapted from Hamilton et al. (2008)

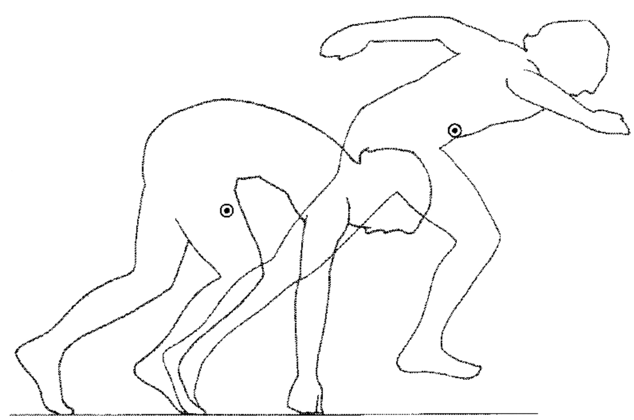
Appendix D presents major human body segment CGs as a percentage of segment lengths, measured from the proximal end.

<u>Human Body Center of Gravity (CG)</u>



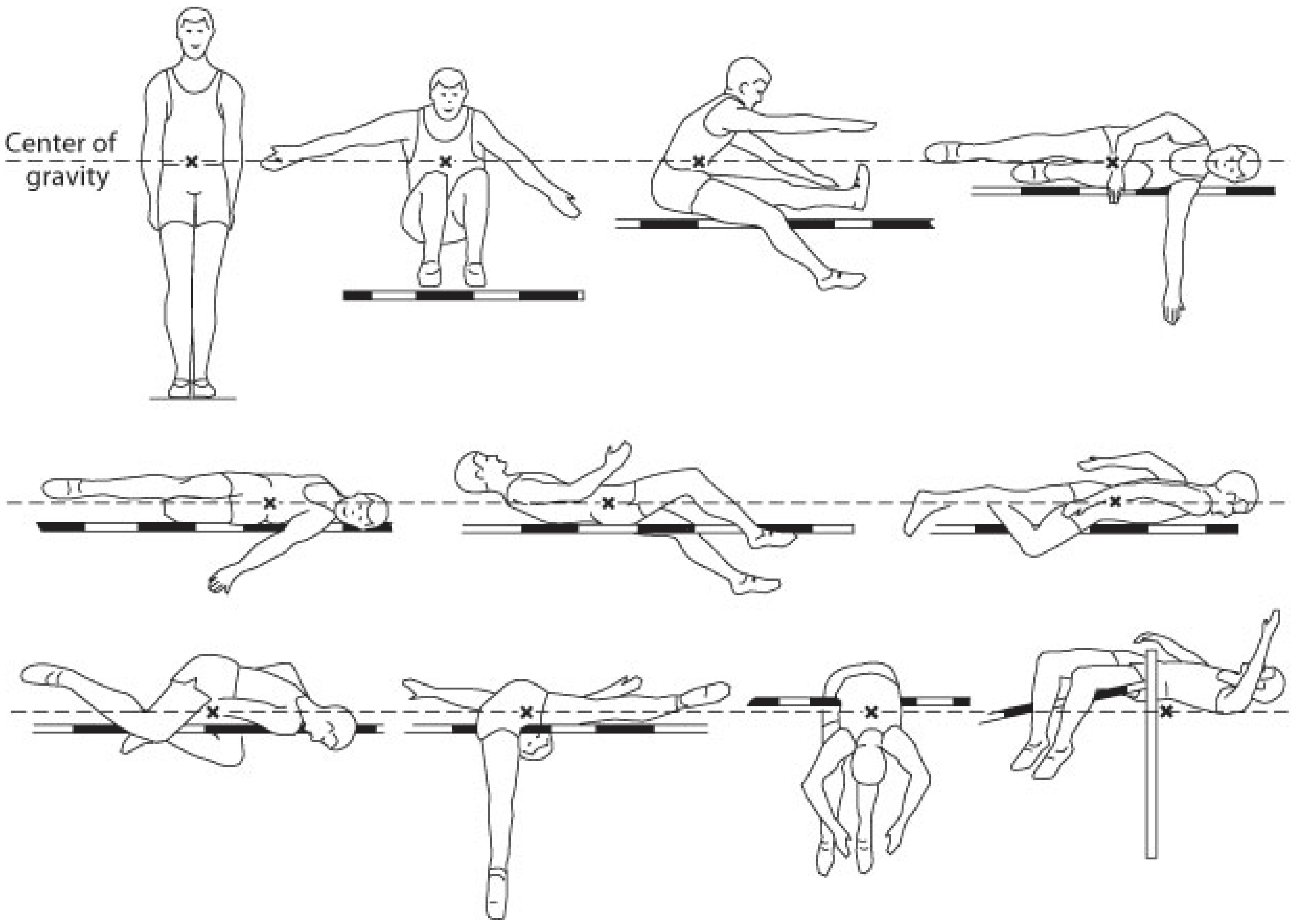
CG of an Irregular Object Found by Suspension





CG Changes with Posture

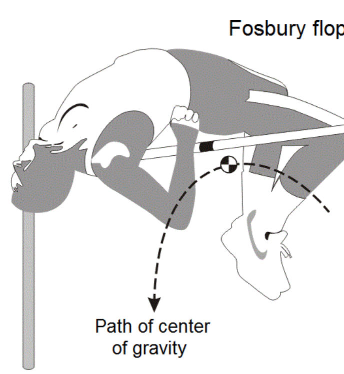
Hamilton et al. (2008)

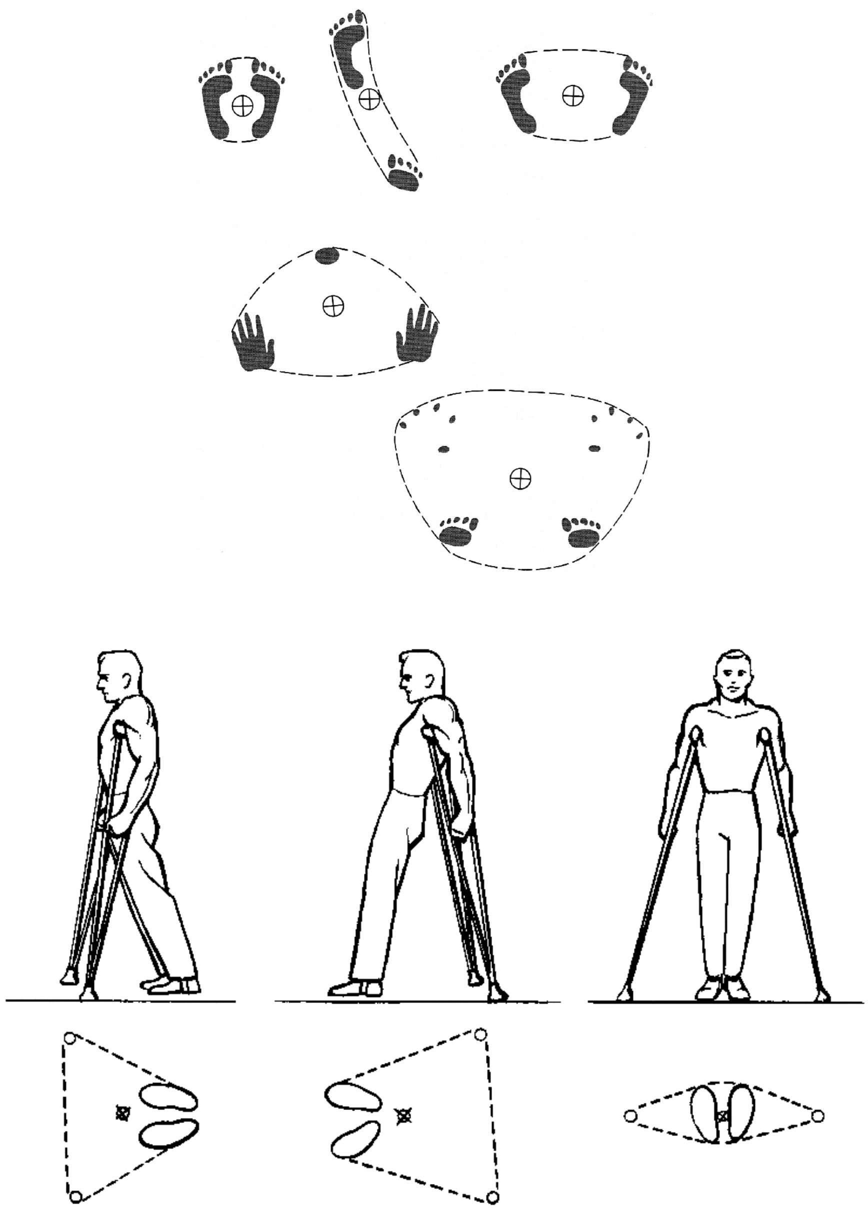


In high-jumping using the Fosbury Flop (bottom right, an innovative technique introduced by Dick Fosbury, who won the gold medal in the 1968 Summer Olympics in Mexico City, now copied by most high jumpers), the human body *CG* remains under the bar by up to 20 cm while the individual segments all clear the bar in turn. All other jumping techniques require the *CG* to clear the bar. This technique was enabled by the use of deep foam mats to land on. Previously only sand pits or thin mats were used.

youtube.com

Dick Fosbury won the gold medal in high jumping in 1968 with a winning height of 2.24 m. Fosbury never held the world record and he failed to qualify for the 1972 Summer Olympics. The current men's world record is 2.45 m, established in 1993 using the Fosbury Flop, the only technique for serious high jumpers.





CG Bounded by Supports for Stability

Hamilton et al. (2008)

Human Body Statics Examples

Example 1

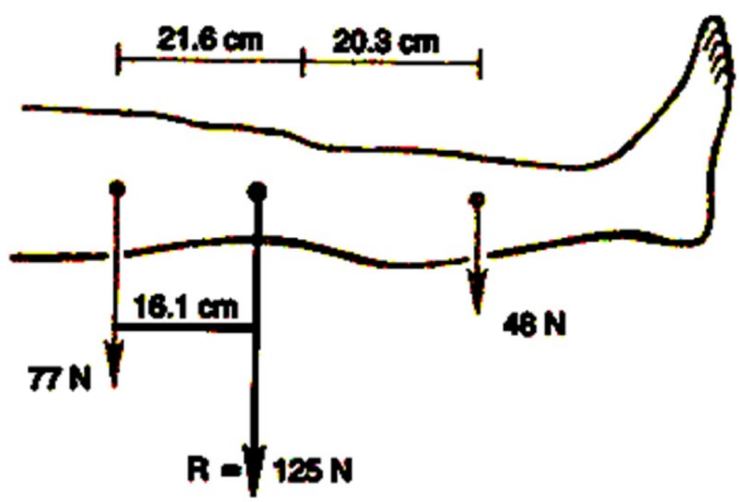
Human Body Composite CG Example. Find the CG of the combined adult human leg shown below (thigh, lower leg/foot), measured in the horizontal direction only, with respect to the thigh CG. The thigh weight is $W_1 = 77$ N and the combined lower leg/foot weight is $W_2 = 48$ N. As shown in the figure, the knee is 21.6 cm distal to the thigh CG and the combined lower leg/foot CG is 20.3 cm distal to the knee. In the solution below, let us establish the X origin at the thigh CG.

Solution

$$\overline{X} = \frac{m_1\overline{x}_1 + m_2\overline{x}_2}{m_T} = \frac{m_1 g\overline{x}_1 + m_2 g\overline{x}_2}{m_T g} = \frac{77(0) + 48(21.6 + 20.3)}{(77 + 48)} = 16.1\,\text{cm}$$

The combined CG thus lies 16.1 cm distal to the thigh CG, as shown, or 5.5 cm proximal to the knee.

Note above we multiplied by a '1' (g/g) in order to use weight data ($W = mg$) directly instead of mass m.



LeVeau (1992)

Alternate Solution

As indicated in the figure, the total resultant weight is $R = W_1 + W_2 = 77 + 48 = 125$ N. We know this resultant weight R must act at the overall CG. Since we are basing our calculation on the thigh CG location, the moment of this resultant force must equal the moment of the lower leg/foot weight acting at its CG.

$$R\overline{X} = W_2(21.6 + 20.3)$$

$$\overline{X} = \frac{W_2(21.6 + 20.3)}{R} = \frac{48(21.6 + 20.3)}{125}$$

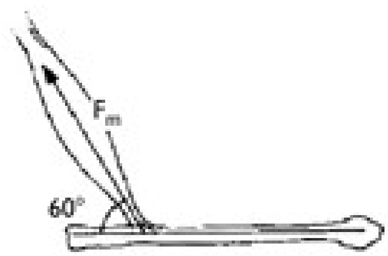
$$\overline{X} = 16.1 \quad \text{cm}$$

which yields the same result.

Example 2

How much torque is generated about the elbow joint by the biceps brachii inserting on the radius bone d = 4 cm from the elbow, at an angle of $\theta = 60^\circ$ as shown in the figure, given a muscle force of $F_m = 500$ N? For this case, if a load is lifted L = 30 cm distal to the elbow joint, what is the load weight? (Ignore the muscle/bone weight; the load is straight down.) For this case, calculate the elbow joint vector reaction force (including the load weight but ignoring the muscle/bone weight).

FBD



Hall (2007)

Solution

$$\tau = F_m d \sin\theta$$

$$\tau = 500(0.04)\sin 60^\circ = 17.3 \text{ Nm}$$

The formal moment definition may also be used to solve the first part of this problem.

$$\mathbf{T_m} = \mathbf{r_m} \times \mathbf{F_m} = \begin{Bmatrix} d \\ 0 \\ 0 \end{Bmatrix} \times \begin{Bmatrix} -F_m \cos\theta \\ F_m \sin\theta \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ F_m d \sin\theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 500(0.04)\sin 60^\circ \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 17.3 \text{ Nm} \end{Bmatrix}$$

To calculate the load weight W that can be lifted L = 30 cm distal to the elbow joint by this muscle force in this configuration, ignoring the muscle/bone weight, the sum of moments about the elbow joint must balance to zero.

$$\sum M_{EZ} = 0$$

$$F_m d \sin\theta - WL = 0$$

$$W = \frac{F_m d \sin\theta}{L} = \frac{500(0.04)\sin 60^\circ}{0.30}$$

$$W = 57.7 \quad \text{N}$$

The elbow joint reaction force is calculated from the static equilibrium condition for vector forces.

$$\sum F_X = 0$$

$$F_{RX} - F_m \cos\theta = 0$$

$$F_{RX} = F_m \cos\theta$$

$$F_{RX} = 500\cos 60^\circ$$

$$F_{RX} = 250 \quad \text{N}$$

$$\sum F_Y = 0$$

$$F_{RY} + F_m \sin\theta - W = 0$$

$$F_{RY} = W - F_m \sin\theta$$

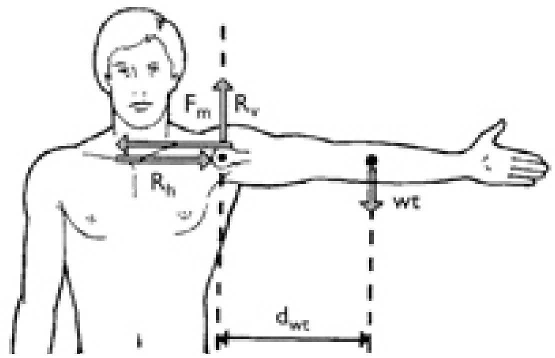
$$F_{RY} = 57.7 - 500\sin 60^\circ$$

$$F_{RY} = -375.3 \quad \text{N}$$

Example 3

Calculate the required deltoid muscle force F_m to maintain static equilibrium of the adult human arm in the configuration shown below. From Appendix D, the total weight of the adult human arm is wt = 51.3 N located at a combined CG of d_{wt} (measured from the shoulder joint). Assume the deltoid muscle moment arm (vertical in this configuration) is d_m = 3 cm. Also calculate the components of the vector reaction force at the shoulder joint, R_h and R_v.

FBD



Hall (2007)

Solution

We first must calculate the overall CG of the arm as in Examples 1 and 2 (see Appendix D for the required data); 1 – upper arm, 2 – forearm, and 3 – hand.

$$d_{wt} = \bar{X} = \frac{m_1 g \bar{x}_1 + m_2 g \bar{x}_2 + m_3 g \bar{x}_3}{m_T g} = \frac{28.91(13.7) + 16.64(31.5 + 12.3) + 5.78(31.5 + 28.7 + 4.9)}{28.91 + 16.64 + 5.78} = 29.2 \quad \text{cm}$$

If we sum moments about the shoulder joint, the unknowns R_h and R_v will not be involved; for static equilibrium, the sum of moments about the shoulder joint is zero.

$$\sum M_{SZ} = 0$$

$$F_m(d_m) - wt(d_{wt}) = 0$$

$$F_m = \frac{wt(d_{wt})}{d_m} = \frac{51.3(29.2)}{3}$$

$$F_m = 499.3 \quad \text{N}$$

The shoulder joint reaction force is calculated from the static equilibrium condition for vector forces.

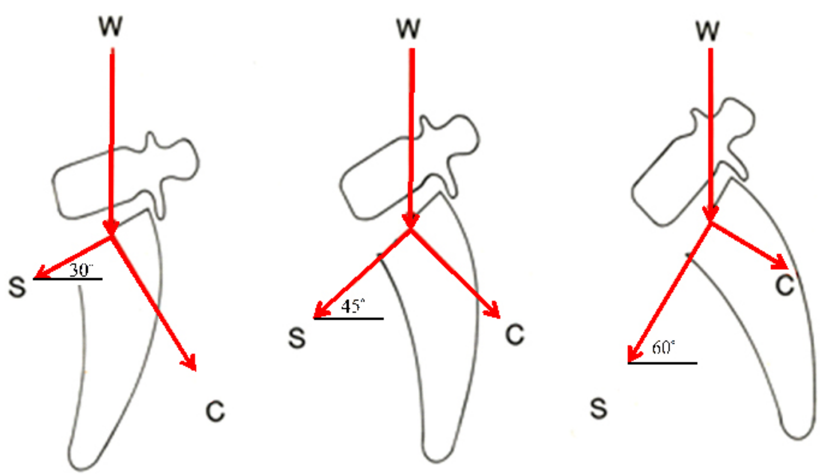
$$\sum F_X = 0 \qquad\qquad \sum F_Y = 0$$

$$R_h - F_m = 0 \qquad\qquad R_v - wt = 0$$

$$R_h = F_m = 499.3 \quad \text{N} \qquad\qquad R_v = wt = 51.3 \quad \text{N}$$

Example 4

The magnitudes of the compressive (F_C) and shear (F_S) forces between the last lumbar vertebra (L5) and the sacrum are dependent on the sacrum angle θ. The body weight above this point is W. Find and plot the statics relationships between the upper body weight W and F_C and F_S as a function of θ.



adapted from LeVeau (1992)

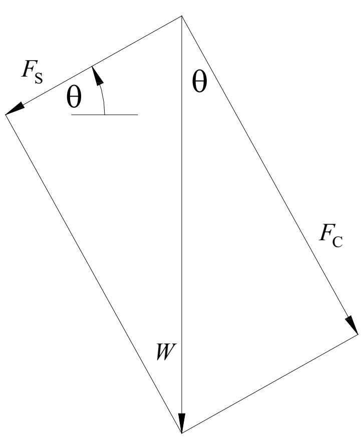
Solution

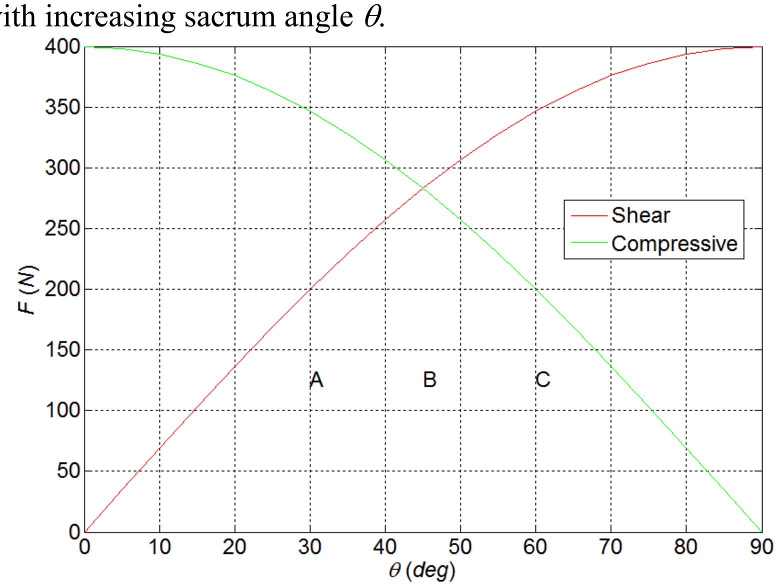
From the vector diagram shown on the left below:

$$F_S = W \sin\theta$$

$$F_C = W \cos\theta$$

For W = 400 N, the plot below shows the shear and compressive force components at the L5-S1 joint vs. sacrum angle θ. The two components are equal at $\theta = 45^\circ$ ($F_S = F_C = 200\sqrt{2}$, the middle case in the figure above). As the sacrum angle approaches $\theta = 90^\circ$ the shear force becomes relatively high, leading to increased risk of low-back injury. Compounding this problem, the moment arm for W, zero in the statics problem here, increases with increasing sacrum angle θ.





F_S and F_C vs. θ

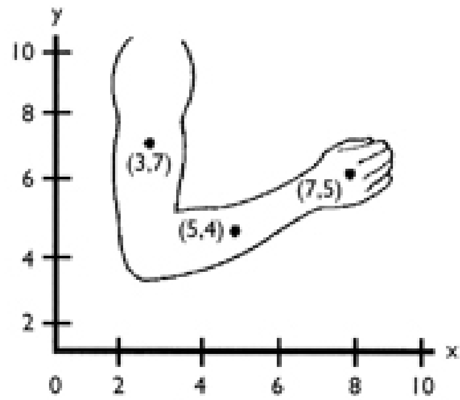
Example 5

Human Body Composite CG Example. Calculate the overall CG for the adult male human arm in the configuration shown. The absolute *XY* coordinates of the individual upper arm, forearm, and hand segment CGs are shown in the figure, with units of inches. From Appendix D, the adult male human arm segment masses are 2.947, 1.696, and 0.589 kg for the upper arm, forearm, and hand, respectively. Combining units of kg and in is unconventional, however, the kg units cancel out.

Solution

$$\overline{X} = \frac{m_1\overline{x}_1 + m_2\overline{x}_2 + m_3\overline{x}_3}{m_T} = \frac{2.947(3) + 1.696(5) + 0.589(7)}{2.947 + 1.696 + 0.589} = 4.099 \quad \text{in}$$

$$\overline{Y} = \frac{m_1\overline{y}_1 + m_2\overline{y}_2 + m_3\overline{y}_3}{m_T} = \frac{2.947(7) + 1.696(4) + 0.589(5)}{2.947 + 1.696 + 0.589} = 5.802 \quad \text{in}$$



Hall (2007)

Alternate Solution

Factoring the above $\overline{X}$ yields:

$$\overline{X} = \frac{m_1\overline{x}_1 + m_2\overline{x}_2 + m_3\overline{x}_3}{m_T} = \frac{m_1}{m_T}\overline{x}_1 + \frac{m_2}{m_T}\overline{x}_2 + \frac{m_3}{m_T}\overline{x}_3$$

$\overline{Y}$ can be factored in a similar manner. Then the CG solution can be found using a spreadsheet using upper arm, forearm, and hand mass percentages of 56.3%, 32,4%, and 11.3%, respectively (calculated from the above mass information):

Segment	mass ratio	Xi	mass ratio x Xi	Yi	mass ratio x Yi
upper arm	0.563	3	1.690	7	3.943
forearm	0.324	5	1.621	4	1.297
hand	0.113	7	0.788	5	0.563
CG		**Xbar**	**4.099 in**	**Ybar**	**5.802 in**

Xbar and **Ybar**, which agree with the first solution method, are found by summing the three numbers in the columns **mass ratio x Xi** and **mass ratio x Yi**, respectively.

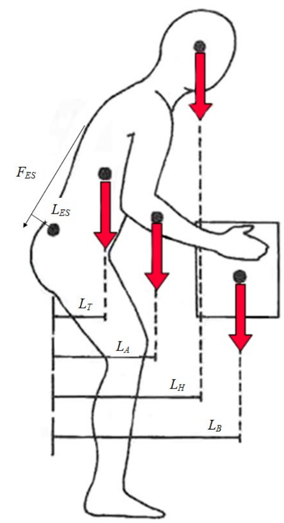
Note that the mass properties for the adult human female are slightly different. The upper arm, forearm, and hand mass percentages are 58.3%, 31,6%, and 10.1%, respectively.

Example 6

How much muscle force F_{ES} must be provided by the erector spinae muscle (with a moment arm of L_{ES} = 6 cm from the center of the L5-S1 joint) to achieve static equilibrium in the lifting position shown with the segment weights and moment arms as given in the table? Assuming the back is 30° from vertical, calculate the L5-S1 reaction force F_R. These segment weights are for a 600 N adult.

Segment	Weight (N)		Moment Arm (cm)	
trunk	W_T	280	L_T	12
arms	W_A	65	L_A	25
head/neck	W_H	50	L_H	22
box	W_B	100	L_B	42

FBD



adapted from *Hall (2007)*

Solution

To maintain static equilibrium, the sum of moments about any point (such as the L5-S1 joint) must be zero. Taking a positive convention according to the right-hand rule:

$$\sum M_{L5-S1_Z} = 0$$
$$F_{ES}L_{ES} - W_T L_T - W_A L_A - W_H L_H - W_B L_B = 0$$
$$F_{ES}(6) - 280(12) - 65(25) - 50(22) - 100(42) = 0$$
$$F_{ES} = 1714 \text{ N}$$

$$\sum F_X = 0$$
$$F_{RX} - F_{ES}\sin 30^\circ = 0$$
$$F_{RX} = F_{ES}\sin 30^\circ = 1714(0.5)$$
$$F_{RX} = 857 \text{ N}$$

To maintain static equilibrium, the sum of vector forces must be zero (*X* above, *Y* below).

$$\sum F_Y = 0$$
$$F_{RY} - F_{ES}\cos 30^\circ - W_T - W_A - W_H - W_B = 0$$
$$F_{RY} = F_{ES}\cos 30^\circ + W_T + W_A + W_H + W_B$$
$$F_{RY} = 1714(0.866) + 280 + 65 + 50 + 100$$
$$F_{RY} = 1980 \text{ N}$$

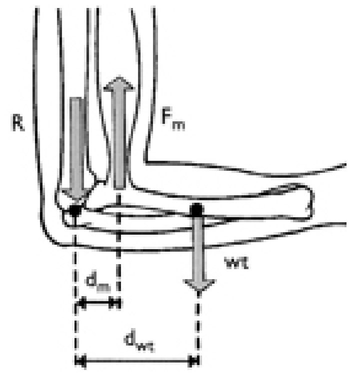
The figure above demonstrates IMPROPER lifting technique!

Example 7

How much biceps muscle force F_m is required to maintain static equilibrium in the position shown? Consider only forearm/hand weight *wt*; additionally the moment arms are given.

wt = 15 N
d_{wt} = 15 cm
d_m = 5 cm

FBD



Hall (2007)

Solution

To maintain static equilibrium, the sum of moments about the elbow point must be zero. Taking a positive convention according to the right-hand rule:

$$\sum M_E = 0$$
$$F_m d_m - wt d_{wt} = 0$$
$$F_m(5) - 15(15) = 0$$
$$F_m = 45 \text{ N}$$

To find the elbow reaction force in this case, assuming static equilibrium, the sum of vector forces must be zero. The positive convention for forces is X right and Y up.

$$\sum F_X = 0$$
$$F_{RX} = 0$$

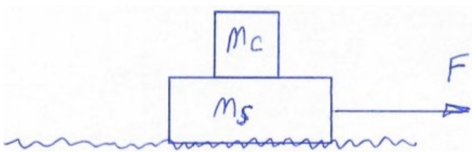
$$\sum F_Y = 0$$
$$F_m - R - wt = 0$$
$$R = F_m - wt = 45 - 15$$
$$F_{RY} = R = 30 \quad \text{N}$$

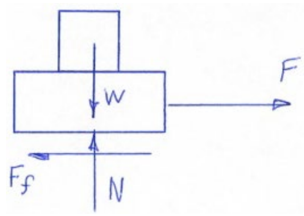
15 N is rather low for the forearm plus hand weight of an adult human male; from Appendix D it is 22.4 N. For the petite female model the forearm plus hand weight is 9.2 N.

Example 8

The static and kinetic coefficients of friction between a sled and the grass are $\mu_s = 0.55$ and $\mu_k = 0.45$, respectively. A $m_C = 27$ kg child sits on a $m_S = 2$ kg sled. How much horizontal dad force is required to initiate motion? How much horizontal dad force is required to maintain motion?

Diagram **FBD**





To initiate motion, the force must exceed the force of static friction.

$$F_s = F_{f_s} = \mu_s N = \mu_s W = \mu_s mg = 0.55(27+2)(9.81) = 156.5 \quad \text{N}$$

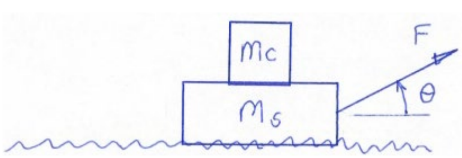
To maintain motion, the force must equal the force of kinetic friction.

$$F_k = F_{f_k} = \mu_k N = \mu_k W = \mu_k mg = 0.45(27+2)(9.81) = 128.0 \quad \text{N}$$

This problem ignores acceleration a_x since it is about impending motion and barely maintaining motion. Therefore, it belongs in this pseudostatics section.

How does this problem change if the dad's applied force is exerted up from the horizontal at angle θ?

Diagram **FBD**



$$\sum F_X = F\cos\theta - F_f = ma_x = 0 \qquad F_f = \mu N = F\cos\theta$$

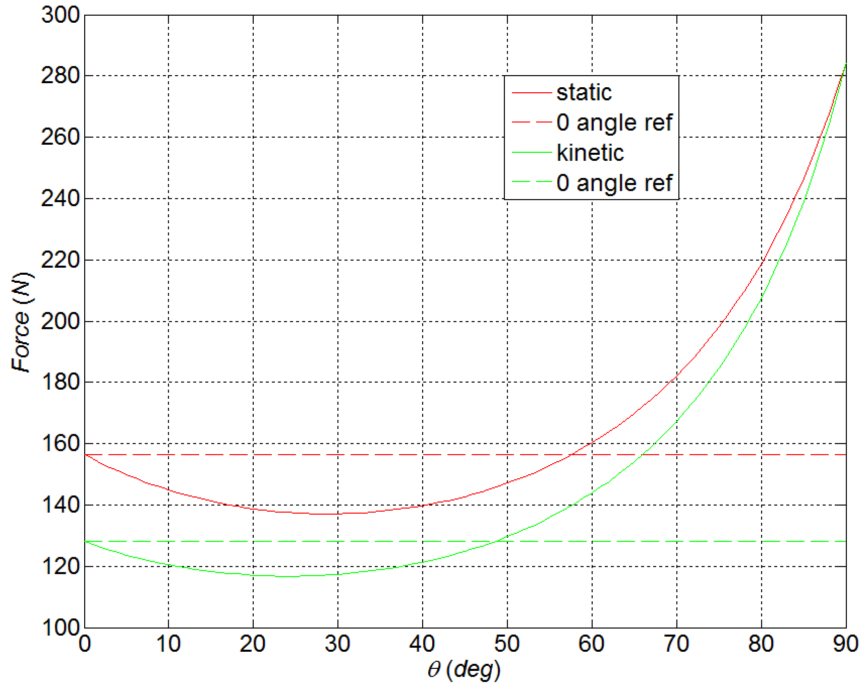
$$\sum F_Y = F\sin\theta + N - W = ma_y = 0 \qquad N = W - F\sin\theta$$

$$\mu(W - F\sin\theta) = F\cos\theta$$

$$F(\cos\theta + \mu\sin\theta) = \mu W$$

$$F = \frac{\mu W}{\cos\theta + \mu\sin\theta}$$

$$F_s = \frac{\mu_s W}{\cos\theta + \mu_s \sin\theta} \qquad\qquad F_k = \frac{\mu_k W}{\cos\theta + \mu_k \sin\theta}$$



What is the optimal pulling angle?

Static friction is commonly termed **stiction**.

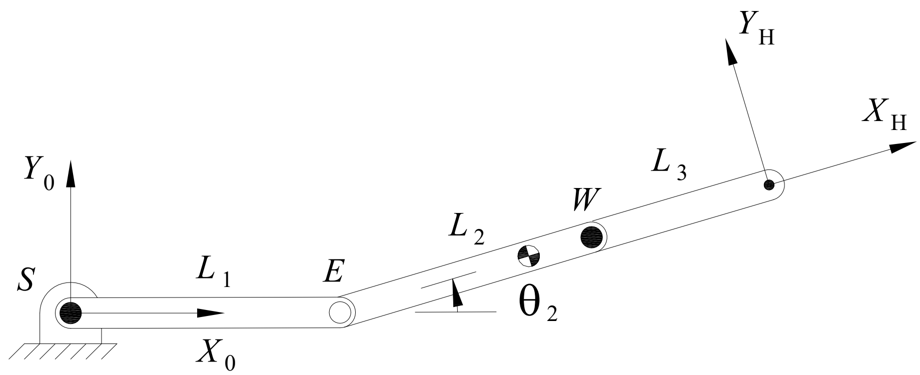
Can we coin a term for kinetic friction, **kinection**?

5.2 Human Arm Statics

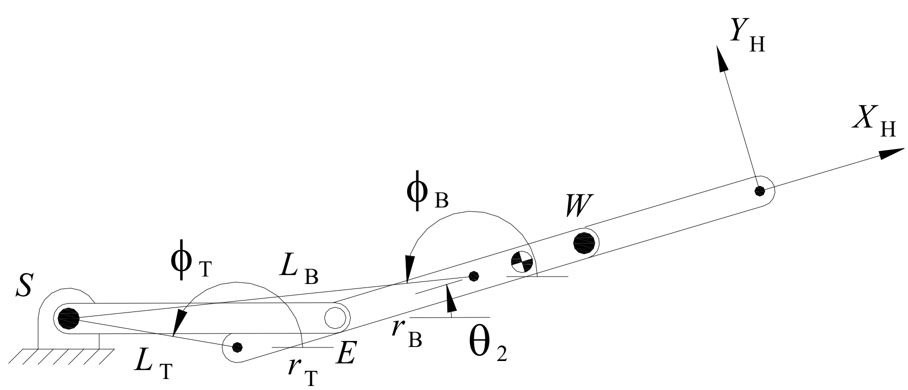
This section presents a simplified model for the pseudostatics of the planar 1-dof human arm, with biceps and triceps muscles inserting near the elbow, and the shoulder and wrist joints fixed to zero angles. Again, **pseudostatics** involves using conditions of static equilibrium for human body segments under motion, where the velocities and accelerations are small enough to ignore their dynamic effects.

Simplified Human Arm Model

The figure below shows the simplified 1-dof planar human arm model with an active R joint at the elbow *E* only. The planar shoulder and wrist pitch motions have been locked (indicated by the solid R joints at *S* and *W*). Note this will also serve as a simplified 1-dof leg model, with the knee joint in motion and the hip and ankle joints locked, with appropriate lengths and joint limits substituted for the arm parameters.



L_1 is the upper arm length, L_2 is the forearm length, and L_3 is the hand length. θ_1=0 is the locked absolute shoulder pitch angle, θ_2 is the variable elbow pitch angle, and θ_3=0 is the locked relative wrist pitch angle. The CG symbol above is for the combined forearm/hand segment, located a distance r_{CG23} from the elbow joint, along the forearm. A weight W_L is lifted at the tip of the hand, at a distance $r_L = L_2 + L_3$ from the elbow joint, along the forearm/hand. The figure below shows biceps and triceps attachments (origins and insertions), lengths, and angles.



1-dof Elbow Motion Pseudostatics Solution

Free-Body Diagram (FBD)

Human Elbow/Forearm Inverse Pseudostatics Problem Statement

Given: L_2, L_3, m_2, m_3, r_{CG23}, r_L, r_B, r_T, ϕ_B, ϕ_T, W_L, and θ_2

Find: t_B, t_T, F_{Ex}, and F_{Ey}

Static equilibrium vector equations:

Planar cross product equations (positive moment according to right-hand rule):

$$\underline{r}\times\underline{t}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ r_x & r_y & 0\\ t_x & t_y & 0\end{vmatrix}=(r_x t_y - r_y t_x)\,\hat{k}=(r_x t\sin\phi - r_y t\cos\phi)\,\hat{k}$$

$$\underline{r}\times\underline{W}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ r_x & r_y & 0\\ 0 & -W & 0\end{vmatrix}=-r_x W\,\hat{k}$$

Static equilibrium *XYZ* scalar equations:

Count number of scalar unknowns and scalar equations.

This set of equations is underconstrained, i.e. there are more unknowns than equations – infinite solutions exist in general. So let us ignore the triceps tension for now.

Inverse Pseudostatics Solution Ignoring Triceps Tension

Given: θ_2, L_2, L_3, m_2, m_3, r_{CG23}, r_B, ϕ_B, r_L, W_L, and $t_T = 0$

Find: t_B, F_{Ex}, and F_{Ey}

Static equilibrium vector equations:

Static equilibrium *XYZ* scalar equations

These linear equations can be written in matrix form.

We don't need a matrix to solve since the equations are partially decoupled.

This solution requires length, weight, and CG terms, plus the biceps muscle angle ϕ_B (solved earlier in the human arm kinematics section). The required moment arm terms are

$$\underline{r}_B = \begin{Bmatrix} r_{Bx} \\ r_{By} \end{Bmatrix} = \begin{Bmatrix} r_B \cos\theta_2 \\ r_B \sin\theta_2 \end{Bmatrix} \qquad \underline{r}_{CG23} = \begin{Bmatrix} r_{23x} \\ r_{23y} \end{Bmatrix} = \begin{Bmatrix} r_{CG23} \cos\theta_2 \\ r_{CG23} \sin\theta_2 \end{Bmatrix} \qquad \underline{r}_L = \begin{Bmatrix} r_{Lx} \\ r_{Ly} \end{Bmatrix} = \begin{Bmatrix} L_{23} \cos\theta_2 \\ L_{23} \sin\theta_2 \end{Bmatrix}$$

<u>Human Elbow Statics Parameters</u>

From Appendix D, we again use the following segment lengths (m).

Subject	L_2 (forearm)	L_3 (hand)	L_{23} (forearm/hand)
adult male	0.287	0.105	0.392
adult female	0.252	0.091	0.343

The upper arm length $L_1 = 0.315$ m for the male ($L_1 = 0.272$ m for the female) is not required for the pseudostatics model. We can use this value for the graphics, where L_1 is a fixed horizontal extension of the fixed torso/shoulder ground link.

From Appendix D, we now need the following segment weights (N). We don't need the upper arm weight W_1 since it is fixed to the ground link.

Subject	W_2 (forearm)	W_3 (hand)	W_{23} (forearm/hand)
adult male	16.64	5.78	22.42
adult female	6.98	2.22	9.20

Appendix D gives the following r_{CG} lengths (m) for the forearm (2) and hand (3), measured from the proximal end of each segment.

Subject	r_{CG2} (forearm)	r_{CG3} (hand)
adult male	0.123	0.049
adult female	0.109	0.043

We calculate the following lumped CG locations (m, measured from the proximal end, i.e. the elbow joint) for the combined forearm (2) and hand (3) segment, with the wrist locked. We see that the combined CG is in the forearm due to the mass distribution in each case.

male

$$r_{CG23} = \frac{16.64(0.123) + 5.78(0.287 + 0.049)}{16.64 + 5.78} = 0.178$$

female

$$r_{CG23} = \frac{6.98(0.109) + 2.22(0.252 + 0.043)}{6.98 + 2.22} = 0.154$$

Subject	r_{23CG} (forearm/hand)
adult male	0.178
adult female	0.154

Additionally, a constant end load (vertical, down) of $W_L = 22.24$ N (5 lb) is given at the fingertips so that $r_L = L_{23} = L_2 + L_3$, with respect to the elbow. Also, r_B and r_T were previously given in the kinematics section, for the adult human male and female.

<u>Human Elbow Inverse Pseudostatics Examples, Biceps Tension Only</u> **Snapshot solution**

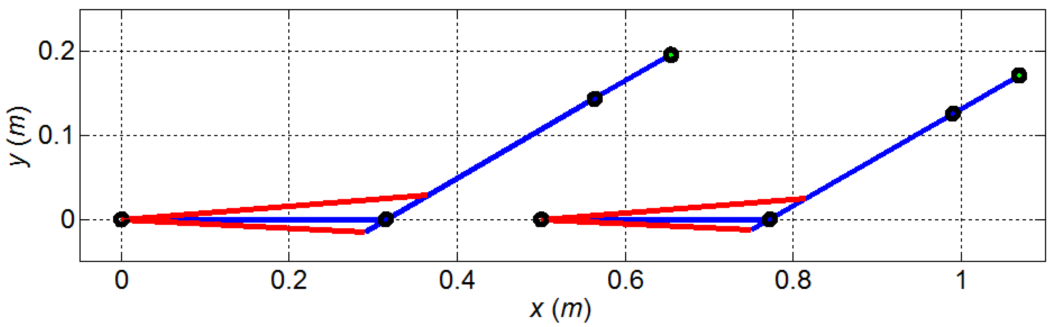
We now give a snapshot result for the biceps-only pseudostatics solution, at $\theta_2 = 30^\circ$.

male

$t_B = 445.3$ N $\quad \phi_B = -175.5^\circ \quad F_E = \begin{Bmatrix} 443.9 \\ 79.6 \end{Bmatrix}$ N

female

$t_B = 361.8$ N $\quad \phi_B = -175.4^\circ \quad F_E = \begin{Bmatrix} 360.7 \\ 60.2 \end{Bmatrix}$ N



Statics Snapshot Example Poses
(male left, female right)

Full Range of Motion Simulation

We simulate the pseudostatics solution from the minimum to maximum elbow joint angle, with the shoulder and wrist joints locked to zero. From Appendix B we use the following elbow angle limits (for both male and female).

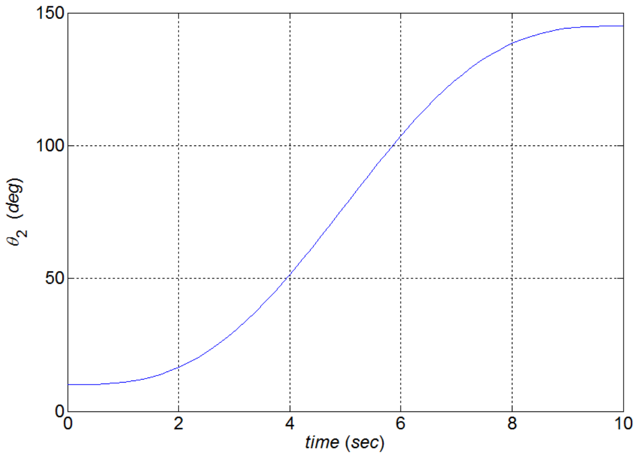
Angle	Min	Max
θ_2	0°	145°

Remember there is a kinematic singularity at $\theta_2 = 0$; this is a statics singularity also, where infinite muscle tension is required to initiate motion, since the biceps muscle is collinear with the radius bone (in the model only – not in the real human arm). Therefore, let us start the simulation at $\theta_{20} = 10^\circ$ instead of 0. Further, we use the cycloidal function below for smooth θ_2 inputs, starting and ending at rest, from t = 0 through t_F = 10 sec.

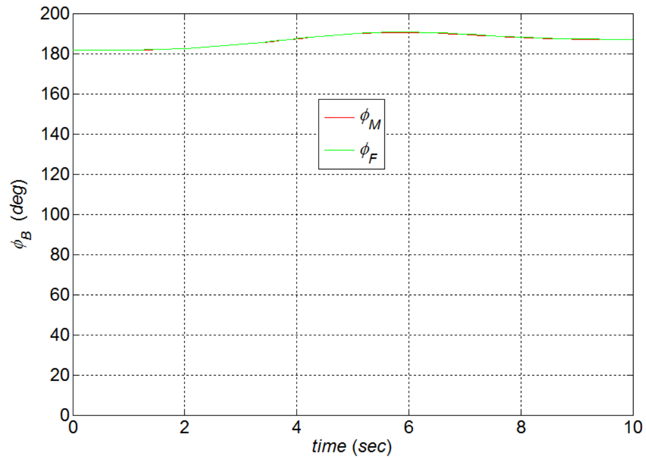
$$\theta_2(t) = \theta_{20} + (\theta_{2F} - \theta_{20})\left[\frac{t}{t_F} - \frac{1}{2\pi}\sin\frac{2\pi t}{t_F}\right]$$

The cycloidal θ_2 plot is given in the left figure below, then the biceps muscle angles for the simulated motion (both required inputs to the pseudostatics problem). Then the simulated muscles length plots are shown; these are not required for the pseudostatics solution, but are included as a kinematics problem check. The results of this biceps-only human arm pseudostatics motion simulation are then given.

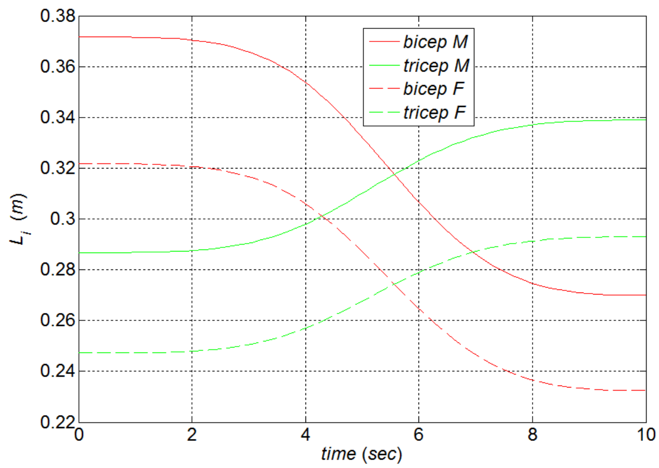
In the results we see biceps tension t_B dominates; since the muscle angle ϕ_B is close to 180°, the X component of the elbow reaction force F_{Ex} is similar in magnitude and shape to t_B. The Y component of the elbow reaction force F_{Ey} depends on the weights, but is much smaller.



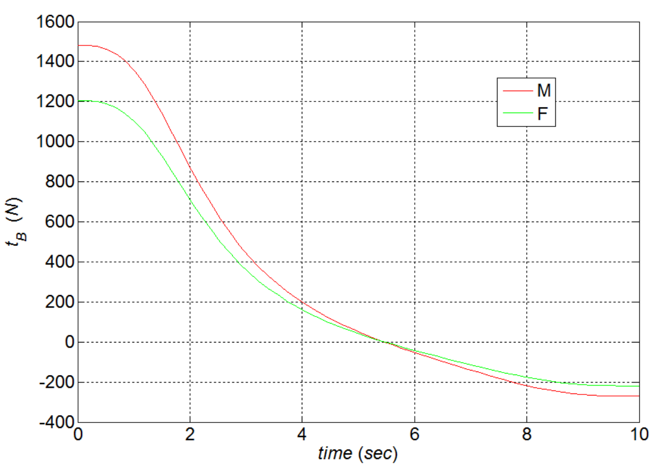
Cycloidal Input Angle θ_2



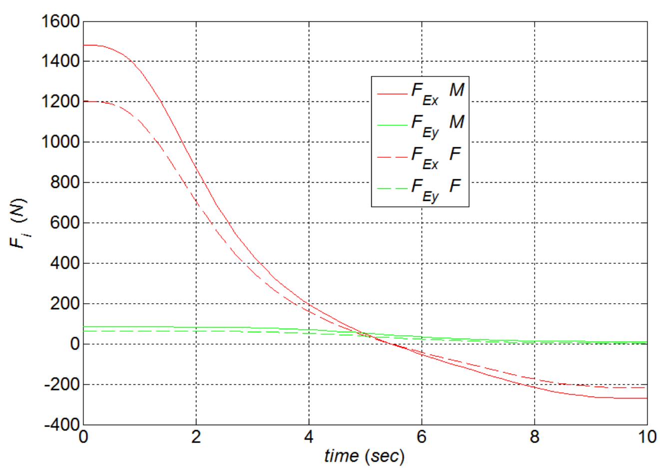
Biceps Muscle Angles (ϕ_B vs. t)



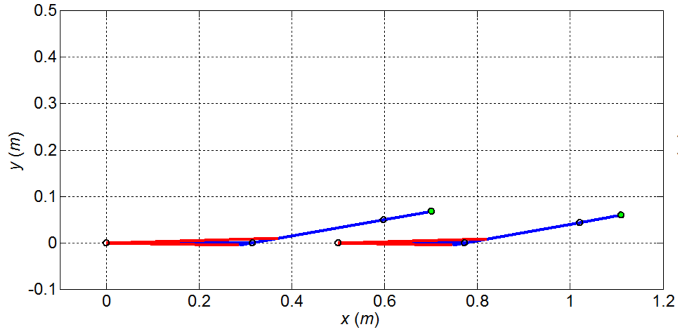
Simulated Muscle Lengths



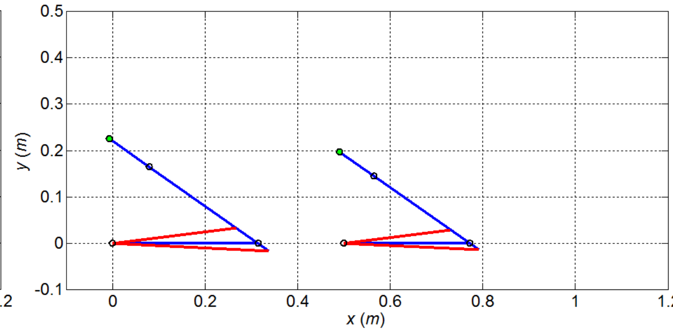
Biceps Pseudostatics Results (t_B vs. t)



Biceps Pseudostatics Results (F_E vs. t)



Initial Poses (male left, female right)



Final Poses (male left, female right)

What is wrong with this solution with only biceps tension, ignoring the triceps tension?

The switch happens after θ_2 crosses through 90°, at about $t = 5.5$ sec in this simulation. Here the effect of gravity switches and the biceps muscle must push, according to our model, which is ***impossible***.

So let's develop an alternate pseudostatics solution, including the triceps tension, this time ignoring the biceps tension.

Inverse Pseudostatics Solution Ignoring Biceps Tension

The Free-Body Diagram (FBD) is identical to that drawn previously, ignoring t_B instead of t_T.

Given: θ_2, L_2, L_3, m_2, m_3, r_{CG23}, r_T, ϕ_T, r_L, W_L, and $t_B = 0$

Find: t_T, F_{Ex}, and F_{Ey}

Static equilibrium vector equations

$$\sum \underline{F} = \underline{t}_T + \underline{F}_E + \underline{W}_{23} + \underline{W}_L = \underline{0}$$
$$\sum \underline{M}_E = \underline{r}_T \times \underline{t}_T + \underline{r}_{CG23} \times \underline{W}_{23} + \underline{r}_L \times \underline{W}_L = \underline{0}$$

Static equilibrium XYZ scalar equations

$$\sum F_x = t_T \cos\phi_T + F_{Ex} = 0$$
$$\sum F_y = t_T \sin\phi_T + F_{Ey} - W_{23} - W_L = 0$$
$$\sum M_{Ez} = r_{Tx} t_T \sin\phi_T - r_{Ty} t_T \cos\phi_T - r_{23x} W_{23} - r_{Lx} W_L = 0$$

These linear equations can be written in matrix form.

$$\begin{bmatrix} \cos\phi_T & 1 & 0 \\ \sin\phi_T & 0 & 1 \\ r_{Tx}\sin\phi_T - r_{Ty}\cos\phi_T & 0 & 0 \end{bmatrix} \begin{Bmatrix} t_T \\ F_{Ex} \\ F_{Ey} \end{Bmatrix} = \begin{Bmatrix} 0 \\ W_{23} + W_L \\ r_{23x} W_{23} + r_{Lx} W_L \end{Bmatrix}$$

We don’t need a matrix to solve since the equations are partially decoupled.

$$t_T = \frac{r_{23x} W_{23} + r_{Lx} W_L}{r_{Tx}\sin\phi_T - r_{Ty}\cos\phi_T}$$
$$F_{Ex} = -t_T \cos\phi_T$$
$$F_{Ey} = -t_T \sin\phi_T + W_{23} + W_L$$

This solution requires length, weight, and CG terms, plus the triceps muscle angle ϕ_T (solved earlier in the human arm kinematics section). The required moment arm terms are given below.

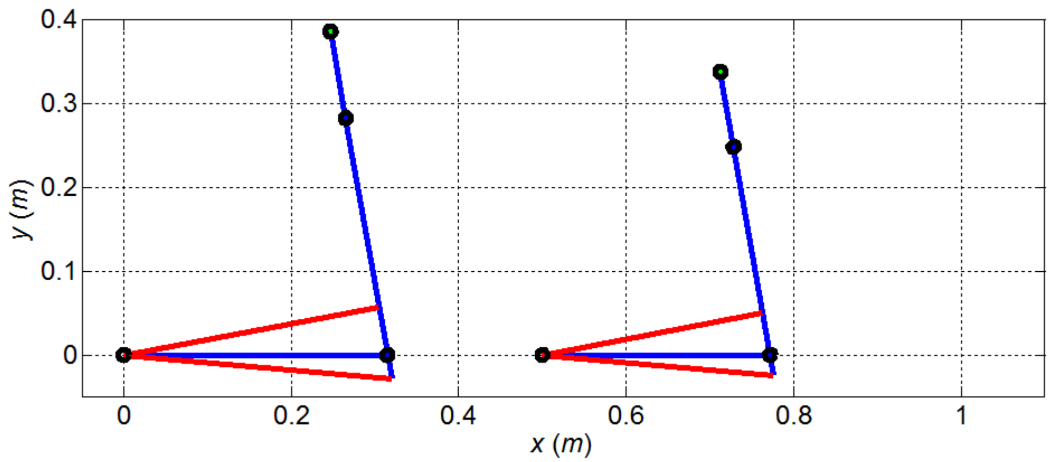
$$\underline{r}_T = \begin{Bmatrix} r_{Tx} \\ r_{Ty} \end{Bmatrix} = \begin{Bmatrix} -r_T \cos\theta_2 \\ -r_T \sin\theta_2 \end{Bmatrix} \qquad \underline{r}_{CG23} = \begin{Bmatrix} r_{23x} \\ r_{23y} \end{Bmatrix} = \begin{Bmatrix} r_{CG23}\cos\theta_2 \\ r_{CG23}\sin\theta_2 \end{Bmatrix} \qquad \underline{r}_L = \begin{Bmatrix} r_{Lx} \\ r_{Ly} \end{Bmatrix} = \begin{Bmatrix} L_{23}\cos\theta_2 \\ L_{23}\sin\theta_2 \end{Bmatrix}$$

<u>Human Elbow Inverse Pseudostatics Examples, Triceps Tension Only</u> **Snapshot solution**

We now give a snapshot result for the triceps-only pseudostatics solution, at $\theta_2 = 100^\circ$

male **female**

$t_T = 79.6$ N $\phi_T = 175.0^\circ$ $F_E = \begin{Bmatrix} 79.3 \\ 37.7 \end{Bmatrix}$ N $t_T = 64.6$ N $\phi_T = 174.9^\circ$ $F_E = \begin{Bmatrix} 64.3 \\ 25.7 \end{Bmatrix}$ N

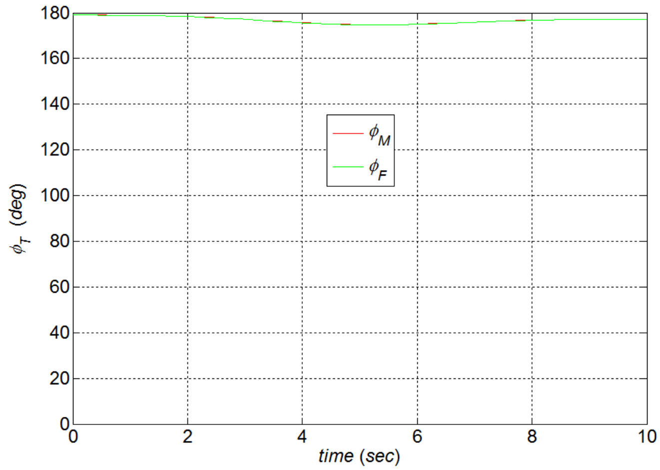


Statics Snapshot Example Poses (male left, female right)

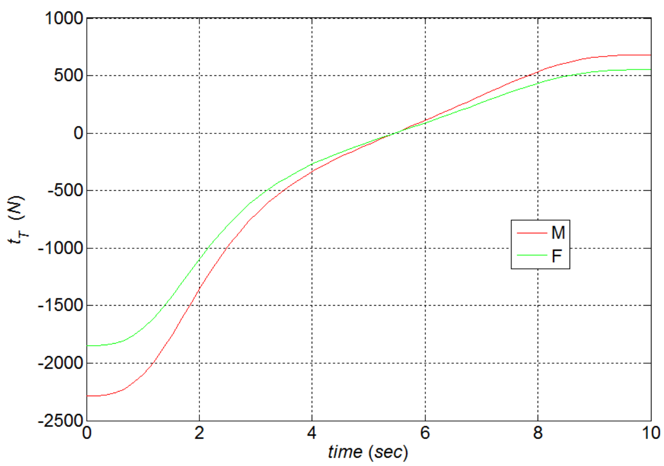
Full Range of Motion Simulation

We now repeat the biceps-tension-only pseudostatics examples for the same male and female human arm models, this time ignoring the biceps tension and including only the triceps tension. All other inputs and parameters are identical to the previous examples.

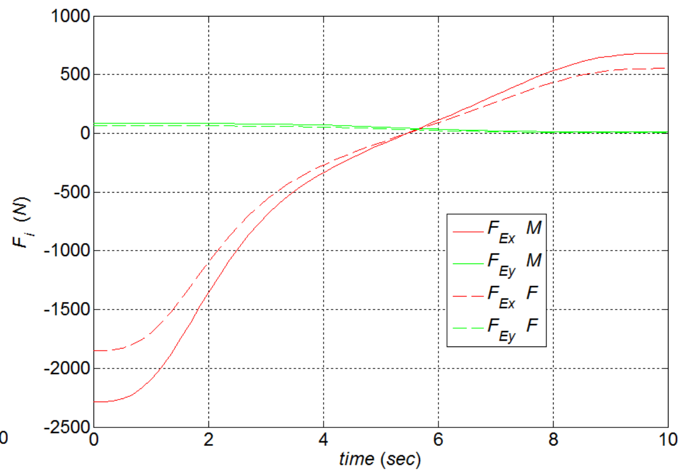
The muscle angles calculations and the arm animations (all kinematics terms) are done in the exact same manner as the biceps-only simulation example above. The input angle and initial and final arm poses plots are not repeated since they are kinematics results that are identical to the previous biceps-tension-only simulation results. The triceps muscle angles plot for the simulated motion, required inputs to the pseudostatics problem, is first given. The results of this triceps-only human arm pseudostatics motion simulation are then given.



Triceps Muscle Angles (ϕ_T vs. t)



Triceps Pseudostatics Results (t_T vs. t)



Triceps Pseudostatics Results (F_E vs. t)

What is wrong with this solution with only triceps tension, ignoring the biceps tension?

To fix this situation, we consider three additional methods, in turn, only the third of which yields acceptable results.

`pinv` Method

The biceps-only and triceps-only pseudostatics solution methods failed due to the requirement of negative muscle tensions in some simulation portions, which is ***impossible***. So let's try to develop an alternate pseudostatics solution, including both the biceps and triceps tensions simultaneously, using a matrix approach for this underconstrained system of equations. We can use the pseudoinverse to solve the equations, which chooses the smallest (magnitudes) for the four unknowns out of the infinite solutions. Here is the underconstrained 3 x 4 set of statics equations:

$$\begin{bmatrix} \cos\phi_B & \cos\phi_T & 1 & 0 \\ \sin\phi_B & \sin\phi_T & 0 & 1 \\ r_{Bx}\sin\phi_B - r_{By}\cos\phi_B & r_{Tx}\sin\phi_T - r_{Ty}\cos\phi_T & 0 & 0 \end{bmatrix} \begin{Bmatrix} t_B \\ t_T \\ F_{Ex} \\ F_{Ey} \end{Bmatrix} = \begin{Bmatrix} 0 \\ W_{23} + W_L \\ r_{23x}W_{23} + r_{Lx}W_L \end{Bmatrix}$$

To solve this underconstrained set of equations, use MATLAB function **`pinv`** (pseudoinverse). Note the pseudoinverse has a completely different meaning than pseudostatics. From the infinite solutions, the pseudoinverse chooses the solution with the smallest vector norm – a realistic, physiological choice for human muscle applications.

Let the 3x4 coefficient matrix in the above equation be called [A]. Then the pseudoinverse of [A] is:

$$[A]^* = [A]^T([A][A]^T)^{-1}$$

This approach is also called the least-squares solution since $\sqrt{t_B^2 + t_T^2 + F_{Ex}^2 + F_{Ey}^2}$ is minimized out of the infinite possible solutions. This is a good approach as the human body tends to minimize muscle forces with the muscle actuation redundancy. The elbow reaction force is also minimized, which may have a biological basis.

We now give a snapshot result for the biceps/triceps statics **`pinv`** solution method, at $\theta_2 = 30^\circ$.

male

$t_B = 289.1$ N $\quad \phi_B = -175.5^\circ \qquad t_T = -248.0$ N $\quad \phi_T = 177.2^\circ \qquad F_E = \begin{Bmatrix} 40.5 \\ 79.6 \end{Bmatrix}$ N

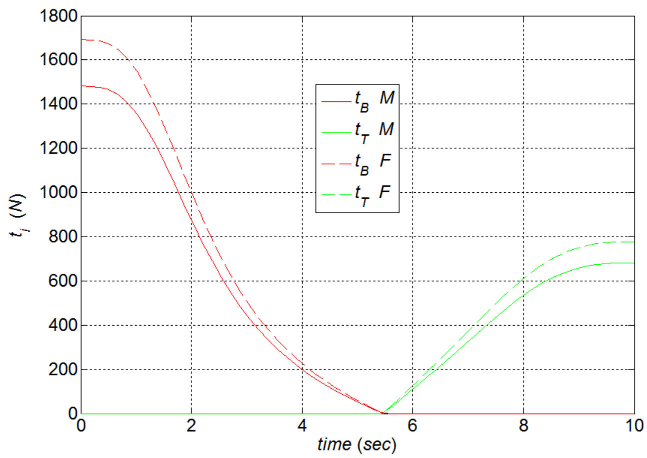
female

$t_B = 329.5$ N $\quad \phi_B = -175.4^\circ \qquad t_T = -283.0$ N $\quad \phi_T = 177.1^\circ \qquad F_E = \begin{Bmatrix} 45.9 \\ 85.1 \end{Bmatrix}$ N

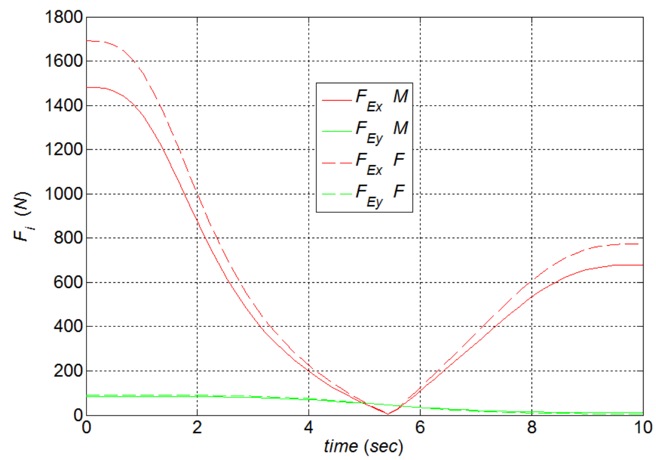
This method is no good since, though the muscle tensions are minimized, the method knows nothing about the need to keep muscle tensions all positive for all motion. Therefore, we will not even simulate the results for all motion.

`lsqnonneg` Method

Instead, we will try MATLAB function **`lsqnonneg`**, which also solves the underconstrained least-squares problem, but enforces that all solution components must be greater than or equal to 0. The results of the human arms pseudostatics motion simulation with simultaneous biceps and triceps tensions from **`lsqnonneg`** are given below.



t_B and t_T vs. t



F_E vs. t

`lsqnonneg` Pseudostatics Results

Did this solution succeed? Not really:

1. The muscle forces are allowed to go to zero – in reality there will always be some small positive antagonist tensions.

2. Also, the elbow reaction forces can be +/–; there is no need to limit those to +, but **`lsqnonneg`** does.

Acceptable Inverse Pseudostatics Simulation

All four pseudostatics modeling methods so far have not succeeded completely (assuming $t_B = 0$, assuming $t_T = 0$, the **`pinv`** method, and the **`lsqnonneg`** method). We cannot use a method in which part of the time either the triceps or the biceps must ***push*** to accomplish the pseudostatic motion, which is ***impossible***.

For a more advanced modeling approach, let us put a pre-tension on the triceps muscle (antagonist) as the biceps muscle (agonist) is working. Before the biceps loses tension, this pre-tensioning will have to be switched to the biceps (as the biceps and triceps reverse their agonist/antagonist roles). This will yield all positive muscle tensions plus realistic elbow reaction forces and would be a better model for real-world human biomechanics.

Here are the statics equations for specifying a pre-tension on t_T and solving for t_B:

$$\begin{bmatrix} \cos\phi_B & 1 & 0 \\ \sin\phi_B & 0 & 1 \\ r_{Bx}\sin\phi_B - r_{By}\cos\phi_B & 0 & 0 \end{bmatrix} \begin{Bmatrix} t_B \\ F_{Ex} \\ F_{Ey} \end{Bmatrix} = \begin{Bmatrix} 0 \\ W_{23} + W_L \\ r_{23x}W_{23} + r_{Lx}W_L \end{Bmatrix} - \begin{Bmatrix} \cos\phi_T \\ \sin\phi_T \\ r_{Tx}\sin\phi_T - r_{Ty}\cos\phi_T \end{Bmatrix} t_T$$

Whose solution is:

$$\begin{aligned} t_B &= \frac{r_{23x}W_{23} + r_{Lx}W_L - (r_{Tx}\sin\phi_T - r_{Ty}\cos\phi_T)t_T}{r_{Bx}\sin\phi_B - r_{By}\cos\phi_B} \\ F_{Ex} &= -t_B\cos\phi_B - t_T\cos\phi_T \\ F_{Ey} &= -t_B\sin\phi_B - t_T\sin\phi_T + W_{23} + W_L \end{aligned}$$

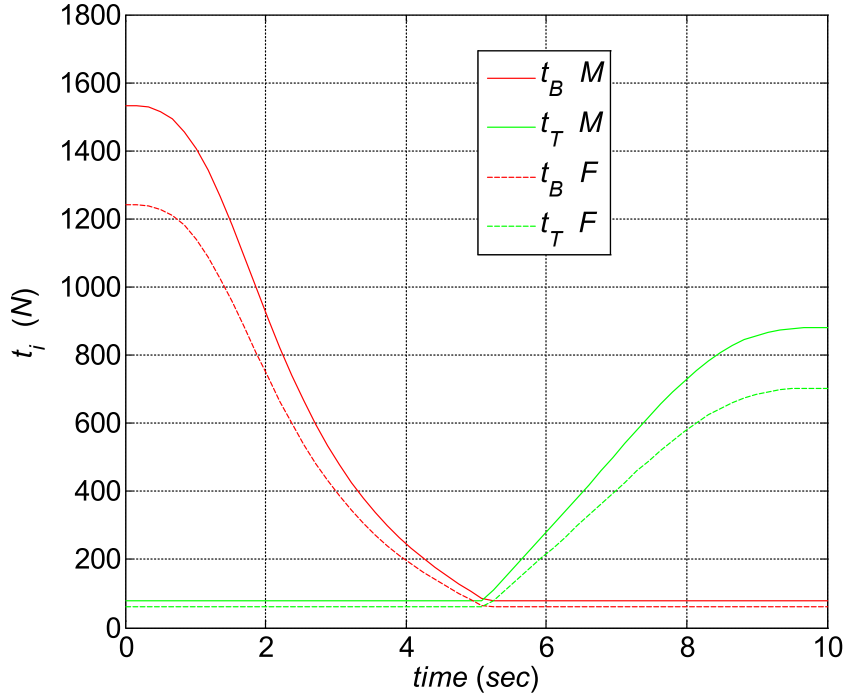
Here are the statics equations for specifying a pre-tension on t_B and solving for t_T:

$$\begin{bmatrix} \cos\phi_T & 1 & 0 \\ \sin\phi_T & 0 & 1 \\ r_{Tx}\sin\phi_T - r_{Ty}\cos\phi_T & 0 & 0 \end{bmatrix} \begin{Bmatrix} t_T \\ F_{Ex} \\ F_{Ey} \end{Bmatrix} = \begin{Bmatrix} 0 \\ W_{23} + W_L \\ r_{23x}W_{23} + r_{Lx}W_L \end{Bmatrix} - \begin{Bmatrix} \cos\phi_B \\ \sin\phi_B \\ r_{Bx}\sin\phi_B - r_{By}\cos\phi_B \end{Bmatrix} t_B$$

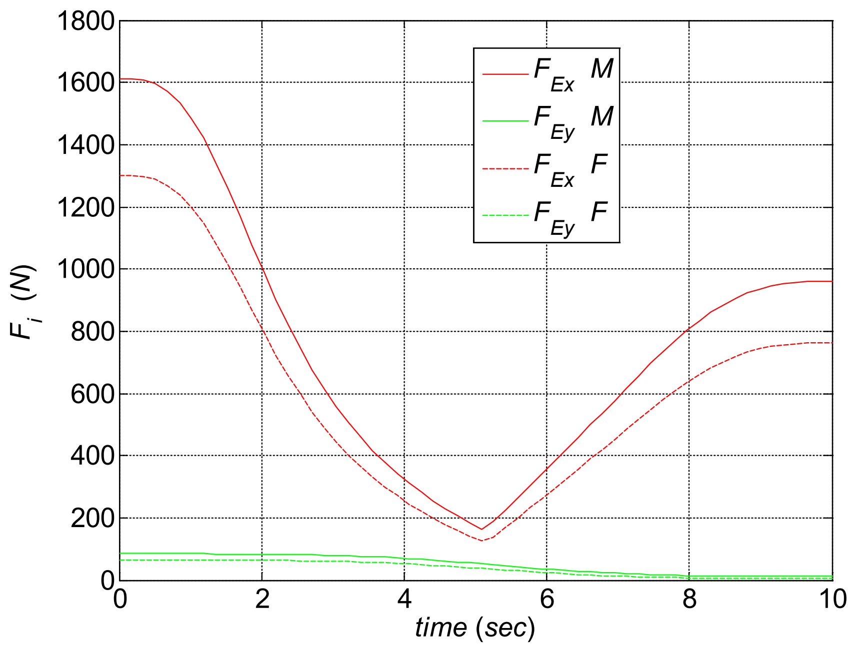
Whose solution is:

$$\begin{aligned} t_T &= \frac{r_{23x}W_{23} + r_{Lx}W_L - (r_{Bx}\sin\phi_B - r_{By}\cos\phi_B)t_B}{r_{Tx}\sin\phi_T - r_{Ty}\cos\phi_T} \\ F_{Ex} &= -t_T\cos\phi_T - t_B\cos\phi_B \\ F_{Ey} &= -t_T\sin\phi_T - t_B\sin\phi_B + W_{23} + W_L \end{aligned}$$

Using the same conditions for the adult male and female arm models as presented in the two previous examples, the pseudostatics results for this new agonist/antagonist method are shown below.



Pseudostatics Results (t_B and t_T vs. t)



Pseudostatics Results (F_E vs. t)

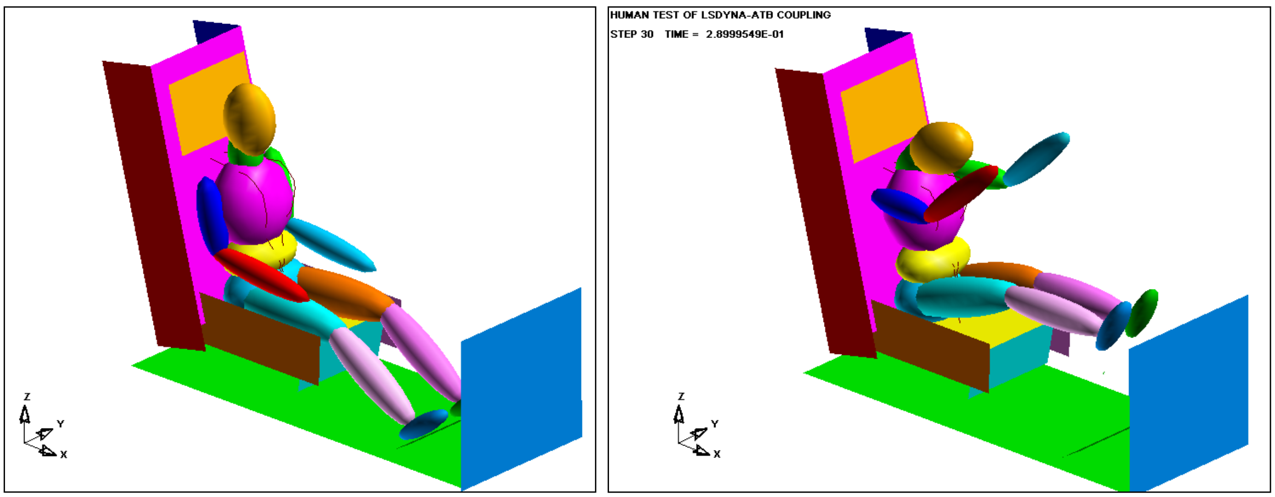
In this simulation the chosen antagonist pre-tensions are 80 and 60 N, for the adult male and female, respectively. We see that the biceps and triceps muscles reverse their agonist/antagonist roles after θ_2 crosses through 83.4°, at about t = 5.3 sec. Note this behavior is different from the zero-antagonist-tension methods, where the muscle roles were reversed when θ_2 crossed through 90° (why?).

6. Human Body Engineering Mechanics: Dynamics

<u>Dynamics</u> is the study of motion <u>with regard</u> to **forces**/**moments**. **Kinematics** motion equations are required prior to **dynamics** analysis. **Kinematics** includes the **translational** and **rotational position**, **velocity**, and **acceleration** analyses. The mathematics of **robotics** dynamics (Craig, 2005) applies well to human body rigid-body segmental dynamics.

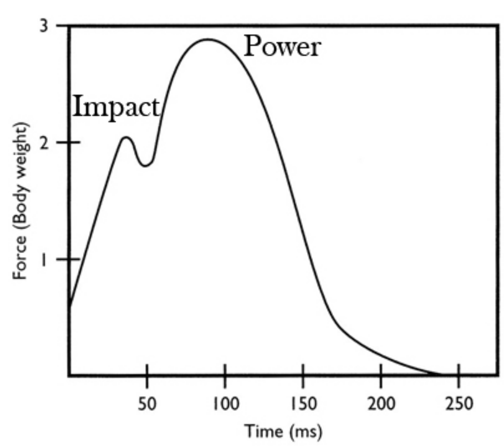
6.1 Human Body Dynamics

Below is shown an example which clearly requires dynamics analysis. The simulation of human body motion during aircraft and/or automobile crashes involves large velocities and accelerations/ decelerations, leading to high dynamic forces and moments. Pseudostatics analysis would be totally inadequate to handle this type of dynamics simulation.



ATB-LSDYNA3D Crash Dynamics Simulation (Williams et al., 2001)

As another example requiring dynamics analysis, consider human running. As the figure below shows, the dynamic force on the human leg during running can be nearly 3 times the body weight. By contrast, using pseudostatic analysis, the maximum load would be about equal to body weight. Therefore, pseudostatics analysis is also inadequate to handle human running dynamics simulation.



Hall (2007)

There are two peaks, the first reaching two times body weight upon impact of the running foot with the ground. An even higher peak follows closely, up to three times body weight during the power stroke on the ground from the leg. How does walking compare? Cycling?

Dynamics Concepts Review

Translational and Rotational Acceleration

$$A = \frac{dV}{dt} = \frac{d^2X}{dt^2} \qquad\qquad \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \text{ (for planar 1-dof rotations only)}$$

Free-Body Diagrams (FBDs) including Internal and External Forces and Moments

Mass *m*

Center of Mass (Center of Gravity, *CG* – see Statics section)

Mass Moment of Inertia *I*

This is rotational inertia, i.e. resistance to change in angular velocity.
The units are kg-m^2.
I is the proportionality constant in Euler's rotational dynamics equation.

Appendix D presents the major human body segment mass properties of mass, CG, and mass moments of inertia. Review the **radius of gyration**.

Dynamics Methods

Newton-Euler Dynamics Equations of Motion

Newton's Second Law (translational)

Euler's Law (rotational)

Conservation of Linear Momentum

The total linear momentum $p = mv$ of a closed system with no external forces is constant. One consequence is that the center of mass of any system of objects will always continue with the same velocity unless acted on by an outside force.

This law is implied by Newton's first law of motion. Newton's Third Law of motion is also due to the conservation of momentum.

$$m_1 v_1 = m_2 v_2$$

Conservation of Angular Momentum

The total angular momentum ($L = r$ x p) of a closed system of objects (with no external torques) is constant.

$$r \times (m_1 v_1) = r \times (m_2 v_2)$$

Work-Energy Method

Work applied to a body from time 1 to 2 equals the change in energy.

$${}^{1}W_2 = E_2 - E_1$$

Work

translational	$F \bullet \Delta x$	rotational	$\tau \bullet \Delta \theta$

Potential energy (spring and gravity)

translational	$\frac{1}{2} k \Delta x^2$	mgh	rotational	$\frac{1}{2} k_R \Delta \theta^2$	$mgLsin\theta$

Kinetic energy

translational	$\frac{1}{2} mv^2$	rotational	$\frac{1}{2} I \omega^2$

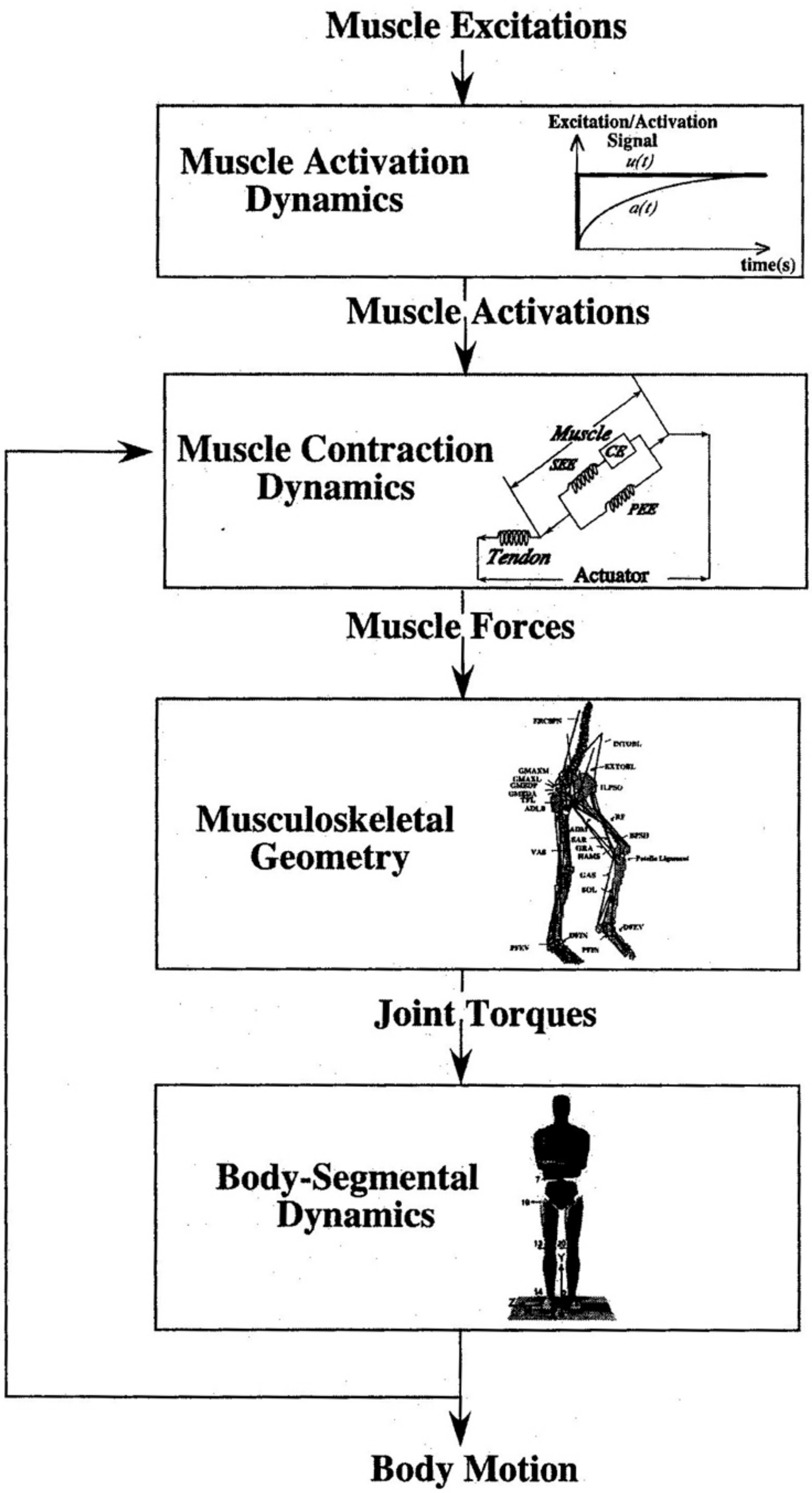
Impulse-Momentum Method

This method is useful for collision dynamics and is derived from Newton's Second Law.

$$F \Delta t = m\left(v_2 - v_1\right)$$

The impulse $F \Delta t$ equals the change in momentum $m\left(v_2 - v_1\right)$.

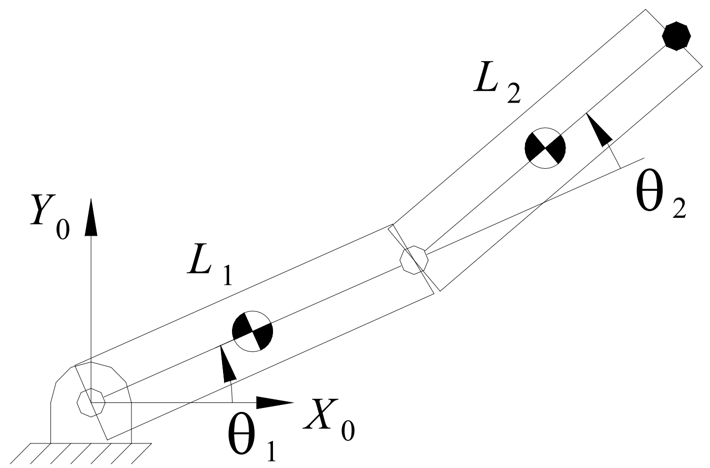
Complexity of Human Body Dynamics



Kutz (2003)

Dynamics Equations of Motion Example (Body Segmental Dynamics)

The **dynamic equations of motion** for the two-link planar 2-dof, 2R arm are given below. This could serve as a planar model for the human arm (shoulder-elbow/hand) or the human leg (hip-knee/foot).



Dynamic equations of motion

$$\tau_1 = \left(I_{zz1} + I_{zz2} + \frac{m_1 L_1^2}{4} + m_2 L_1^2 + m_2 L_1 L_2 c_2 + \frac{m_2 L_2^2}{4} \right) \ddot{\theta}_1 + \left(I_{zz2} + \frac{m_2 L_1 L_2 c_2}{2} + \frac{m_2 L_2^2}{4} \right) \ddot{\theta}_2 + \left(-\frac{m_2 L_1 L_2 s_2}{2} \right) \dot{\theta}_2^2$$

$$+ \left(-m_2 L_1 L_2 s_2 \right) \dot{\theta}_1 \dot{\theta}_2 + \left(\frac{m_1 L_1 c_1}{2} + m_2 L_1 c_1 + \frac{m_2 L_2 c_{12}}{2} \right) g$$

$$\tau_2 = \left(I_{zz2} + \frac{m_2 L_1 L_2 c_2}{2} + \frac{m_2 L_2^2}{4} \right) \ddot{\theta}_1 + \left(I_{zz2} + \frac{m_2 L_2^2}{4} \right) \ddot{\theta}_2 + \left(\frac{m_2 L_1 L_2 s_2}{2} \right) \dot{\theta}_1^2 + \left(\frac{m_2 L_2 c_{12}}{2} \right) g$$

Dynamic equations of motion expressed in matrix/vector form

$$\tau = M(\Theta)\ddot{\Theta} + B(\Theta)\left[\dot{\Theta}\dot{\Theta}\right] + C(\Theta)\left[\dot{\Theta}^2\right] + G(\Theta) \qquad \tau = \begin{Bmatrix} \tau_1 \\ \tau_2 \end{Bmatrix} \qquad \ddot{\Theta} = \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix}$$

$$M(\Theta) = \begin{bmatrix} I_{zz1} + I_{zz2} + \frac{m_1 L_1^2}{4} + m_2 L_1^2 + m_2 L_1 L_2 c_2 + \frac{m_2 L_2^2}{4} & I_{zz2} + \frac{m_2 L_1 L_2 c_2}{2} + \frac{m_2 L_2^2}{4} \\ I_{zz2} + \frac{m_2 L_1 L_2 c_2}{2} + \frac{m_2 L_2^2}{4} & I_{zz2} + \frac{m_2 L_2^2}{4} \end{bmatrix}$$

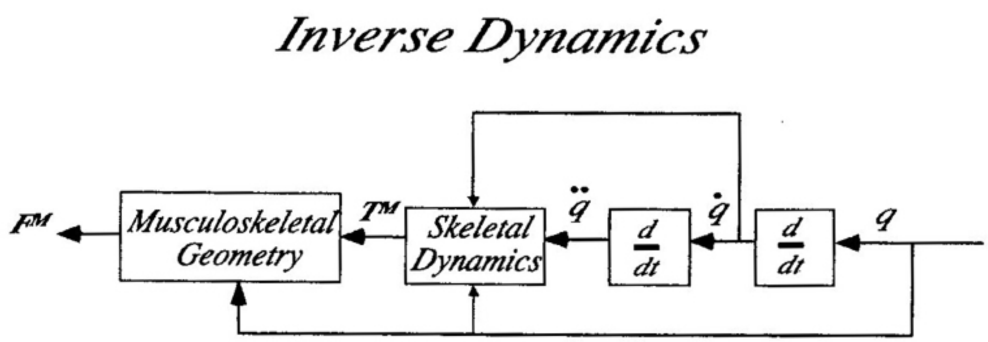
$$B(\Theta)\left[\dot{\Theta}\dot{\Theta}\right] = \begin{Bmatrix} -m_2 L_1 L_2 s_2 \\ 0 \end{Bmatrix} \dot{\theta}_1 \dot{\theta}_2$$

$$C(\Theta)\left[\dot{\Theta}^2\right] = \begin{bmatrix} 0 & -\frac{m_2 L_1 L_2 s_2}{2} \\ \frac{m_2 L_1 L_2 s_2}{2} & 0 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{Bmatrix} \qquad G(\Theta) = \begin{Bmatrix} \frac{m_1 L_1 c_1}{2} + m_2 L_1 c_1 + \frac{m_2 L_2 c_{12}}{2} \\ \frac{m_2 L_2 c_{12}}{2} \end{Bmatrix} g$$

Dynamics Problems

We see the dynamics equations of motion are a set of n complicated, highly coupled and highly non-linear ordinary differential equations – try to imagine the symbolic dynamics terms for a 7-dof spatial human arm!

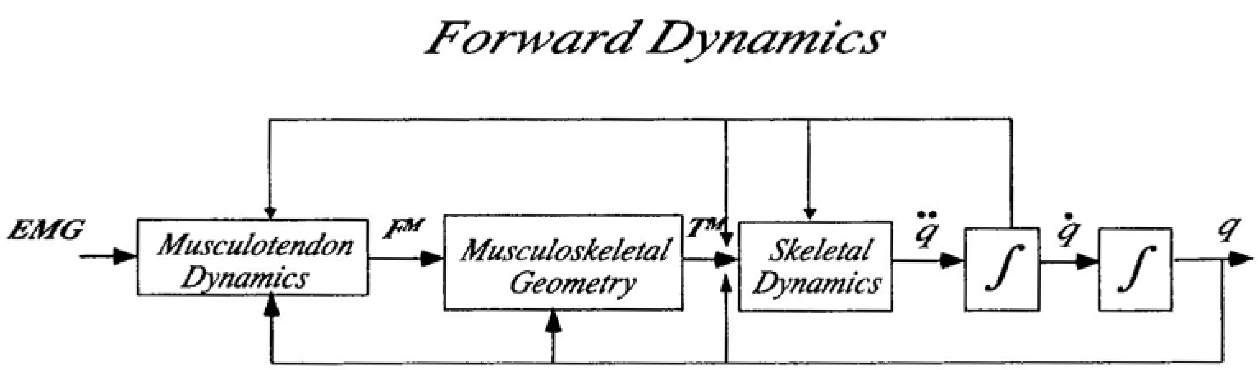
The same set of equations may be used in two ways (q stands for θ in these figures).



Kutz (2003)

Inverse Dynamics

Given the motion, calculate the muscle actuation forces



Kutz (2003)

Forward Dynamics

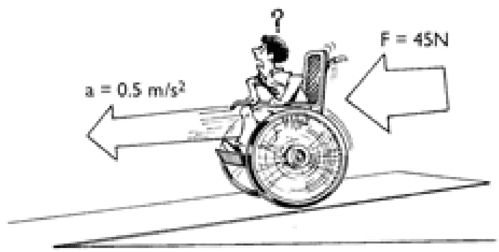
Given the muscle actuation forces, calculate the motion

Which problem is easier? Why?

<u>Human Body Dynamics Examples</u> **Translational dynamics problems**

<u>Example 1</u>

A human sits in a 20 kg wheelchair at the top of an inclined ramp as shown. Upon release from rest, the wheelchair accelerates down the ramp with a constant acceleration of $a = 0.5$ m/s^2 along the ramp. If the resultant force causing this motion is $F = 45$ N, what is the human's mass? What is the ramp angle θ? How much time t is required for the wheelchair to travel 10 m down the ramp?



Hall (2007)

Solution

FBD (align X down along the ramp and Y up perpendicular to the ramp)

F_H is the external force acting on the human/wheelchair

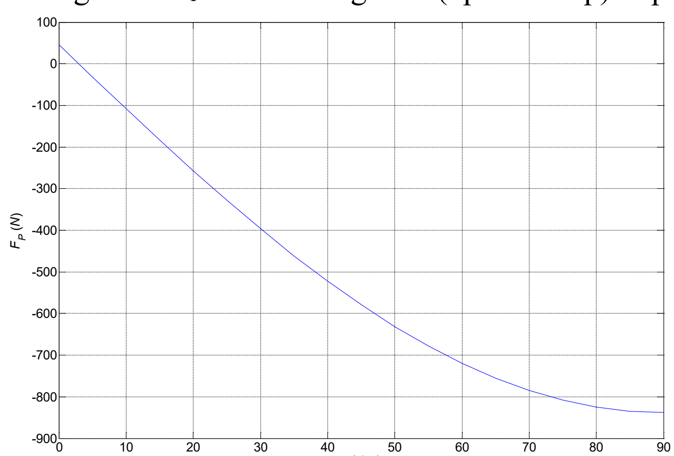
F_f is the friction force

$$\sum F_X = ma_X$$

$$F = (m_H + m_W)a$$
$$45 = (m_H + 20)0.5$$
$$m_H = \frac{45}{0.5} - 20 = 70 \quad \text{kg}$$

$$F = F_H + W\sin\theta - F_f = ma$$
$$45 = F_H - F_f + mg\sin\theta = F_P + 882.9\sin\theta$$
$$F_P = 45 - 882.9\sin\theta$$

Regarding ramp angle θ, we need a more detailed equation (above right), where the net pushing force $F_H - F_f$ has been named F_P. So we see there is a range of allowable θ. For ramp angles greater than 2.9°, ($\theta = \sin^{-1}(a/g)$) the net pushing force F_P must be negative (up the ramp) to provide this motion.

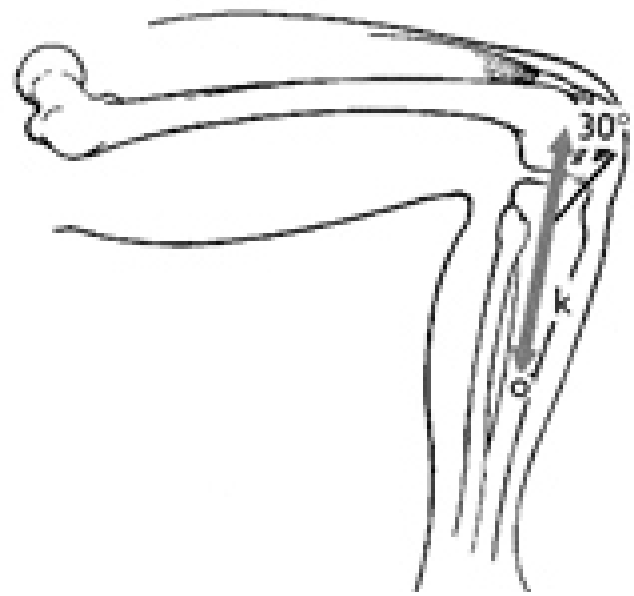


For constant acceleration a, the final time may be found from the distance traveled.

$$s(t) = s_0 + v_0 t + \frac{1}{2}at^2 \qquad s(t) = 10 = 0 + 0(t) + \frac{0.5}{2}t^2 \Rightarrow t = \sqrt{40} = 6.3 \quad \text{sec}$$

<u>Example 2</u> **Rotational dynamics problem**

The knee extensors insert on the tibia at an angle of 30°, with a moment arm of 3 cm (from the knee to the insertion point along the tibia. At the instant where the combined lower leg/foot CG is directly below the knee hinge joint, how much muscle force F_m is required to generate a lower leg/foot angular acceleration of $\alpha = 2$ rad/s^2? The combined lower leg/foot mass and radius of gyration are $m = 5$ kg and $k = 25$ cm, respectively.



Hall (2007)

Solution

This problem is solved by using Euler's Rotational Dynamics Law, the rotational equivalent of Newton's Second Law. The lower leg/foot weight does not contribute to the moment since its moment arm is zero at this instant (the drawing is not quite correct; the lower leg should be perfectly vertical).

$$\sum M_{KZ} = I_{KZ}\alpha$$

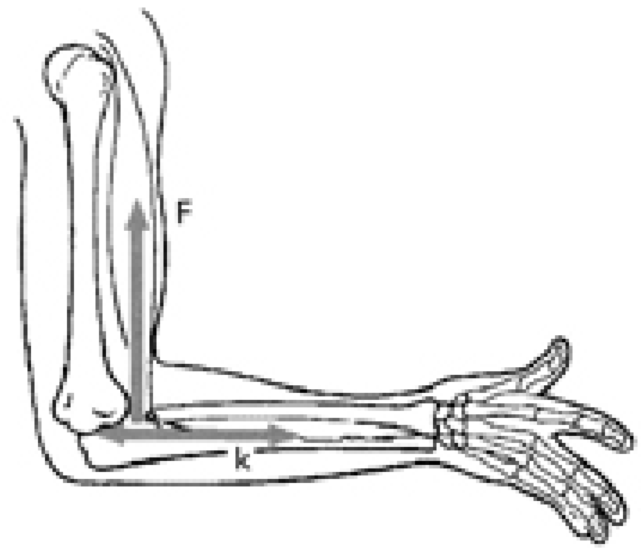
$$F_m \sin\theta (d_m) = mk^2\alpha$$

$$F_m = \frac{mk^2\alpha}{d_m \sin\theta}$$

$$F_m = \frac{5(0.25)^2(2)}{(0.03)\sin 30^\circ} = 41.7 \quad \text{N}$$

Example 3 **Rotational impulse-momentum problem**

How much average force F must be exerted by the biceps brachii inserting d = 3.5 cm from the elbow hinge joint to stop the combined forearm/hand segment in 0.5 sec with an initial angular velocity of $\omega = -3$ rad/s (CW)? Assume the forearm/hand mass is m = 4 kg and the radius of gyration is k = 22 cm. Further assume that the biceps brachii is perpendicular to the radius for this short time span.



Hall (2007)

Solution

This problem is solved with the angular impulse-momentum method. The angular impulse applied by the biceps brachii equals the change in angular momentum for the forearm/hand.

$$\int \tau dt = \Delta H$$

$$\tau(t_2 - t_1) = I_{EZ}(\omega_2 - \omega_1)$$

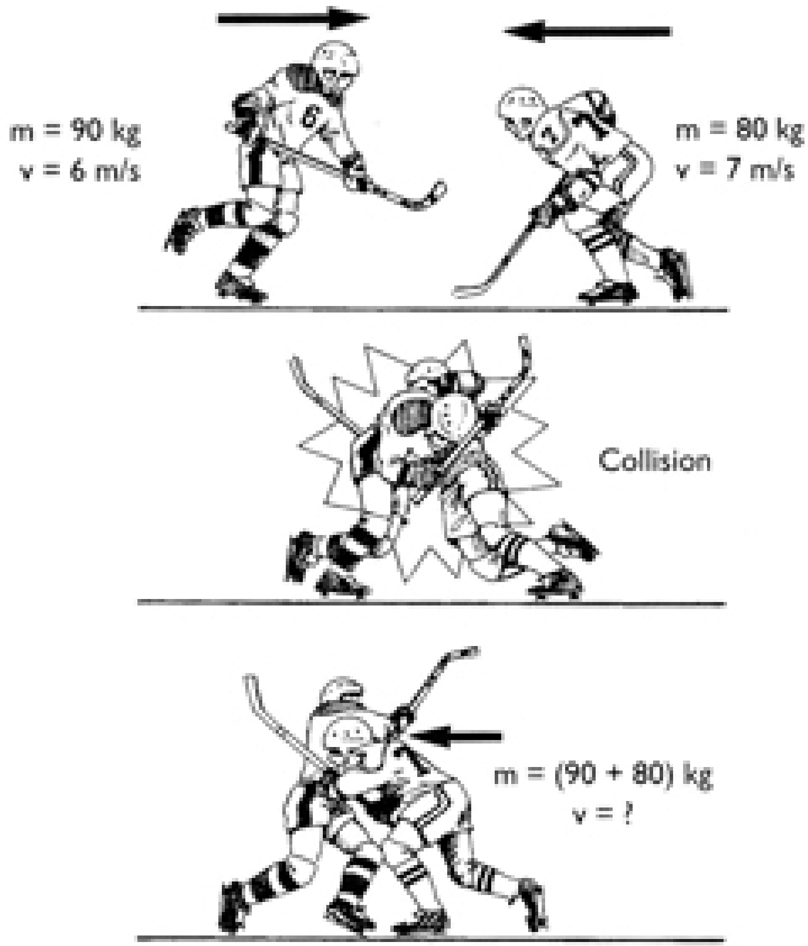
$$Fd(t_2 - t_1) = mk^2(\omega_2 - \omega_1)$$

$$F = \frac{mk^2(\omega_2 - \omega_1)}{d(t_2 - t_1)}$$

$$F = \frac{4(0.22)^2(0-(-3))}{0.035(0.5-0)} = 33.2 \quad \text{N}$$

Example 4 **Translational conservation of momentum problem**

A 90 kg hockey player skating to the right at 6 m/s collides with an 80 kg hockey player skating to the left at 7 m/s. What is their combined velocity after the entangling collision?



Hall (2007)

Solution

If we assume a perfect collision with the two players traveling as one following the collision, plus zero non-conservative forces, the principle of conservation of momentum applies. The combined momentum of the entangled players after the collision equals the sum of their individual momenta prior to the collision. Taking positive to the right:

$$p_{\text{before}} = p_{\text{after}}$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$$

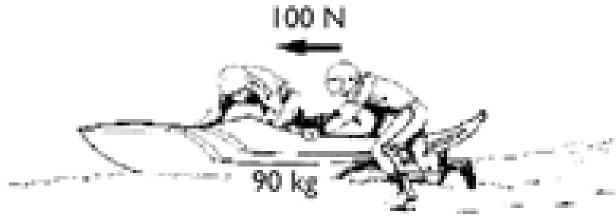
$$v = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$v = \frac{90(6) - 80(7)}{90 + 80} = -0.12 \quad \text{m/s}$$

The hockey players move to the left after the collision, in the direction of the smaller, but faster player.

Example 5 **Translational impulse-momentum problem**

A pair of toboggan crew members exert an average force of $F = 100$ N for 7 sec to a 90 kg toboggan at rest to start their run. Neglecting friction, what is the toboggan speed after that 7 sec?



Hall (2007)

Solution

This problem is solved with the impulse-momentum method. The impulse applied by the crew equals the change in momentum for the toboggan (the velocity direction is the same as the applied force direction):

$$\int F dt = \Delta(mv)$$

$$F(t_2 - t_1) = m_2 v_2 - m_1 v_1$$

$$100(7-0) = 90(v_2 - 0)$$

$$v_2 = \frac{700}{90} = 7.78 \quad \text{m/s}$$

Alternate Solution

Like many engineering mechanics problems, this problem may be solved using a different method. If we assume that the given average force is constant, the resulting constant acceleration is found from Newton's Second Law.

$$F = ma$$

$$100 = 90a$$

$$a = \frac{100}{90} = 1.11 \quad \text{m/s}^2$$

Then kinematics is used to calculate the toboggan speed at the end of 7 seconds.

$$v_2 - v_1 = at$$

$$v_2 - 0 = 1.11(7)$$

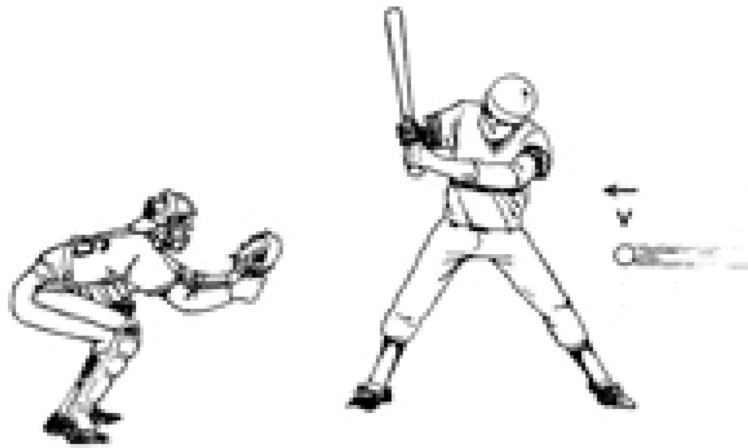
$$v_2 = 7.78 \quad \text{m/s}$$

We see the alternate solution yields the same result. What is the distance traveled during this 7 second acceleration phase of motion?

$$s_2 = s_1 + v_1 t + \frac{at^2}{2} = 0 + 0(7) + \frac{1.11(7)^2}{2} = 27.22 \quad \text{m}$$

Example 6 **Translational work-energy problem**

How much work is required to stop a 0.145 kg baseball* moving at a speed of 44 m/s?



Hall (2007)

Solution

This problem is solved with the work-energy method. The total energy of the baseball must be balanced by the work of the catcher's mitt on the ball. Assume the ball energy is kinetic T only, i.e. no significant change in elevation occurs during the pitch.

$$W = \Delta T$$

$$W = \frac{1}{2}m_2 v_2^2 - \frac{1}{2}m_1 v_1^2 = \frac{1}{2}m(v_2^2 - v_1^2)$$

$$W = \frac{1}{2}(0.145)(0 - 44^2) = -140.4 \quad \text{J}$$

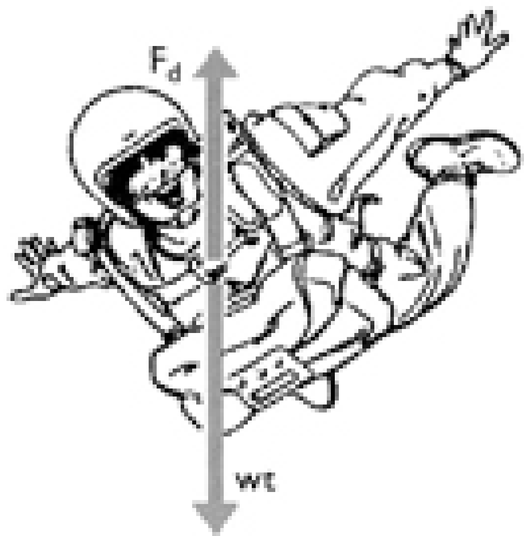
The required work is negative, indicating that the work must be done by the catcher on the ball, in the opposite direction of the ball velocity, to stop the ball.

A baseball speed of 44 m/s is equal to 98.4 mph, really smoking! The world record is 105.1 mph, set in 2010. At youth and amateur levels, the energy required to stop a baseball is much less, due to the squaring of the speed.

*The mass of a standard major-league baseball must be between 142 and 149 grams.

Example 7 **Translational dynamics problem**

An 80 kg free-falling skydiver is accelerating down at 8 m/s^2 rather than g = 9.81 m/s^2 due to the air resistance. Calculate the drag force acting on the skydiver at this instant.



Hall (2007)

Solution

The FBD is shown above. We use Newton's Second Law in the vertical direction, taking down to be positive:

$$\sum F_Y = ma_Y$$

$$wt - F_d = ma_Y$$

$$mg - F_d = ma_Y$$

$$80(9.81) - F_d = 80(8)$$

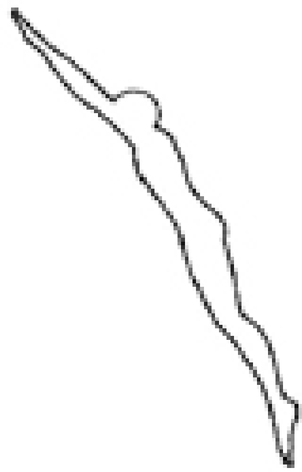
$$F_d = 80(9.81 - 8) = 144.8 \quad \text{N}$$

Terminal velocity is the final speed (down vertically) of a free-falling object after the resultant forces acting on it are zero. That is, a falling body will cease to accelerate down when the drag force acting up equals the weight force acting down. Once this condition occurs, the remainder of the descent motion will be under constant velocity (the terminal velocity), until the ground is reached.

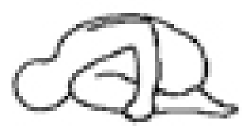
Example 8 **Conservation of angular momentum problem**

A 70 kg diver has a radius of gyration of 0.6 m upon leaving the diving board (Position 1) with an angular velocity of 2 rad/s. What is the diver's angular velocity in the tuck phase (Position 2), with a radius of gyration of 0.3 m?

Position 1



Position 2



Hall (2007)

Solution

Assuming zero non-conservative external forces and moments acting on the diver, the principle of conservation of angular momentum applies. In Position 1 the angular momentum is equal to the angular momentum of Position 2.

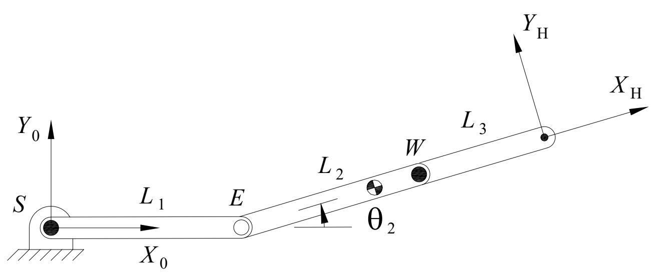
$$
\begin{aligned}
H_1 &= H_2 \\
I_1\omega_1 &= I_2\omega_2 \\
m_1k_1^2\omega_1 &= m_2k_2^2\omega_2 \\
\omega_2 &= \frac{m_1k_1^2\omega_1}{m_2k_2^2} = \frac{mk_1^2\omega_1}{mk_2^2} = \frac{k_1^2\omega_1}{k_2^2} \\
\omega_2 &= \frac{(0.6)^2(2)}{(0.3)^2} = 8 \quad \text{rad/s}
\end{aligned}
$$

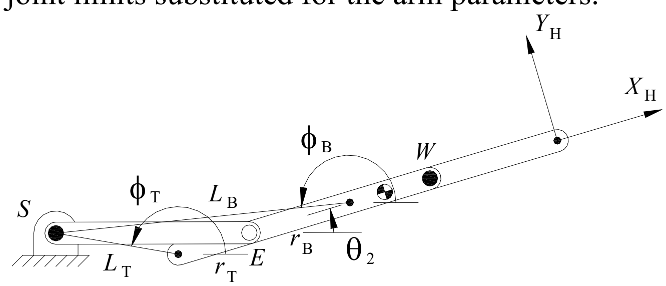
6.2 Human Arm Dynamics

This section presents a simplified model for the dynamics of the planar 1-dof human arm, with biceps and triceps muscles inserting near the elbow, and the shoulder and wrist angles fixed to zero.

Biceps/Triceps Dynamics Examples

The left figure below shows the simplified 1-dof planar human arm model with an R joint at the elbow only. The planar shoulder and wrist pitch motions have been locked (indicated by the solid R joints). Note this will also serve as a simplified 1-dof leg model, with the knee joint in motion and the hip and ankle joints locked, with appropriate lengths and joint limits substituted for the arm parameters.





L_1 is the upper arm length, L_2 is the forearm length, and L_3 is the hand length. $\theta_1 = 0$ is the locked absolute shoulder pitch angle, θ_2 is the variable elbow pitch angle, and $\theta_3 = 0$ is the locked relative wrist pitch angle. The CG symbol above is for the combined forearm/hand segment, located a distance r_{CG23} from the elbow joint, along the forearm. A weight W_L is lifted at the tip of the hand (not shown), at a distance r_L from the elbow joint, along the forearm/hand. The right figure above shows biceps and triceps attachments (origins and insertions), lengths, and angles.

This is identical to the model used for pseudostatics. The dynamics FBD is identical and the dynamics solutions will be similar to the pseudostatics solutions, but we have to include inertial terms.

$$\sum \underline{F} = m\underline{A}_G \qquad\qquad \sum \underline{M}_E = I_{EZ}\underline{\alpha}$$

Human Elbow/Forearm Inverse Dynamics Problem Statement

Given: L_2, L_3, m_2, m_3, r_{CG23}, I_{EZ23}, r_L, r_B, r_T, ϕ_B, ϕ_T, W_L, θ_2, ω_2, and α_2

Find: t_B, t_T, F_{Ex}, and F_{Ey}

1-dof Dynamics Solution

Free-Body Diagram (FBD)

From the FBD we derive the Newton-Euler equations. These three scalar equations in four scalar unknowns are underconstrained, with infinite solutions.

$$\cos\phi_B t_B + \cos\phi_T t_T + F_{Ex} = m_{23} A_{G23x}$$
$$\sin\phi_B t_B + \sin\phi_T t_T + F_{Ey} - W_{23} - W_L = m_{23} A_{G23y}$$
$$(r_{Bx}\sin\phi_B - r_{By}\cos\phi_B)t_B + (r_{Tx}\sin\phi_T - r_{Ty}\cos\phi_T)t_T - r_{23x}W_{23} - r_{Lx}W_L = I_{EZ23}\alpha_2$$

Below are the matrix-vector equations from the pseudostatics model. The resulting dynamics equations are nearly identical to the pseudostatics equations – we need only include the three inertial terms.

$$\begin{bmatrix} \cos\phi_B & \cos\phi_T & 1 & 0 \\ \sin\phi_B & \sin\phi_T & 0 & 1 \\ r_{Bx}\sin\phi_B - r_{By}\cos\phi_B & r_{Tx}\sin\phi_T - r_{Ty}\cos\phi_T & 0 & 0 \end{bmatrix} \begin{Bmatrix} t_B \\ t_T \\ F_{Ex} \\ F_{Ey} \end{Bmatrix} = \begin{Bmatrix} \\ +W_{23} + W_L \\ +r_{23x}W_{23} + r_{Lx}W_L \end{Bmatrix}$$

There are 4 scalar unknowns and 3 scalar equations. This set of equations is underconstrained, i.e. there are more unknowns than equations – infinite solutions to the 4 scalar unknowns exist in general. So let us once again **ignore the triceps tension** for now.

Here are the dynamics equations ignoring triceps tension.

$$\begin{bmatrix} \cos\phi_B & 1 & 0 \\ \sin\phi_B & 0 & 1 \\ r_{Bx}\sin\phi_B - r_{By}\cos\phi_B & 0 & 0 \end{bmatrix} \begin{Bmatrix} t_B \\ F_{Ex} \\ F_{Ey} \end{Bmatrix} = \begin{Bmatrix} m_{23}A_{G23x} \\ m_{23}A_{G23y} + W_{23} + W_L \\ I_{EZ23}\alpha_2 + r_{23x}W_{23} + r_{Lx}W_L \end{Bmatrix}$$

The solution is

$$t_B = \frac{I_{EZ23}\alpha_2 + r_{23x}W_{23} + r_{Lx}W_L}{r_{Bx}\sin\phi_B - r_{By}\cos\phi_B}$$
$$F_{Ex} = m_{23}A_{G23x} - \cos\phi_B t_B$$
$$F_{Ey} = m_{23}A_{G23y} + W_{23} + W_L - \sin\phi_B t_B$$

For the required kinematics terms, we can again use the cycloidal function below (with its first two time derivatives) for smooth θ_2 inputs, starting and ending at rest (zero velocity and acceleration).

$$\theta_2(t) = \theta_{20} + (\theta_{2F} - \theta_{20})\left[\frac{t}{t_F} - \frac{1}{2\pi}\sin\frac{2\pi t}{t_F}\right]$$

$$\omega_2(t) = \frac{(\theta_{2F} - \theta_{20})}{t_F}\left[1 - \cos\frac{2\pi t}{t_F}\right]$$

$$\alpha_2(t) = \frac{2\pi(\theta_{2F} - \theta_{20})}{t_F^2}\left[\sin\frac{2\pi t}{t_F}\right]$$

Most of the required terms for this dynamics solution are identical to those from the pseudostatics solution, derived and presented previously. The inertial terms in the dynamics equations, not needed in pseudostatics, require the additional terms $A_{G23x}(t)$, $A_{G23y}(t)$, $\omega_2(t)$, and $\alpha_2(t)$.

$$\underline{A}_{23CG} = \begin{Bmatrix} A_{G23x}(t) \\ A_{G23y}(t) \end{Bmatrix}$$

$$= \frac{d^2}{dt^2}\{\underline{r}_{23CG}(t)\}$$

$$= \begin{Bmatrix} \dfrac{d^2}{dt^2}(r_{23CG}\cos\theta_2(t)) \\ \dfrac{d^2}{dt^2}(r_{23CG}\sin\theta_2(t)) \end{Bmatrix}$$

$$\begin{Bmatrix} A_{G23x}(t) \\ A_{G23y}(t) \end{Bmatrix} = \begin{Bmatrix} -r_{23CG}\alpha_2(t)\sin\theta_2(t) - r_{23CG}\omega_2^2(t)\cos\theta_2(t) \\ r_{23CG}\alpha_2(t)\cos\theta_2(t) - r_{23CG}\omega_2^2(t)\sin\theta_2(t) \end{Bmatrix}$$

Human Elbow Inverse Dynamics Examples

We use the same segment lengths, biceps and triceps insertion lengths, segment weights, and combined CG locations from the pseudostatics simulation, so this data is not repeated here. From Appendix D, we now need the following segment masses (kg) and mass moments of inertia (kg-m^2).

Subject	m_2 **(forearm)**	m_3 **(hand)**
adult male	1.696	0.589
adult female	0.712	0.227

Subject	I_{G2} **(forearm)**	I_{G3} **(hand)**
adult male	0.0128	0.0005
adult female	0.0042	0.0002

From this information we calculate the following lumped I_{EZ23} mass moments of inertia (kg-m^2, about the elbow joint Z axis, since that is the axis we summed moments about). This information was not required in the pseudostatics solution.

male

$$I_{EZ23} = I_{GZ2} + m_2 r_{CG2}^2 + I_{GZ3} + m_3 \left(L_2 + r_{CG3}\right)^2$$
$$= 0.0128 + 1.696(0.123)^2 + 0.0005 + 0.589(0.287 + 0.049)^2 = 0.1055$$

female

$$I_{EZ23} = I_{GZ2} + m_2 r_{CG2}^2 + I_{GZ3} + m_3 \left(L_2 + r_{CG3}\right)^2$$
$$= 0.0042 + 0.712(0.109)^2 + 0.0002 + 0.227(0.252 + 0.043)^2 = 0.0326$$

Subject	I_{EZ23}
adult male	0.1055
adult female	0.0326

We simulate the dynamics solution from the minimum to maximum elbow joint angle, with the shoulder and wrist joints locked to zero. We again use the elbow angle limits from statics, from 0 to 145°, the same for both male and female. Again, a constant end load (vertical, down) of W_L = 22.24 N (5 lb) is given at the fingertips so that $r_L = L_{23} = L_2 + L_3$.

Dynamics is also affected by the kinematic singularity at $\theta_2 = 0$, where infinite muscle tension is required to initiate motion. Therefore, let us start the dynamics simulation at 10° instead of 0 to avoid this singularity. In order to have a truly dynamic motion, let us simulate from 0 – 0.5 sec. Note the pseudostatics simulation occurred for the same motion range, but over a much longer time span, 10 sec. The results of this human arms dynamics motion simulation are given below.

Human Elbow Inverse Dynamics Examples, Biceps Tension Only **Snapshot solution**

We now give a snapshot result for the biceps-only dynamics solution, at $\theta_2 = 31^\circ$, ω_2 = 6.31 rad/s, and α_2 = 55.71 rad/s^2 (this is a snapshot for the range of motion simulation that follows).

male

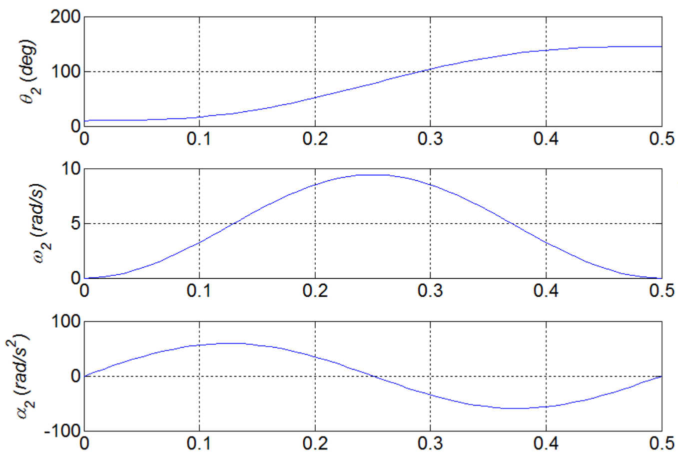
t_B = 658.5 N $\quad \phi_B = -175.4^\circ \quad F_E = \begin{Bmatrix} 630.8 \\ 109.0 \end{Bmatrix}$ N

female

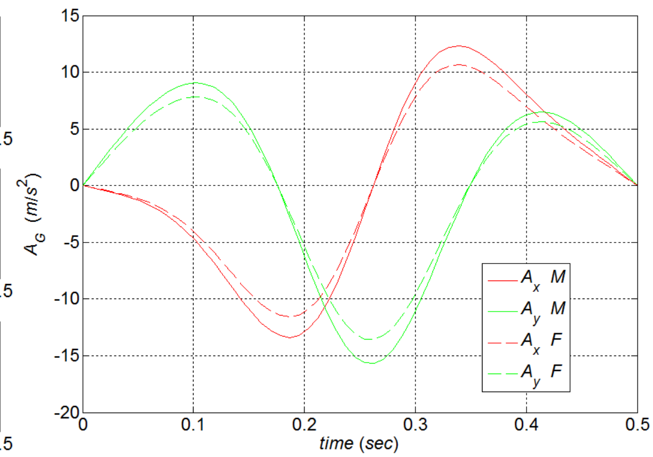
t_B = 429.1 N $\quad \phi_B = -175.3^\circ \quad F_E = \begin{Bmatrix} 418.5 \\ 70.6 \end{Bmatrix}$ N

Range of motion simulation

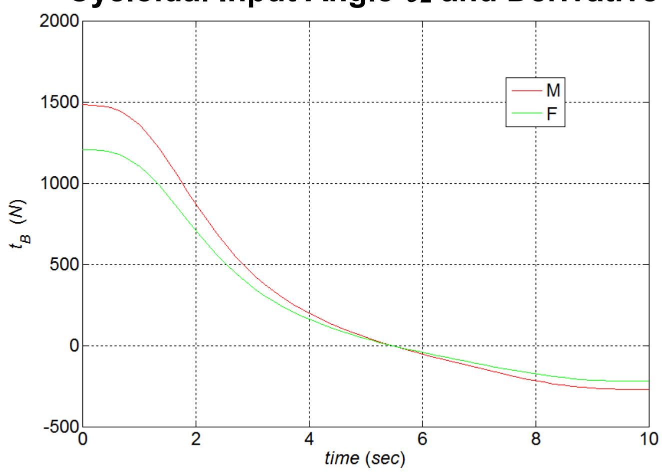
We simulate the dynamics solution from the minimum to maximum elbow joint angle, from 10 to 145° like in the pseudostatics solution, to avoid the singularity at $\theta_2 = 0$; this is also a dynamics singularity, where infinite muscle tension is required to initiate motion, since the biceps muscle is collinear with the radius bone. We again use the cycloidal function, plotted below along with its first two time derivatives, for smooth θ_2 inputs, starting and ending at rest, from 0 – 0.5 sec. Again, the shoulder and wrist joints are locked to zero.



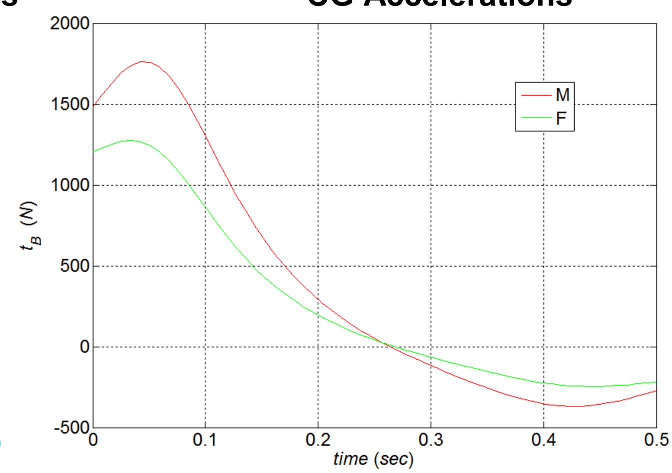
Cycloidal Input Angle θ_2 and Derivatives



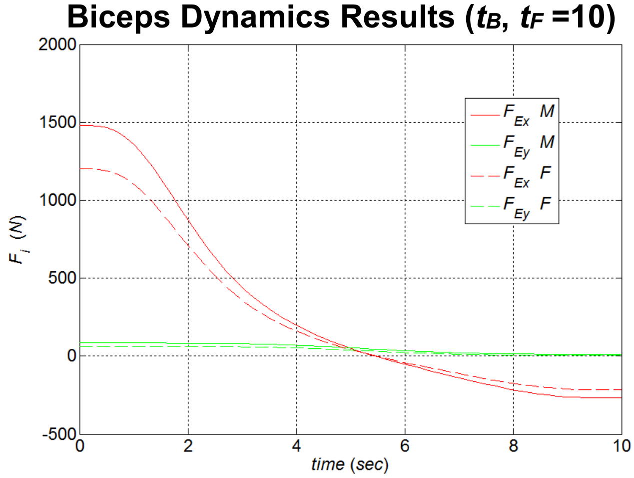
CG Accelerations



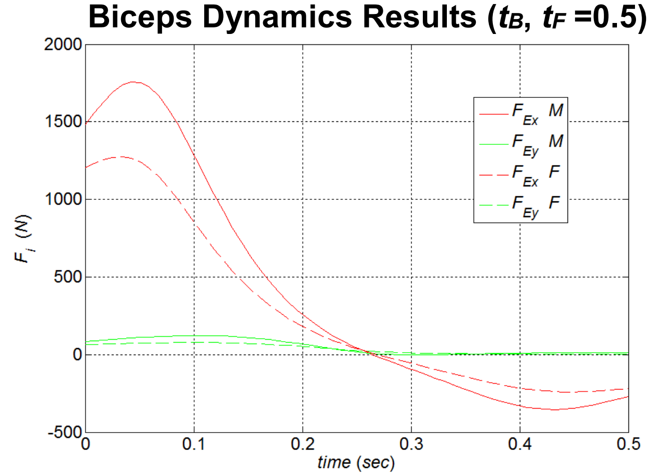
Biceps Dynamics Results (t_B, t_F =10)



Biceps Dynamics Results (t_B, t_F =0.5)



Biceps Dynamics Results (F_E, t_F =10)



Biceps Dynamics Results (F_E, t_F =0.5)

The previous plots compared the dynamics solution (t_F = 0.5 sec) to the dynamics solution (t_F = 10 sec), for the biceps tension and the elbow reaction force. When the dynamics solution is run for a final time of 10 sec, the results are indistinguishable from the pseudostatics solution presented earlier. Indeed, when the final time for the dynamics solution is 1 sec, the results are very similar to the pseudostatics solution. This leads us to conclude that this problem is dominated by statics, except at extremely high velocities and accelerations.

The kinematic results are identical to those for the pseudostatic solution shown earlier in that section, for muscle lengths and angles, plus initial and final arm poses (not repeated here).

Now, what is wrong with this solution with only biceps tension, ignoring the triceps tension?

So let's develop an alternate dynamics solution, including the triceps tension, **this time ignoring the biceps tension**.

Here are the dynamics equations ignoring biceps tension.

$$\begin{bmatrix} \cos\phi_T & 1 & 0 \\ \sin\phi_T & 0 & 1 \\ r_{Tx}\sin\phi_T - r_{Ty}\cos\phi_T & 0 & 0 \end{bmatrix} \begin{Bmatrix} t_T \\ F_{Ex} \\ F_{Ey} \end{Bmatrix} = \begin{Bmatrix} m_{23}A_{G23x} \\ m_{23}A_{G23y} + W_{23} + W_L \\ I_{EZ23}\alpha_2 + r_{23x}W_{23} + r_{Lx}W_L \end{Bmatrix}$$

The solution is

$$t_T = \frac{I_{EZ23}\alpha_2 + r_{23x}W_{23} + r_{Lx}W_L}{r_{Tx}\sin\phi_T - r_{Ty}\cos\phi_T}$$

$$F_{Ex} = m_{23}A_{G23x} - \cos\phi_T t_T$$

$$F_{Ey} = m_{23}A_{G23y} + W_{23} + W_L - \sin\phi_T t_T$$

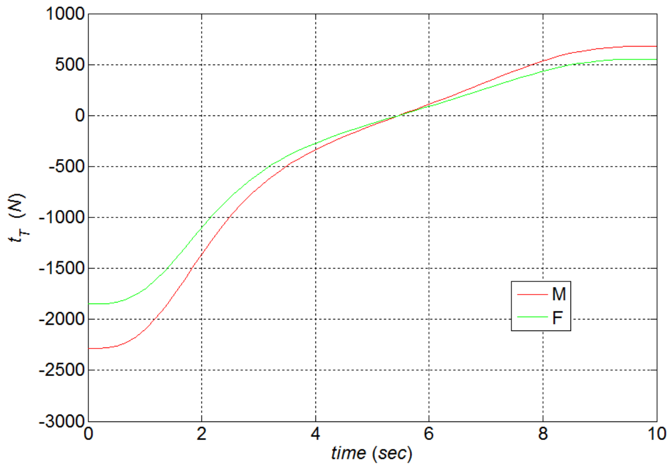
<u>Human Elbow Inverse Dynamics Examples, Triceps Tension Only</u>

We now repeat the biceps-tension only dynamics examples for the same male and female human arm models, this time ignoring the biceps and including only the triceps tension. All other inputs and parameters are identical to the previous examples.

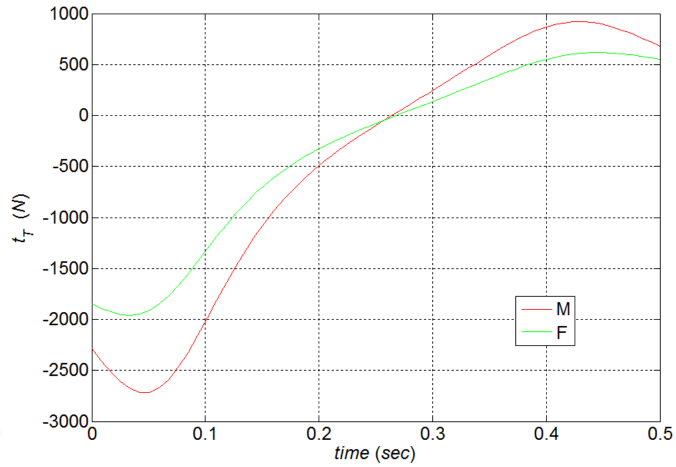
The muscle vector and length calculations and the arm animations are done in the exact same manner since these are kinematics terms.

The results of this human arms dynamics motion simulation are given on the next page. The input angle, muscle lengths, and initial and final arm poses plots are not repeated since they are kinematics results identical to the previous case, and identical to that for the pseudostatics solutions, where these plots were shown.

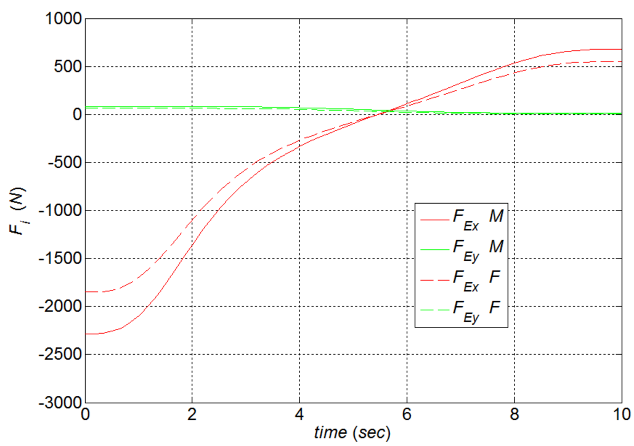
Once again we compare the dynamics solution (t_F = 0.5 sec) to the dynamics solution (t_F = 10 sec), for the triceps tension and the elbow reaction force. Once again the dynamics results for t_F = 10 sec are indistinguishable from the triceps-only pseudostatics solution presented earlier.



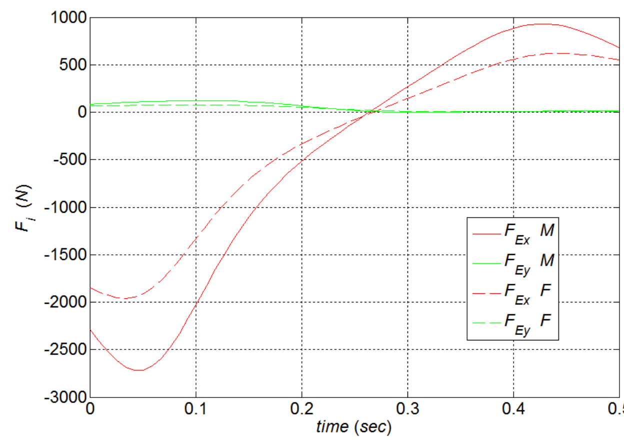
Triceps Dynamics Results (t_T, t_F =10)



Triceps Dynamics Results (t_T, t_F =0.5)



Triceps Dynamics Results (F_E, t_F =10)



Triceps Dynamics Results (F_E, t_F =0.5)

Now, what is wrong with this solution with only triceps tension, ignoring the biceps tension?

`pinv` Method

The biceps-only and triceps-only dynamics solution methods failed due to the requirement of negative muscle tensions in some simulation portions, which is ***impossible***. So let's try to develop an alternate dynamics solution, including both the biceps and triceps tensions, using a matrix approach for this underconstrained system of equations. We can use the pseudoinverse to solve the equations, which chooses the smallest (magnitudes) for the four unknowns out of the infinite solutions. Here is the underconstrained 3 x 4 set of dynamics equations:

$$\begin{bmatrix} \cos\phi_B & \cos\phi_T & 1 & 0 \\ \sin\phi_B & \sin\phi_T & 0 & 1 \\ r_{Bx}\sin\phi_B - r_{By}\cos\phi_B & r_{Tx}\sin\phi_T - r_{Ty}\cos\phi_T & 0 & 0 \end{bmatrix} \begin{Bmatrix} t_B \\ t_T \\ F_{Ex} \\ F_{Ey} \end{Bmatrix} = \begin{Bmatrix} m_{23}A_{G23x} \\ m_{23}A_{G23y} + W_{23} + W_L \\ I_{EZ23}\alpha_2 + r_{23x}W_{23} + r_{Lx}W_L \end{Bmatrix}$$

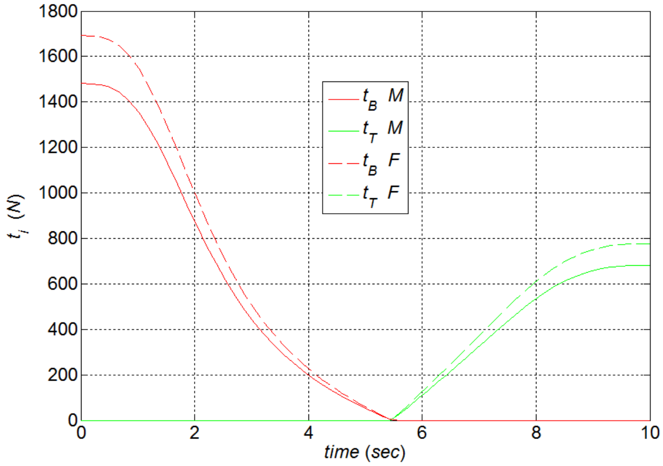
To solve this underconstrained set of equations, use MATLAB function **`pinv`** (pseudoinverse). From the infinite solutions, this chooses the solution with the smallest vector norm – a decent choice for human muscle applications.

This approach is also called the **least-squares solution** since $\sqrt{t_B^2 + t_T^2 + F_{Ex}^2 + F_{Ey}^2}$ is minimized out of the infinite possible solutions. This makes good sense as the human body tends to minimize muscle forces with the muscle actuation redundancy. The elbow reaction force is also minimized, which may have a biological basis.

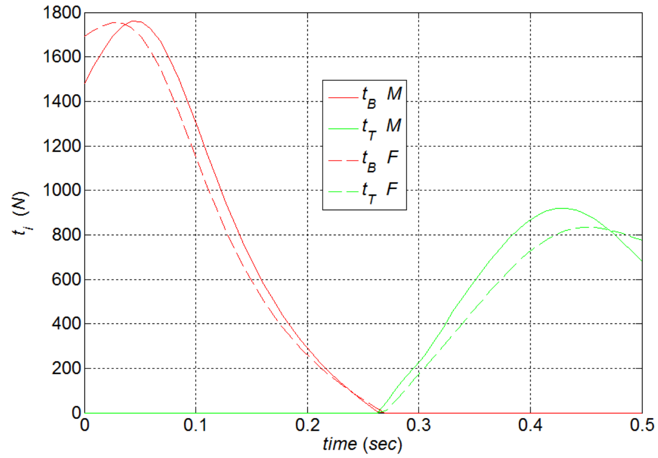
This method is no good since, though the muscle tensions are minimized, the method knows nothing about the need to keep muscle tensions all positive for all motion. Therefore, we will not even simulate the results for all motion.

`lsqnonneg` Method

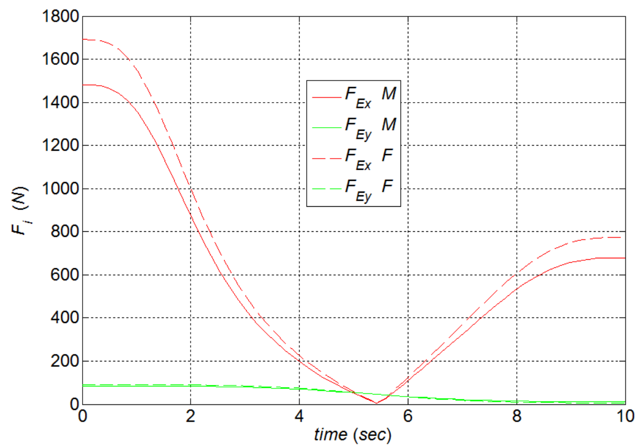
Instead, we will try MATLAB function **`lsqnonneg`**, which also solves the underconstrained least-squares problem, but enforces all solution components to be greater than or equal to 0. The results of the human arms pseudostatics motion simulation with simultaneous biceps and triceps tensions from **`lsqnonneg`** are given next.



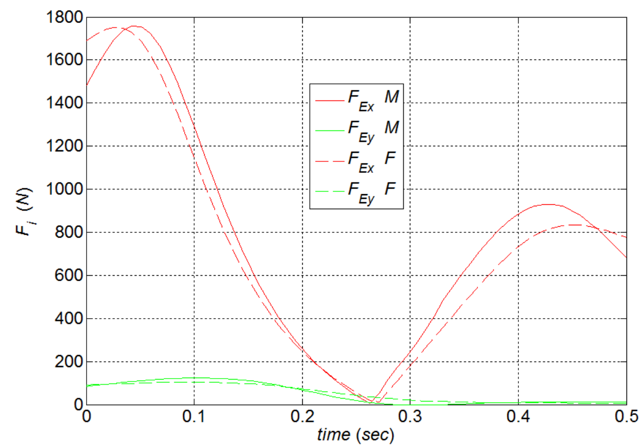
t_{BT} vs. t, t_F =10



t_{BT} vs. t, t_F =0.5



F_E vs. t, t_F =10



F_E vs. t, t_F =0.5

`lsqnonneg` Dynamics Results

Did this solution succeed? Not really:

1. The muscle forces are allowed to go to zero – in reality there will always be some small positive antagonist tensions.

2. The elbow reaction forces can be +/–; there is no need to limit those to +, but **`lsqnonneg`** does.

Acceptable Inverse Dynamics Simulation

All four dynamics modeling methods so far have not succeeded completely (assuming $t_B = 0$, assuming $t_T = 0$, the `pinv` method, and the `lsqnonneg` method. We cannot use a method in which part of the time either the triceps or the biceps must ***push*** to accomplish the dynamic motion, which is ***impossible***.

For a more advanced modeling approach, let us put a pre-tension on the triceps muscle (antagonist) as the biceps muscle (agonist) is working. Before the biceps loses tension, this pre-tensioning will have to be switched to the biceps (as the biceps and triceps reverse their agonist/antagonist roles). This will yield all positive muscle tensions plus realistic elbow reaction forces and would be a better model for real-world human biomechanics.

Here are the dynamics equations for specifying a pre-tension on t_T and solving for t_B:

$$\begin{bmatrix} \cos\phi_B & 1 & 0 \\ \sin\phi_B & 0 & 1 \\ r_{Bx}\sin\phi_B - r_{By}\cos\phi_B & 0 & 0 \end{bmatrix} \begin{Bmatrix} t_B \\ F_{Ex} \\ F_{Ey} \end{Bmatrix} = \begin{Bmatrix} m_{23}A_{G23x} \\ m_{23}A_{G23y} + W_{23} + W_L \\ I_{EZ23}\alpha_2 + r_{23x}W_{23} + r_{Lx}W_L \end{Bmatrix} - \begin{Bmatrix} \cos\phi_T \\ \sin\phi_T \\ r_{Tx}\sin\phi_T - r_{Ty}\cos\phi_T \end{Bmatrix} t_T$$

Whose solution is:

$$t_B = \frac{I_{EZ23}\alpha_2 + r_{23x}W_{23} + r_{Lx}W_L - (r_{Tx}\sin\phi_T - r_{Ty}\cos\phi_T)t_T}{r_{Bx}\sin\phi_B - r_{By}\cos\phi_B}$$

$$F_{Ex} = m_{23}A_{G23x} - t_B\cos\phi_B - t_T\cos\phi_T$$

$$F_{Ey} = m_{23}A_{G23y} - t_B\sin\phi_B - t_T\sin\phi_T + W_{23} + W_L$$

Here are the statics equations for specifying a pre-tension on t_B and solving for t_T:

$$\begin{bmatrix} \cos\phi_T & 1 & 0 \\ \sin\phi_T & 0 & 1 \\ r_{Tx}\sin\phi_T - r_{Ty}\cos\phi_T & 0 & 0 \end{bmatrix} \begin{Bmatrix} t_T \\ F_{Ex} \\ F_{Ey} \end{Bmatrix} = \begin{Bmatrix} m_{23}A_{G23x} \\ m_{23}A_{G23y} + W_{23} + W_L \\ I_{EZ23}\alpha_2 + r_{23x}W_{23} + r_{Lx}W_L \end{Bmatrix} - \begin{Bmatrix} \cos\phi_B \\ \sin\phi_B \\ r_{Bx}\sin\phi_B - r_{By}\cos\phi_B \end{Bmatrix} t_B$$

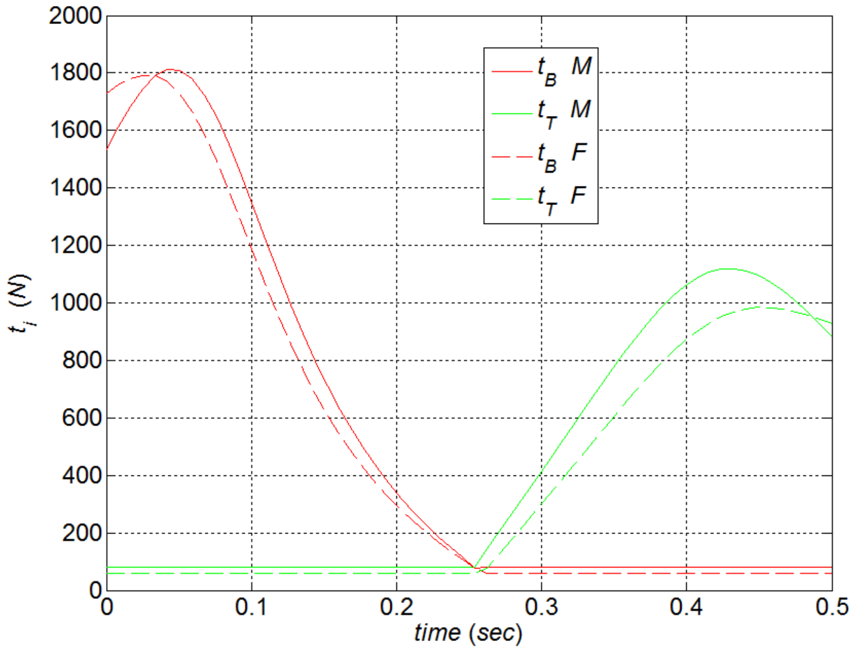
Whose solution is:

$$t_T = \frac{I_{EZ23}\alpha_2 + r_{23x}W_{23} + r_{Lx}W_L - (r_{Bx}\sin\phi_B - r_{By}\cos\phi_B)t_B}{r_{Tx}\sin\phi_T - r_{Ty}\cos\phi_T}$$

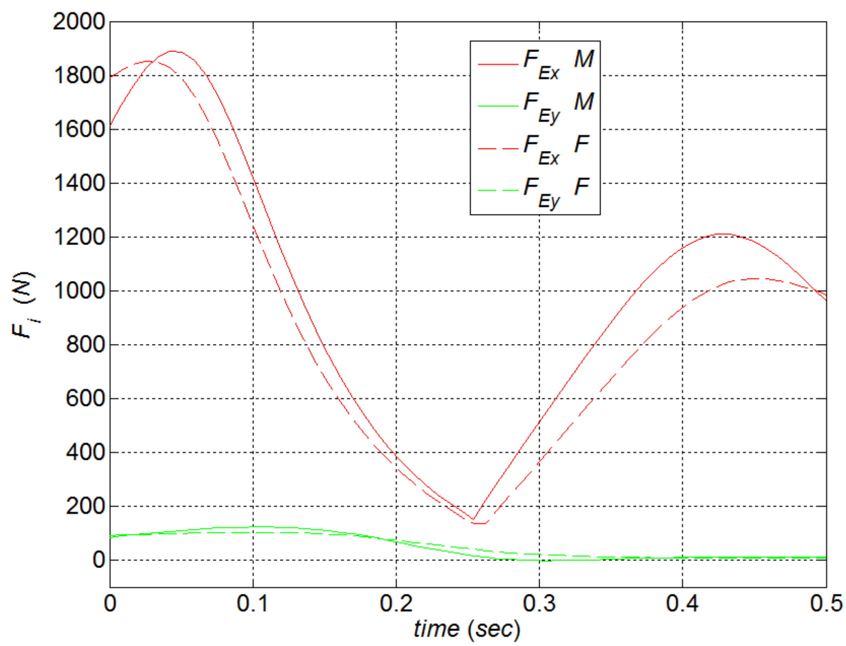
$$F_{Ex} = m_{23}A_{G23x} - t_T\cos\phi_T - t_B\cos\phi_B$$

$$F_{Ey} = m_{23}A_{G23y} - t_T\sin\phi_T - t_B\sin\phi_B + W_{23} + W_L$$

Using the same conditions for the adult male and female arm models as presented in the two previous examples, the dynamics results for this new agonist/antagonist method are shown below.



Dynamics Results (t_B and t_T vs. t)



Dynamics Results (F_E vs. t)

In this simulation the chosen antagonist pre-tensions are 80 and 60 N, for the adult male and female, respectively. We see that the biceps and triceps muscles reverse their agonist/antagonist roles after θ_2 crosses through 84.3°, at about $t = 0.26$ sec. In this case the elbow reaction forces approach zero but do not become negative since they mirror the muscle tensions closely and the weights are always negative. The pseudostatics results presented earlier were all at or below 1600 N.

References

American Medical Association (AMA), 1990, Guide to the Evaluation of Permanent Impairment, Third edition, Chicago, IL.

D.L. Bartel, D.T. Davy, and T.M. Keaveny, 2006, Orthopaedic Biomechanics: Mechanics and Design in Musculoskeletal Systems, Pearson-Prentice Hall, Upper Saddle River, NJ.

S.A. Berger, W. Goldsmith, and E.R. Lewis, editors, 2004, Introduction to Bioengineering, Oxford University Press, Oxford, UK.

R.M. Berne, M.N. Levy, B.M. Koeppen, and B.A. Stanton, editors, 2004, Physiology, 5th edition, C.V. Mosby, St. Louis, MO.

Bodies: The Exhibition, Easton Market, Columbus, OH, December 3, 2007, bodiestheexhibition.com/intro.html.

J.J. Craig, 2005, Introduction to Robotics: Mechanics and Control, Third edition, Pearson Prentice Hall, Upper Saddle River, NJ.

S.L. Delp, F.C. Anderson, A.S. Arnold, P. Loan, A. Habib, C.T. John, E. Guendelman, D.G. Thelen, 2007, “OpenSim: Open-source Software to Create and Analyze Dynamic Simulations of Movement”, IEEE Transactions on Biomedical Engineering, in press.

J. Denavit and R.S. Hartenberg, 1955, "A Kinematic Notation for Lower-Pair Mechanisms Based on Matrices", Journal of Applied Mechanics: 215 - 221.

Y.C. Fung, 1993, Biomechanics: Mechanical Properties of Living Tissue, Second edition, Springer, New York.

J.J. Gerhardt and O.A. Russe, 1975, International SFTR Method of Measuring and Recording Joint Motion, Hans Huber Publishers, Bern, Switzerland.

S.J. Hall, 2007, Basic Biomechanics, 5th edition, McGraw-Hill, Boston, MA.

J. Hamill and K.M. Knutzen, 1995, Biomechanical Basics of Human Movement, Second edition, Lippincott, Williams, & Wilkins, Baltimore, MD.

N. Hamilton, W. Weimar, and K. Luttgens, 2008, Kinesiology: Scientific Basis of Human Motion, 11th edition, McGraw-Hill, Boston, MA.

N. Jones, N. McCartney, and A. McComas, 1986, Human Muscle Power, Human Kinetics Publishers, Champaign, IL.

I.A. Kapandji, 1974a, The Physiology of The Joints: Volume 1, The Upper Limb, Churchill Livingstone.

I.A. Kapandji, 1974b, The Physiology of The Joints: Volume 3, The Trunk and the Vertebral Column, Churchill Livingstone.

H.O. Kendall, 1971, Muscles: Testing and Function, Second edition, Williams, & Wilkins, Baltimore, MD.

M. Kutz, editor, 2003, Standard Handbook of Biomedical Engineering & Design, McGraw-Hill, New York, NY.

B.F. LeVeau, 1992, Williams & Lissner's Biomechanics of Human Motion, Third edition, W.B. Saunders, Philadelphia, PA.

H.K. Lum, M. Zribi, and Y.C. Soh, 1999, "Planning and Control of a Biped Robot", International Journal of Engineering Science, 37: 1319-1349.

F.H. Martini, M.J. Timmons, and R.B. Tallitsch, 2003, Human Anatomy, Fourth edition, Pearson-Prentice Hall, Upper Saddle River, NJ.

Medical University of Ohio, 2006, Anatomy & Physiology Revealed CD, McGraw-Hill, New York, NY.

M. Nordin and V.H. Frankel, 2001, Basic Biomechanics of the Musculoskeletal System, Third edition, Lippincott, Williams, & Wilkins, Baltimore, MD.

R. Norton, 2008, Design of Machinery: An Introduction to the Synthesis and Analysis of Mechanisms and Machines, McGraw-Hill, Fourth edition.

S. Plagenhoef, F.G. Evans, and T. Abdelnour, 1983, "Anatomical Data for Analyzing Human Motion", Research Quarterly for Exercise and Sport, 54: 169-178.

L.K. Smith and L.D. Lehmkuhl, 1983, Brunnstom's Clinical Kinesiology, Fourth edition, F.A. Davis, Philadelphia, PA.

A.P. Spence, 1982, Basic Human Anatomy, The Benjamin Cummings Publishing Company, Inc., Menlo Park, CA.

T.K. Uchida and S.L. Delp, 2021, Biomechanics of Movement: The Science of Sports, Robotics, and Rehabilitation, The MIT Press, Cambridge MA.

USAF, dtic.mil/dticasd/ddsm/srch/ddsm0097.html for GEBOD and ATB.

C. Venkatayogi, 2007, "Simulation of a Humanoid Robot", MS Thesis, Mechanical Engineering, Ohio University, Robert L. Williams II, advisor.

WCB, 1998, The Dynamic Human CD, McGraw-Hill, New York, NY.

A.A. White and M.M. Panjabi, 1990, Clinical Biomechanics of the Spine, Lippincott Williams & Wilkins.

Wikipedia, en.wikipedia.org/wiki/ for many of the definitions and figures.

R.L. Williams II, B.V. Mehta, and L.P. Bindeman, 2001, "Economical Occupant/Seat Restraint Model", Final Project Technical and Business Reports, submitted to Dr. Joseph Pellettiere, AFRL/HEPA, WPAFB, December 31.

D.A. Winter, 1990, Biomechanics and Motor Control of Human Movement, Second edition, John Wiley & Sons, New York, NY.

iadeaf.k12.ia.us/The%20Human%20Bones%20&%20Muscles.htm/ for muscle anatomy.

answers.com/topic/skeletal-muscle?cat=health for muscle physiology.

Appendices

These four appendices are given in the on-line ME 4670 / 5670 NotesBook Supplement, to augment the material in this NotesBook:

people.ohio.edu/williams/html/PDF/Supplement4670.pdf

Appendix A. Main Joints of the Human Body

Appendix B. Human Synovial Joint Motion Ranges

Appendix C. Skeletal Muscles for the Major Joints

Appendix D. Human Body Anthropomorphic Parameters